Multi-User Multi-Cell MIMO towards 5G

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Huawei Workshop :
Mathematical Algorithms for Future Networks,
Paris, Jan. 22, 2015

Some Lessons Learned in Wireless Com

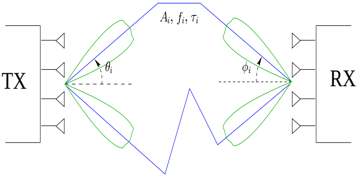
- SDMA (Spatial Division Multiple Access) why did it not take off in the early 90's?
  - No cross-layer design (proper scheduling) at that time.
  - Only feedback was for Power Control.
- At the start of 3G+ activities: it was said that no new PHY development was required, only integration of existing systems. What happened? A lot of PHY work! Dimensions of multi-antenna, multi-user and increasing bandwidth (equalization) were underestimated.
- Wireless standardization starts with the PHY layer. Should become more crosslayer though.
- User Selection: $\Rightarrow$ diversity, simplified transceiver designs.
- Channel Feedback: the return of analog transmission?
- Smart phones: location information everywhere.

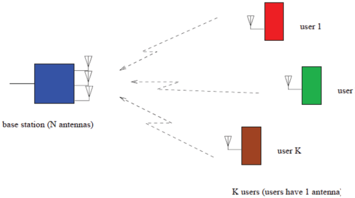
Outline

- interference single cell: Broadcast Channel (BC)
  - utility functions: SINR balancing, (weighted) sum rate (WSR)
  - BC with user selection: DPC vs BF, MIMO: role of Rx antennas
- interference multi-cell/HetNets: Interference Channel (IC)
  - Degrees of Freedom (DoF) and Interference Alignment (IA)
  - multi-cell multi-user: Interfering Broadcast Channel (IBC)
  - Weighted Sum Rate (WSR) maximization and UL/DL duality
  - Deterministic Annealing to find global max WSR
- Max WSR with Partial CSIT
  - CSIT: perfect, partial, LoS
  - EWSMSE, Massive MIMO limit, large MIMO asymptotics
- CSIT acquisition and distributed designs
  - distributed CSIT acquisition, netDoF
  - topology, rank reduced, decoupled Tx/Rx design, local CSIT
  - Massive MIMO and mmWave aspects, covariance CSIT
  - distributed designs

SDMA considerations

- Whereas single user (SU) MIMO communications represented a big breakthrough and are now integrated in a number of wireless communication standards, the next improvement is indeed multi-user MIMO (MU MIMO).
- This topic is nontrivial as e.g. illustrated by the fact that standardization bodies were not able to get an agreement on the topic until recently to get it included in the LTE-A standard.
- MU MIMO is a further evolution of SDMA, which was THE hot wireless topic throughout the nineties.





MU MIMO key elements

- SDMA is a suboptimal approach to MU MIMO, with transmitter precoding limited to linear beamforming, whereas optimal MU MIMO requires Dirty Paper Coding (DPC).
- Channel feedback has gained much more acceptance, leading to good Channel State Information at the Transmitter (CSIT), a crucial enabler for MU MIMO, whereas SDMA was either limited to TDD systems (channel CSIT through reciprocity) or Covariance CSIT. In the early nineties, the only feedback that existed was for slow power control.
- Since SDMA, the concepts of multiuser diversity and user selection have emerged and their impact on the MU MIMO sum rate is now well understood. Furthermore, it is now known that user scheduling allows much simpler precoding schemes to be close to optimal.

MU MIMO key elements (2)

- Whereas SU MIMO allows to multiply transmission rate by the spatial multiplexing factor, when mobile terminals have multiple antennas, MU MIMO allows to reach this same gain with single antenna terminals.
- Whereas in SU MIMO, various degrees of CSIT only lead to a variation in coding gain (the constant term in the sum rate), in MU MIMO however CSIT affects the spatial multiplexing factor (= Degrees of Freedom (DoF)) (multiplying the log(SNR) term in the sum rate).

CSIT considerations

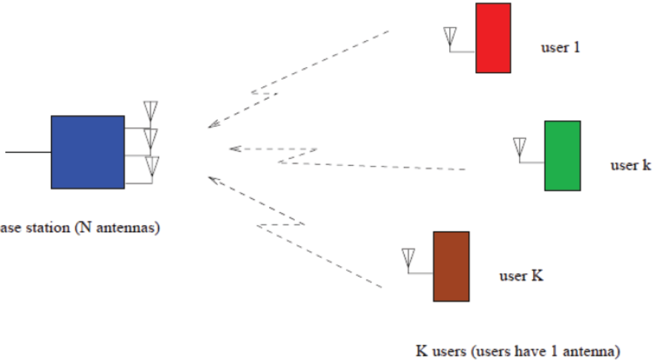
In the process attempting to integrate MU-MIMO into the LTE-A standard, a number of LTE-A contributors had recently become extremely sceptical about the usefulness of the available MU-MIMO proposals. The isue is that they currently do MU-MIMO in the same spirit as SU-MIMO, i.e. with feedback of CSI limited to just a few bits! However, MU-MIMO requires very good CSIT! Some possible solutions:

- Increase CSI feedback enormously (possibly using analog transmission).
- Exploit channel reciprocity in TDD (electronics calibration issue though).
- Limit MU-MIMO to LOS users and extract essential CSIT from DoA or location information.

SDMA system model

- Rx signal at user k:

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \qquad k = 1, 2, \ldots, K$$



Utility Functions

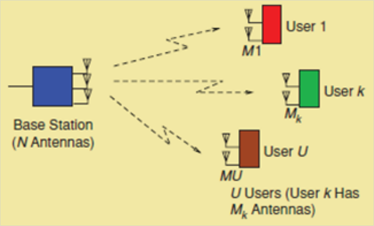
- single user (MIMO) in Gaussian noise: Gaussian signaling optimal (avg. power constr.)
- rate stream k : $R_k = \ln(1 + \text{SINR}_k)$
- SINR balancing: $\max_{BF} \min_k \text{SINR}_k / \gamma_k$ under Tx power P, fairness
- related: min Tx power under $\text{SINR}_k \geq \gamma_k$ GREEN
- max Weighted Sum Rate (WSR): $\max_{BF} \sum_k u_k R_k$, given P

  weights u_k may reflect state of queues (to minimize queue overflow)

  weights also allow to vary orientation of normal to Pareto boundary of rate region and hence to explore whole Pareto boundary if rate region convex
  Pareto boundary: cannot increase an R_k without decreasing some R_i.

MIMO BroadCast

- MIMO BC = Multi-User MIMO Downlink
- N_t transmission antennas.
- K users with N_k receiving antennas.
- Assume perfect CSI
- Possibly multiple streams/user d_k.
- Power constraint P
- Noise variance $\sigma^2 = 1$.
- $\mathbf{H}_k$ the MIMO channel for user k.



$$\mathbf{F}_k\mathbf{y}_k = \mathbf{F}_k\mathbf{H}_k\sum_{i=1}^{K}\mathbf{G}_i\mathbf{s}_i + \mathbf{F}_k\mathbf{z}_k$$

$$= \underbrace{\mathbf{F}_k\mathbf{H}_k\mathbf{G}_k\mathbf{s}_k}_{\text{useful signal}} + \underbrace{\sum_{i=1, i\neq k}^{K}\mathbf{F}_k\mathbf{H}_k\mathbf{G}_i\mathbf{s}_i}_{\text{inter-user interference}} + \underbrace{\mathbf{F}_k\mathbf{z}_k}_{\text{noise}}$$

System Model (2)

- Rx signal: $\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{z}_k = \mathbf{H}_k \sum_{i=1}^{K} \mathbf{G}_i \mathbf{s}_i + \mathbf{z}_k$
- $\underbrace{\mathbf{F}_k}_{d_k \times N_k} \underbrace{\mathbf{y}_k}_{N_k \times 1} = \underbrace{\mathbf{F}_k}_{d_k \times N_k} \underbrace{\mathbf{H}_k}_{N_k \times N_t} \sum_{i=1}^{K} \underbrace{\mathbf{G}_i}_{N_t \times d_i} \underbrace{\mathbf{s}_i}_{d_i \times 1} + \underbrace{\mathbf{F}_k}_{d_k \times N_k} \underbrace{\mathbf{z}_k}_{N_k \times 1}$
- [Christensen etal:T-WC08]: use of linear receivers in MIMO BC is not suboptimal (full CSIT, // SU MIMO): can prefilter $\mathbf{G}_k$ with a $d_k \times d_k$ unitary matrix to make interference plus noise prewhitened channel matrix - precoder cascade of user k orthogonal (columns)

User Selection Motivation

- Optimal MIMO BC design requires DPC, which is significantly more complicated than BF.
- User selection allows to
  - improve the rates of DPC
  - bring the rate of BF close to those of DPC
- Optimal user/stream selection requires selection of optimal combination of N_t streams: too complex. Greedy user/stream selection (GUS): select one stream at a time $\Rightarrow$ complexity $\approx$ N_t times the complexity of selecting one stream ($K \gg N_t$).
- Multiple receive antennas cannot improve the sum rate prelog.
  So what benefit can they bring?
  Of course: cancellation of interference from other transmitters (spatially colored noise): not considered here.

Zero-Forcing (ZF)

- ZF-BF $\mathbf{F}_{1:i}\mathbf{H}_{1:i}\mathbf{G}_{1:i} =$

$$\begin{bmatrix} \mathbf{F}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{F}_2 & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{F}_i \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_i \end{bmatrix} [\mathbf{G}_1 \ \mathbf{G}_2 \cdots \mathbf{G}_i] = \begin{bmatrix} \mathbf{F}_1\mathbf{H}_1\mathbf{G}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{F}_2\mathbf{H}_2\mathbf{G}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{F}_i\mathbf{H}_i\mathbf{G}_i \end{bmatrix}$$

- ZF-DPC (modulo reordering issues) $\mathbf{F}_{1:i}\mathbf{H}_{1:i}\mathbf{G}_{1:i} =$

$$\begin{bmatrix} \mathbf{F}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{F}_2 & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{F}_i \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_i \end{bmatrix} [\mathbf{G}_1 \ \mathbf{G}_2 \cdots \mathbf{G}_i] = \begin{bmatrix} \mathbf{F}_1\mathbf{H}_1\mathbf{G}_1 & 0 & \cdots & 0 \\ * & \mathbf{F}_2\mathbf{H}_2\mathbf{G}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ * & \cdots & * & \mathbf{F}_i\mathbf{H}_i\mathbf{G}_i \end{bmatrix}$$

- BF-style selection, DPC-style selection: as if it's going to be used in BF/DPC

Stream Selection Criterion from Sum Rate

- At high SNR, both
  - optimized (MMSE style) filters vs. ZF filters
  - optimized vs. uniform power allocation

  only leads to $\frac{1}{\text{SNR}}$ terms in rates.
- At high SNR, the sum rate is of the form

$$\underbrace{N_t}_{\text{DoF}} \log(\text{SNR}/N_t) + \underbrace{\sum_i \log\det(\mathbf{F}_i \mathbf{H}_i \mathbf{G}_i)}_{\text{constant}} + O(\frac{1}{\text{SNR}}) + \underbrace{O(\log\log(\text{SNR}))}_{\text{noncoherent Tx}}$$

for properly normalized ZF Rx $\mathbf{F}_i$ and ZF Tx $\mathbf{G}_i$ (BF or DPC).

Role of Rx antennas?

- Different distributions of ZF between Tx and Rx give different ZF channel gains! If Rx ZF's k streams, hence Tx only has to ZF $M-1-k$ streams! So, number of possible solutions (assuming $d_k \equiv 1$):

$$\prod_{k=1}^{M} \left(\sum_{i=0}^{N_k-1} \frac{(M-1)!}{k!(M-1-k)!} \right)$$

  for each user, Rx can ZF k between 0 and $N_k - 1$ streams, to choose among $M-1$.
  Explains non-convexity of MIMO SR at high SNR.
- ZF by Rx can alternatively be interpreted as IA by Tx (Rx adapts Rx-channel cascades to lie in reduced dimension subspace).
- SESAM (and all existing MIMO stream selection algorithms): assumes that all ZF is done by Tx only. Hence, Rx can be a MF, matched to channel-BF cascade.

Concluding Remarks MIMO BC User Selection

- Introduced new MISO BC BF-style GUS criterion/interpretation.
- Extension to MIMO BC with receiver design.
- Plenty of room for asymptotic analysis of transient regime of stream selection.
- Here, did not touch upon CSIT FB issues, user preselection schemes to reduce pool size etc.
- Joint Tx-Rx ZF (IA) provides more opportunities (but hence also larger search space and complexity).

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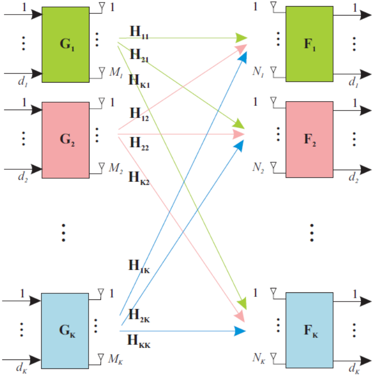
MIMO IFC Introduction

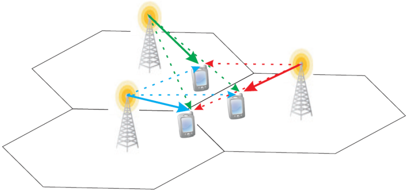
- Interference Alignment (IA) was introduced in [Cadambe,Jafar 2008]
- The objective of IA is to design the Tx beamforming matrices such that the interference at each non intended receiver lies in a common interference subspace
- If alignment is complete at the receiver simple Zero Forcing (ZF) can suppress interference and extract the desired signal
- In [SPAWC2010] we derive a set of interference alignment (IA) feasibility conditions for a K-link frequency-flat MIMO interference channel (IFC)
- $d = \sum_{k=1}^{K} d_k$



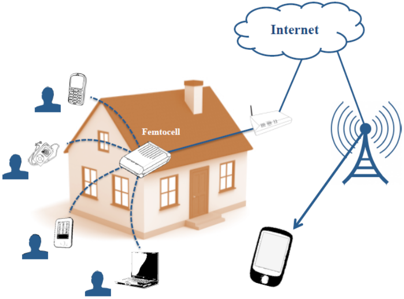
MIMO Interference Channel

Possible Application Scenarios

- Multi-cell cellular systems, modeling intercell interference.
  Difference from Network MIMO: no exchange of signals, "only" of channel impulse responses.



- HetNets: Coexistence of macrocells and small cells, especially when small cells are considered part of the cellular solution.



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Why IA?

- The number of streams (degrees of freedom (dof)) appearing in a feasible IA scenario correspond to prelogs of feasible multi-user rate tuples in the multi-user rate region.
  Max Weighted Sum Rate (WSR) becomes IA at high SNR.
- Noisy IFC: interfering signals are not decoded but treated as (Gaussian) noise.
  Apparently enough for dof.
- Lots of recent work more generally on rate prelog regions: involves time sharing, use of fractional power.

Various IA Flavors

- *linear* IA [GouJafar:IT1210], also called *signal space* IA, only uses the spatial dimensions introduced by multiple antennas.
- *asymptotic* IA [CadambeJafar:IT0808] uses symbol extension (in time and/or frequency), leading to (infinite) symbol extension involving diagonal channel matrices, requiring infinite channel diversity in those dimensions. This leads to infinite latency also. The (sum) DoF of asymptotic MIMO IA are determined by the *decomposition* bound [WangSunJafar:isit12].
- *ergodic* IA [NazerGastparJafarVishwanath:IT1012] explains the factor 2 loss in DoF of SISO IA w.r.t. an interference-free Tx scenario by transmitting the same signal twice at two paired channel uses in which all cross channel links cancel out each other: group channel realizations H_1, H_2 s.t. $\text{offdiag}(H_2) = -\text{offdiag}(H_1)$. Ergodic IA also suffers from uncontrolled latency but provides the factor 2 rate loss at any SNR. The DoF of ergodic MIMO IA are also determined by the decomposition bound [LejosneSlockYuan:icassp14].
- *real* IA [MotahariGharanMaddah-AliKhandani:arxiv09], also called it signal scale IA, exploits discrete signal constellations and is based on the Diophantine equation. Although this approach appears still quite exploratory, some related work based on lattices appears promising.

IA as a Constrained Compressed SVD

- $F_k^H: \ d_k \times N_k, \ \ H_{ki}: \ N_k \times M_i, \ \ G_i: \ M_i \times d_i \qquad F^H H G =$

$$\begin{bmatrix} F_1^H & 0 & \cdots & 0 \\ 0 & F_2^H & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & F_K^H \end{bmatrix} \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1K} \\ H_{21} & H_{22} & \cdots & H_{2K} \\ \vdots & & \ddots & \vdots \\ H_{K1} & H_{K2} & \cdots & H_{KK} \end{bmatrix} \begin{bmatrix} G_1 & 0 & \cdots & 0 \\ 0 & G_2 & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & G_K \end{bmatrix} = \begin{bmatrix} F_1^H H_{11} G_1 & 0 & \cdots & 0 \\ 0 & F_2^H H_{22} G_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & F_K^H H_{KK} G_K \end{bmatrix}$$

  F^H, G can be chosen to be unitary for IA

- per user vs per stream approaches:

  IA: can absorb the $d_k \times d_k$ $F_k^H H_{kk} G_k$ in either F_k^H (per stream LMMSE Rx) or G_k or both.

  WSR: can absorb unitary factors of SVD of $F_k^H H_{kk} G_k$ in F_k^H, G_k without loss in rate $\Rightarrow F^H H G =$ diagonal.

Interference Alignment: Feasibility Conditions (1)

- To derive the existence conditions we consider the ZF conditions

$$\underbrace{\mathbf{F}_k^H}_{d_k \times N_k} \underbrace{\mathbf{H}_{kl}}_{N_k \times M_l} \underbrace{\mathbf{G}_l}_{M_l \times d_l} = \mathbf{0} \ , \quad \forall l \neq k$$

$$\text{rank}(\mathbf{F}_k^H \mathbf{H}_{kk} \mathbf{G}_k) = d_k \ , \quad \forall k \in \{1, 2, \ldots, K\}$$

- rank requirement $\Rightarrow$ SU MIMO condition: $\quad d_k \leq \min(M_k, N_k)$
- The total number of variables in $\mathbf{G}_k$ is
  $d_k M_k - d_k^2 = d_k(M_k - d_k)$
  Only the subspace of $\mathbf{G}_k$ counts, it is determined up to a $d_k \times d_k$ mixture matrix.
- The total number of variables in $\mathbf{F}_k^H$ is
  $d_k N_k - d_k^2 = d_k(N_k - d_k)$
  Only the subspace of $\mathbf{F}_k^H$ counts, it is determined up to a $d_k \times d_k$ mixture matrix.

Interference Alignment: Feasibility Conditions (2)

- A solution for the interference alignment problem can only exist if the total number of variables is greater than or equal to the total number of constraints i.e.,

$$\sum_{k=1}^{K} d_k(M_k - d_k) + \sum_{k=1}^{K} d_k(N_k - d_k) \geq \sum_{i \neq j=1}^{K} d_i\, d_j$$

$$\Rightarrow \;\; \sum_{k=1}^{K} d_k(M_k + N_k - 2d_k) \geq (\sum_{k=1}^{K} d_k)^2 - \sum_{k=1}^{K} d_k^2$$

$$\Rightarrow \;\; \sum_{k=1}^{K} d_k(M_k + N_k) \geq (\sum_{k=1}^{K} d_k)^2 + \sum_{k=1}^{K} d_k^2$$

- In the symmetric case: $d_k = d$, $M_k = M$, $N_k = N$:
  $d \;\leq\; \frac{M+N}{K+1}$
- For the $K = 3$ user case ($M = N$): $\quad d \;=\; \frac{M}{2}$.
  With 3 parallel MIMO links, half of the (interference-free) resources are available!
  However $d \leq \frac{1}{(K+1)/2} M < \frac{1}{2} M$ for $K > 3$.

Interference Alignment: Feasibility Conditions (3)

- [BreslerTse:arxiv11]: counting equations and variables not the whole story!
- appears in very "rectangular" ($\neq$ square) MIMO systems
- example: $(M, N, d)^K = (4, 8, 3)^3$ MIMO IFC system comparing variables and ZF equations: $d = \frac{M+N}{K+1} = \frac{4+8}{3+1} = \frac{12}{4} = 3$ should be possible
- supportable interference subspace dim. $= N - d =$ 8-3 $= 5$
- however, the 2 interfering 8×4 cross channels generate 4-dimensional subspaces which in an 8-dimensional space do not intersect w.p. 1 !
- hence, the interfering 4×3 transmit filters cannot massage their 6-dimensional joint interference subspace into a 5-dimensional subspace!
- This issue is not captured by # variables vs # equations: $d = \frac{M+N}{K+1}$ only depends on $M + N$: $(5, 7, 3)^3$, $(6, 6, 3)^3$ work.

Feasibility Linear IA

- We shall focus here on linear IA, in which the spatial Tx filters align their various interference terms at a given user in a common subspace so that a Rx filter can zero force (ZF) it. Since linear IA only uses spatial filtering, it leads to low latency.
- The DoF of linear IA are upper bounded by the so-called *proper bound* [Negro:eusipco09], [Negro:spawc10], [YetisGouJafarKayran:SP10], which simply counts the number of filter variables vs. the number of ZF constraints.
- The proper bound is not always attained though because to make interference subspaces align, the channel subspaces in which they live have to sufficiently overlap to begin with, which is not always the case, as captured by the so-called *quantity bound* [Tingting:arxiv0913] and first elucidated in [BreslerCartwrightTse:allerton11], [BreslerCartwrightTse:itw11], [WangSunJafar:isit12].
- The transmitter coordination required for DL IA in a multi-cell setting corresponds to the Interfering Broadcast Channel (IBC). Depending on the number of interfering cells, the BS may run out of antennas to serve more than one user, which then leads to the Interference Channel (IC).

I and Q components: IA with Real Symbol Streams

- Using real signal constellations in place of complex constellations, transmission over a complex channel of any given dimension can be interpreted as transmission over a real channel of double the original dimensions (by treating the I and Q components as separate channels).
- This doubling of dimensions provides additional flexibility in achieving the total DoF available in the network.
- Split complex quantities in I and Q components:

$$\mathbf{H}_{ij} = \begin{bmatrix} \mathrm{Re}\{\mathbf{H}_{ij}\} & -\mathrm{Im}\{\mathbf{H}_{ij}\} \\ \mathrm{Im}\{\mathbf{H}_{ij}\} & \mathrm{Re}\{\mathbf{H}_{ij}\} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \mathrm{Re}\{\mathbf{x}\} \\ \mathrm{Im}\{\mathbf{x}\} \end{bmatrix}$$

- Example: GMSK in GSM: was considered as wasting half of the resources, but in fact unknowingly anticipated interference treatment: 3 interfering GSM links can each support one GMSK signal without interference by proper joint Tx/Rx design! (SAIC: handles 1 interferer, requires only Rx design).

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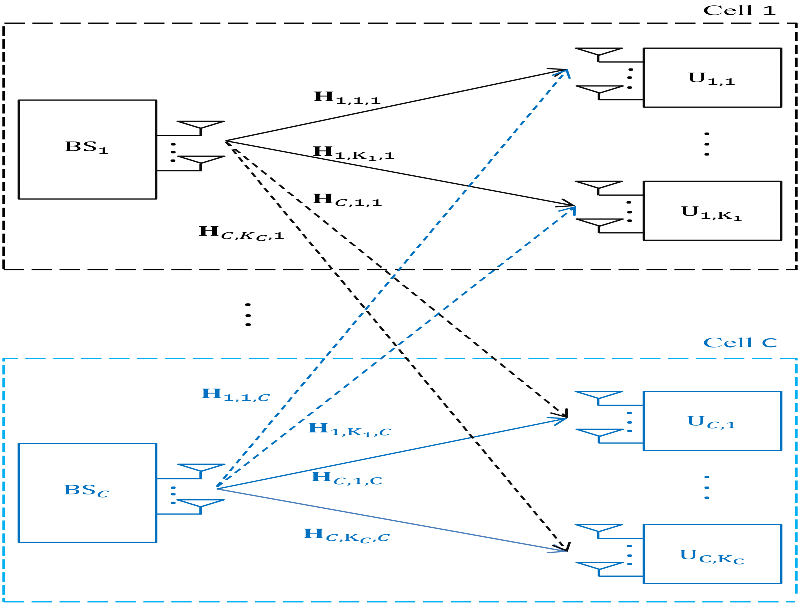
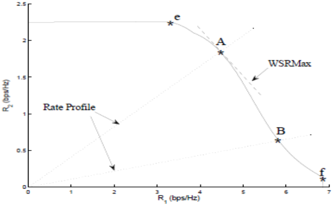
MIMO Interfering Broadcast Channel (IBC)



From IA to Optimized IFC's

- from Interference Alignment (=ZF) to max Sum Rate (SR) for the "Noisy IFC".
- to vary the point reached on the rate region boundary: SR $\rightarrow$ Weighted SR (WSR)
- problem: IFC rate region not convex $\Rightarrow$ multiple (local) optima for WSR (multiple boundary points with same tangent direction)
- solution of [CuiZhang:ita10]: WSR $\rightarrow$ max SR under rate profile constraint: $\frac{R_1}{\alpha_1} = \frac{R_2}{\alpha_2} = \cdots = \frac{R_K}{\alpha_K}$: $K-1$ constraints.
  Pro: explores systematically rate region boundary.
  Con: for a fixed rate profile,
  bad links drag down good links.
  $\Rightarrow$ stick to (W)SR
  (monitoring global opt issues).
  Note: multiple WSR solutions
  $\Leftrightarrow$ multiple IA solutions.



Max WSR Tx BF Design Outline

- max WSR Tx BF design with perfect CSIT
  - using WSR - WSMSE relation
  - from difference of concave to linearized concave
  - MIMO BC: local optima, deterministic annealing
- Gaussian partial CSIT
- max EWSR Tx design w partial CSIT
- Line of Sight (LoS) based partial CSIT
- max EWSR Tx design with LoS based CSIT

MIMO IBC with Linear Tx/Rx, single stream

- IBC with C cells with a total of K users. System-wide user numbering: the $N_k \times 1$ Rx signal at user k in cell b_k is

$$y_k = \underbrace{\mathbf{H}_{k,b_k}\,\mathbf{g}_k\,x_k}_{\text{signal}} + \underbrace{\sum_{\substack{i \neq k \\ b_i = b_k}} \mathbf{H}_{k,b_k}\,\mathbf{g}_i\,x_i}_{\text{intracell interf.}} + \underbrace{\sum_{j \neq b_k} \sum_{i: b_i = j} \mathbf{H}_{k,j}\,\mathbf{g}_i\,x_i}_{\text{intercell interf.}} + v_k$$

  where x_k = intended (white, unit variance) scalar signal stream, $\mathbf{H}_{k,b_k} = N_k \times M_{b_k}$ channel from BS b_k to user k. BS b_k serves $K_{b_k} = \sum_{i:b_i=b_k} 1$ users. Noise whitened signal representation $\Rightarrow$ $v_k \sim \mathcal{CN}(0, I_{N_k})$.
- The $M_{b_k} \times 1$ spatial Tx filter or beamformer (BF) is $\mathbf{g}_k$.
- Treating interference as noise, user k will apply a linear Rx filter $\mathbf{f}_k$ to maximize the signal power (diversity) while reducing any residual interference that would not have been (sufficiently) suppressed by the BS Tx. The Rx filter output is $\widehat{x}_k = \mathbf{f}_k^H y_k$

$$\begin{aligned}
\widehat{x}_k &= \mathbf{f}_k^H \mathbf{H}_{k,b_k}\,\mathbf{g}_k\,x_k + \sum_{i=1,\neq k}^{K} \mathbf{f}_k^H \mathbf{H}_{k,b_i}\,\mathbf{g}_i\,x_i + \mathbf{f}_k^H v_k \\
&= \mathbf{f}_k^H \mathbf{h}_{k,k}\,x_k + \sum_{i \neq k} \mathbf{f}_k^H \mathbf{h}_{k,i}\,x_i + \mathbf{f}_k^H v_k
\end{aligned}$$

  where $\mathbf{h}_{k,i} = \mathbf{H}_{k,b_i}\,\mathbf{g}_i$ is the channel-Tx cascade vector.

Max Weighted Sum Rate (WSR)

- Weighted sum rate (WSR)

$$WSR = WSR(\mathbf{g}) = \sum_{k=1}^{K} u_k \ \ln \frac{1}{e_k}$$

where $\mathbf{g} = \{\mathbf{g}_k\}$, the u_k are rate weights

- MMSEs $e_k = e_k(\mathbf{g})$

$$\frac{1}{e_k} = 1 + \mathbf{g}_k^H \mathbf{H}_k^H R_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k = (1 - \mathbf{g}_k^H \mathbf{H}_k^H R_k^{-1} \mathbf{H}_k \mathbf{g}_k)^{-1}$$
$$R_k = R_{\overline{k}} + \mathbf{H}_k \mathbf{g}_k \mathbf{g}_k^H \mathbf{H}_k^H \ , \ R_{\overline{k}} = \sum_{i \neq k} \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H + I_{N_k} \ ,$$

R_k, $R_{\overline{k}}$ = total, interference plus noise Rx cov. matrices resp.

- MMSE e_k obtained at the output $\widehat{x}_k = \mathbf{f}_k^H y_k$ of the optimal (MMSE) linear Rx

$$\mathbf{f}_k = R_k^{-1} \mathbf{H}_k \mathbf{g}_k \ .$$

From max WSR to min WSMSE

- For a general Rx filter $\mathbf{f}_k$ we have the MSE $e_k(\mathbf{f}_k, \mathbf{g})$
$= (1 - \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_k)(1 - \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{f}_k) + \sum_{i \neq k} \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H \mathbf{f}_k + ||\mathbf{f}_k||^2$
$= 1 - \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_k - \mathbf{g}_k^H \mathbf{H}_k^H \mathbf{f}_k + \sum_i \mathbf{f}_k^H \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H \mathbf{f}_k + ||\mathbf{f}_k||^2.$
- The $WSR(\mathbf{g})$ is a non-convex and complicated function of $\mathbf{g}$. Inspired by [Christensen:TW1208], we introduced [Negro:ita10],[Negro:ita11] an augmented cost function, the Weighted Sum MSE, $WSMSE(\mathbf{g}, \mathbf{f}, w)$

$$= \sum_{k=1}^{K} u_k (w_k \; e_k(\mathbf{f}_k, \mathbf{g}) - \ln w_k) + \lambda (\sum_{k=1}^{K} ||\mathbf{g}_k||^2 - P)$$

where λ = Lagrange multiplier and P = Tx power constraint.
- After optimizing over the aggregate auxiliary Rx filters $\mathbf{f}$ and weights w, we get the WSR back:

$$\min_{\mathbf{f}, w} WSMSE(\mathbf{g}, \mathbf{f}, w) = -WSR(\mathbf{g}) + \overbrace{\sum_{k=1}^{K} u_k}^{\text{constant}}$$

From max WSR to min WSMSE (2)

- Advantage augmented cost function: alternating optimization $\Rightarrow$ solving simple quadratic or convex functions

$$\min_{w_k} WSMSE \;\Rightarrow\; w_k = 1/e_k$$

$$\min_{\mathbf{f}_k} WSMSE \;\Rightarrow\; \mathbf{f}_k = (\sum_i \mathbf{H}_k \mathbf{g}_i \mathbf{g}_i^H \mathbf{H}_k^H + I_{N_k})^{-1} \mathbf{H}_k \mathbf{g}_k$$

$$\min_{\mathbf{g}_k} WSMSE \;\Rightarrow$$

$$\mathbf{g}_k = (\sum_i u_i w_i \mathbf{H}_i^H \mathbf{f}_i \mathbf{f}_i^H \mathbf{H}_i + \lambda I_M)^{-1} \mathbf{H}_k^H \mathbf{f}_k u_k w_k$$

- UL/DL duality: optimal Tx filter g_k of the form of a MMSE linear Rx for the dual UL in which λ plays the role of Rx noise variance and $u_k w_k$ plays the role of stream variance.

Optimal Lagrange Multiplier λ

- (bisection) line search on $\sum_{k=1}^{K} ||\mathbf{g}_k||^2 - P = 0$ [Luo:SP0911].
- Or updated analytically as in [Negro:ita10],[Negro:ita11] by exploiting $\sum_k \mathbf{g}_k^H \frac{\partial WSMSE}{\partial \mathbf{g}_k^*} = 0$.
- This leads to the same result as in [Hassibi:TWC0906]: λ avoided by reparameterizing the BF to satisfy the power constraint: $\mathbf{g}_k = \sqrt{\frac{P}{\sum_{i=1}^{K} ||\mathbf{g}_i^{'}||^2}}\, \mathbf{g}_k^{'}$ with $\mathbf{g}_k^{'}$ now unconstrained

$$\mathrm{SINR}_k = \frac{|\mathbf{f}_k \mathbf{H}_k \mathbf{g}_k^{'}|^2}{\sum_{i=1,\neq k}^{K} |\mathbf{f}_k \mathbf{H}_k \mathbf{g}_i^{'}|^2 + \frac{1}{P} ||\mathbf{f}_k||^2 \sum_{i=1}^{K} ||\mathbf{g}_i^{'}||^2} \quad .$$

- This leads to the same Lagrange multiplier expression obtained in [Christensen:TW1208] on the basis of a heuristic that was introduced in [Joham:isssta02] as was pointed out in [Negro:ita10].

From WSR to WSSINR

- The WSR can be rewritten as

$$WSR = WSR(\mathbf{g}) = \sum_{k=1}^{K} u_k \ \ln(1+\text{SINR}_k)$$

where $1+\text{SINR}_k = 1/e_k$ or for general $\mathbf{f}_k$:

$$\text{SINR}_k = \frac{|\mathbf{f}_k \mathbf{H}_k \mathbf{g}_k|^2}{\sum_{i=1,\neq k}^{K} |\mathbf{f}_k \mathbf{H}_k \mathbf{g}_i|^2 + ||\mathbf{f}_k||^2} \ .$$

- WSR variation

$$\partial WSR = \sum_{k=1}^{K} \frac{u_k}{1+\text{SINR}_k} \ \partial\text{SINR}_k$$

interpretation: variation of a weighted sum SINR (WSSINR)

- The BFs obtained: same as for WSR or WSMSE criteria. But this interpretation shows: WSR = optimal approach to the SLNR or SJNR heuristics. WSSINR approach = [KimGiannakis:IT0511] below.

[KimGiannakis:IT0511]: Difference of Convex Fns

- Let $Q_k = \mathbf{g}_k \mathbf{g}_k^H$ be the transmit covariance for stream k $\Rightarrow$

$$WSR = \sum_{k=1}^{K} u_k [\ln\det(R_k) - \ln\det(R_{\overline{k}})]$$

w $R_k = \mathbf{H}_k (\sum_i Q_i) \mathbf{H}_k^H + I_{N_k}$, $R_{\overline{k}} = \mathbf{H}_k (\sum_{i \neq k} Q_i) \mathbf{H}_k^H + I_{N_k}$.

- Consider the dependence of WSR on Q_k alone:

$$WSR = u_k \ln\det(R_{\overline{k}}^{-1} R_k) + WSR_{\overline{k}}, \;\; WSR_{\overline{k}} = \sum_{i=1, \neq k}^{K} u_i \ln\det(R_{\overline{i}}^{-1} R_i)$$

where $\ln\det(R_{\overline{k}}^{-1} R_k)$ is concave in Q_k and $WSR_{\overline{k}}$ is convex in Q_k. Since a linear function is simultaneously convex and concave, consider the first order Taylor series expansion in Q_k around $\widehat{Q}$ (i.e. all $\widehat{Q}_i$) with e.g. $\widehat{R}_i = R_i(\widehat{Q})$, then

$$WSR_{\overline{k}}(Q_k, \widehat{Q}) \approx WSR_{\overline{k}}(\widehat{Q}_k, \widehat{Q}) - \mathrm{tr}\{(Q_k - \widehat{Q}_k)\widehat{A}_k\}$$

$$\widehat{A}_k = - \left. \frac{\partial WSR_{\overline{k}}(Q_k, \widehat{Q})}{\partial Q_k} \right|_{\widehat{Q}_k, \widehat{Q}} = \sum_{i=1, \neq k}^{K} u_i \mathbf{H}_i^H (\widehat{R}_{\overline{i}}^{-1} - \widehat{R}_i^{-1}) \mathbf{H}_i$$

- Note that the linearized (tangent) expression for $WSR_{\overline{k}}$ constitutes a lower bound for it.
- Now, dropping constant terms, reparameterizing $Q_k = \mathbf{g}_k \mathbf{g}_k^H$ and performing this linearization for all users,

$$WSR(\mathbf{g}, \widehat{\mathbf{g}}) = \sum_{k=1}^{K} u_k \ln(1+\mathbf{g}_k^H \mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k) - \mathbf{g}_k^H (\widehat{A}_k + \lambda I) \mathbf{g}_k + \lambda P \,.$$

The gradient of this concave WSR lower bound is actually still the same as that of the original WSR or of the WSMSE criteria! Allows generalized eigenvector interpretation:

$$\mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k = \frac{1 + \mathbf{g}_k^H \mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k}{u_k} (\widehat{A}_k + \lambda I) \mathbf{g}_k$$

or hence $\mathbf{g}_k^{'} = V_{max}(\mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k, \widehat{A}_k + \lambda I)$
which is proportional to the "LMMSE" $\mathbf{g}_k$,
with max eigenvalue $\sigma_k = \sigma_{max}(\mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k, \widehat{A}_k + \lambda I)$.

[KimGiannakis:IT0511] = Optimally Weighted SLNR

- Again, [KimGiannakis:IT0511] BF:

$$\mathbf{g}_k^{'} = V_{max}(\mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k, \sum_{i=1,\neq k}^{K} u_i \mathbf{H}_i^H (\widehat{R}_{\overline{i}}^{-1} - \widehat{R}_i^{-1}) \mathbf{H}_i + \lambda I)$$

- This can be viewed as an optimally weighted version of SLNR (Signal-to-Leakage-plus-Noise-Ratio) [Sayed:SP0507]

$$SLNR_k = \frac{||\mathbf{H}_k \mathbf{g}_k||^2}{\sum_{i\neq k} ||\mathbf{H}_i \mathbf{g}_k||^2 + \sum_i ||\mathbf{g}_i||^2/P} \text{ vs}$$

$$SINR_k = \frac{||\mathbf{H}_k \mathbf{g}_k||^2}{\sum_{i\neq k} ||\mathbf{H}_k \mathbf{g}_i||^2 + \sum_i ||\mathbf{g}_i||^2/P}$$

- SLNR takes as Tx filter

$$\mathbf{g}_k^{'} = V_{max}(\mathbf{H}_k^H \mathbf{H}_k, \sum_{i\neq k} \mathbf{H}_i^H \mathbf{H}_i + I)$$

- Let $\sigma_k^{(1)} = \mathbf{g}_k^{'H} \mathbf{H}_k^H \widehat{R}_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k^{'}$ and $\sigma_k^{(2)} = \mathbf{g}_k^{'H} \widehat{A}_k \mathbf{g}_k^{'}$.
- The advantage of this formulation is that it allows straightforward power adaptation: substituting $\mathbf{g}_k = \sqrt{p_k}\, \mathbf{g}_k^{'}$ yields

$$WSR = \lambda P + \sum_{k=1}^{K} \{ u_k \ln(1 + p_k \sigma_k^{(1)}) - p_k(\sigma_k^{(2)} + \lambda) \}$$

which leads to the following interference leakage aware water filling

$$p_k = \left(\frac{u_k}{\sigma_k^{(2)} + \lambda} - \frac{1}{\sigma_k^{(1)}} \right)^{+}.$$

- For a given λ, $\mathbf{g}$ needs to be iterated till convergence.
- And λ can be found by duality (line search):

$$\min_{\lambda \geq 0} \max_{\mathbf{g}} \lambda P + \sum_{k} \{ u_k \ln\det(R_{\overline{k}}^{-1} R_k) - \lambda p_k \} = \min_{\lambda \geq 0} WSR(\lambda).$$

High/Low SNR Behavior

- At high SNR, max WSR BF converges to ZF solutions with uniform power

$$\mathbf{g}_k^H = \mathbf{f}_k \mathbf{H}_k P^{\perp}_{(\mathbf{fH})^H_{\overline{k}}} / ||\mathbf{f}_k \mathbf{H}_k P^{\perp}_{(\mathbf{fH})^H_{\overline{k}}}||$$

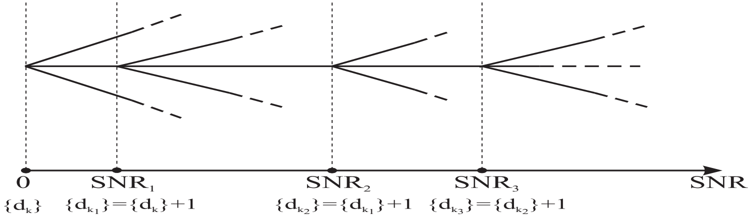
  where $P^{\perp}_{\mathbf{X}} = I - P_{\mathbf{X}}$ and $P_{\mathbf{X}} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H$ projection matrices
  $(\mathbf{fH})_{\overline{k}}$ denotes the (up-down) stacking of $\mathbf{f}_i\mathbf{H}_i$ for users $i = 1, \ldots, K,\ i \neq k$.
- At low SNR, matched filter for user with largest $||\mathbf{H}_k||_2$ (max singular value)

Deterministic Annealing

- At high SNR: max WSR solutions are ZF. When ZF is possible (IA feasible), multiple ZF solutions typically exist.
  Homotopy on the MIMO channel SVD:
  $$H_{ji} = \sum_{k=1}^{d} \sigma_{jik} \mathbf{u}_{jik} \mathbf{v}_{jik}^H + t \sum_{k=d+1} \sigma_{jik} \mathbf{u}_{jik} \mathbf{v}_{jik}^H$$
  The IA (ZF) condition for rank 1 link $i - j$ can be written as
  $$\sigma_{ji} \mathbf{f}_j^H \mathbf{u}_{ji} \mathbf{v}_{ji}^H \mathbf{g}_i = 0$$
- Two configurations are possible: $\mathbf{f}_j^H \mathbf{u}_{ji} = 0$ or $\mathbf{v}_{ji}^H \mathbf{g}_i = 0$
  Either the Tx or the Rx suppresses one particular interfering stream
- These different ZF solutions are the possible local optima for max WSR at infinite SNR. By homotopy, this remains the number of max WSR local optima as the SNR decreases from infinity. As the SNR decreases further, a stream for some user may get turned off until only a single stream remains at low SNR. Hence, the number of local optima reduces as streams disappear at finite SNR.
- At intermediate SNR, the number of streams may also be larger than the DoF though.

Deterministic Annealing (2)

- Homotopy for finding global optimum: at low SNR, noise dominates interference ⇒ optimal: one stream per power constraint, matched filter Tx/Rx. Gradually increasing SNR allows lower SNR solution to be in region of attraction of global optimum at next higher SNR. Phase transitions: add a stream.
- As a corollary, in the MISO case, the max WSR optimum is unique, since there is only one way to perform ZF BF.



Outline

- interference single cell: Broadcast Channel (BC)
  - utility functions: SINR balancing, (weighted) sum rate (WSR)
  - BC with user selection: DPC vs BF, MIMO: role of Rx antennas
- interference multi-cell/HetNets: Interference Channel (IC)
  - Degrees of Freedom (DoF) and Interference Alignment (IA)
  - multi-cell multi-user: Interfering Broadcast Channel (IBC)
  - Weighted Sum Rate (WSR) maximization and UL/DL duality
  - Deterministic Annealing to find global max WSR
- Max WSR with Partial CSIT
  - CSIT: perfect, partial, LoS
  - EWSMSE, Massive MIMO limit, large MIMO asymptotics
- CSIT acquisition and distributed designs
  - distributed CSIT acquisition, netDoF
  - topology, rank reduced, decoupled Tx/Rx design, local CSIT
  - Massive MIMO and mmWave aspects, covariance CSIT
  - distributed designs

Mean and Covariance Gaussian CSIT

- Mean information about the channel can come from channel feedback or reciprocity, and prediction, or it may correspond to the non fading (e.g. LoS) part of the channel (note that an unknown phase factor $e^{j\phi}$ in the overall channel mean does not affect the BF design).
- Covariance information may correspond to channel estimation (feedback, prediction) errors and/or to information about spatial correlations. The separable (or Kronecker) correlation model (for the channel itself, as opposed to its estimation error or knowledge) below is acceptable when the number of propagation paths N_p becomes large ($N_p \gg MN$) as possibly in indoor propagation.
- Given only mean and covariance information, the fitting maximum entropy distribution is Gaussian.

Mean and Covariance Gaussian CSIT (2)

- Hence consider

$$\text{vec}(\mathbf{H}) \sim \mathcal{CN}(\text{vec}(\overline{\mathbf{H}}), C_t^T \otimes C_r) \text{ or } \mathbf{H} = \overline{\mathbf{H}} + C_r^{1/2}\, \widetilde{\mathbf{H}}\, C_t^{1/2}$$

where $C_r^{1/2}$, $C_t^{1/2}$ are Hermitian square-roots of the Rx and Tx side covariance matrices

$$\begin{aligned} \text{E}\, (\mathbf{H}-\overline{\mathbf{H}})(\mathbf{H}-\overline{\mathbf{H}})^H &= \text{tr}\{C_t\}\; C_r \\ \text{E}\, (\mathbf{H}-\overline{\mathbf{H}})^H(\mathbf{H}-\overline{\mathbf{H}}) &= \text{tr}\{C_r\}\; C_t \end{aligned}$$

and the elements of $\widetilde{\mathbf{H}}$ are i.i.d. $\sim \mathcal{CN}(0,1)$. A scale factor needs to be fixed in the product $\text{tr}\{C_r\}\text{tr}\{C_t\}$ for unicity.

- In what follows, it will also be of interest to consider the total Tx side correlation matrix

$$R_t = \text{E}\; \mathbf{H}^H\mathbf{H} = \overline{\mathbf{H}}^H\overline{\mathbf{H}} + \text{tr}\{C_r\}\; C_t\; .$$

- Gaussian CSIT model could be considered an instance of Ricean fading in which the ratio $\text{tr}\{\overline{\mathbf{H}}^H\overline{\mathbf{H}}\}/(\text{tr}\{C_r\}\text{tr}\{C_t\}) =$ Ricean factor.

Location Aided Partial CSIT LoS Channel Model

- Assuming the Tx disposes of not much more than the LoS component information, model

$$\mathbf{H} = \mathbf{h}_r \, \mathbf{h}_t^H(\theta) + \widetilde{\mathbf{H}}'$$

where θ is the LoS AoD and the Tx side array response is normalized: $||\mathbf{h}_t(\theta)||^2 = 1$.

- Since the orientation of the MT is random, model the Rx side LoS array response $\mathbf{h}_r$ as vector of i.i.d. complex Gaussian

$$\begin{aligned} \mathbf{h}_r \;\; &\text{i.i.d.} \;\; \sim \mathcal{CN}(0, \tfrac{\mu}{\mu+1}) \quad \text{and} \\ \widetilde{\mathbf{H}}' \;\; &\text{i.i.d.} \;\; \sim \mathcal{CN}(0, \tfrac{1}{\mu+1} \tfrac{1}{M}) \text{ , independent of } \mathbf{h}_r, \end{aligned}$$

where the matrix $\widetilde{\mathbf{H}}$ represents the aggregate NLoS components.

Location Aided Partial CSIT LoS Channel Model (2)

- Note that

$$\mathrm{E}\ ||\mathbf{H}||_F^2 = \mathrm{E}\ \mathrm{tr}\{\mathbf{H}^H\mathbf{H}\} = \\ ||\mathbf{h}_t(\theta)||^2\ \mathrm{E}\ ||\mathbf{h}_r||^2 + \mathrm{E}\ ||\widetilde{\mathbf{H}}'||_F^2 = \tfrac{\mu N}{\mu+1} + \tfrac{N}{\mu+1} = N\,,$$

$(\mathrm{E}\ ||\mathbf{h}_r\,\mathbf{h}_t^T(\theta)||_F^2)/(\mathrm{E}\ ||\widetilde{\mathbf{H}}'||_F^2) = \mu =$ a Rice factor.

- In fact the only parameter additional to the LoS AoD θ is μ.
- So, this is a case of zero mean CSIT and Tx side covariance CSIT

$$R_t = \mathrm{E}\ \mathbf{H}^H\mathbf{H} = \frac{\mu N}{\mu+1}\mathbf{h}_t(\theta)\mathbf{h}_t^H(\theta) + \frac{N}{\mu+1}\frac{1}{M}I_M\ .$$

Location Aided LoS ZF BF

- For ZF BF, the BS shall use for user k a spatial filter $\mathbf{g}_k = \sqrt{p_k}\,\mathbf{g}_k^{'}$ such that $\mathbf{g}_k^{'} = \mathbf{g}_k^{''}/||\mathbf{g}_k^{''}||$

$$\mathbf{g}_k^{''} = P^{\perp}_{\mathbf{h}_{t,\overline{k}}}\mathbf{h}_{t,k}$$

  where $\mathbf{h}_{t,\overline{k}} = [\mathbf{h}_{t,1} \cdots \mathbf{h}_{t,k-1}\,\mathbf{h}_{t,k+1} \cdots \mathbf{h}_{t,K}]$.
- And uniform power distribution $p_k = P/K,\ k = 1,\ldots,K$.
- The $\mathbf{g}_k^{''}$ can also be computed from

$$\mathbf{g}^{''} = [\mathbf{g}_1^{''} \cdots \mathbf{g}_K^{''}] = \mathbf{h}_t(\mathbf{h}_t^H\mathbf{h}_t)^{-1}\ ,\ \ \mathbf{h}_t = [\mathbf{h}_{t,1} \cdots \mathbf{h}_{t,K}]\,.$$

LoS based Partial CSIT: Beyond ZF

- Go beyond the asymptotics of high SNR and high Ricean factor: even if the Tx ignores the multipath and the Rx can handle it, it would be better to have a multipath aware Tx design. Note that the Ricean factor μ satisfies uplink/downlink (UL/DL) reciprocity, even in a FDD. Solution: previous partial CSIT design.

Max Expected WSR (EWSR)

- scenario of interest: perfect CSIR, partial (LoS) CSIT
- Imperfect CSIT $\Rightarrow$ various possible optimization criteria: outage capacity,.... Here: expected weighted sum rate $\text{E}_{\mathbf{H}} WSR(\mathbf{g}, \mathbf{H}) =$

$$EWSR(\mathbf{g}) = \text{E}_{\mathbf{H}} \sum_k u_k \ln(1 + \mathbf{g}_k^H \mathbf{H}_k^H R_{\overline{k}}^{-1} \mathbf{H}_k \mathbf{g}_k)$$

perfect CSIR: optimal Rx filters $\mathbf{f}_k$ (fn of aggregate $\mathbf{H}$) have been substituted: $WSR(\mathbf{g}, \mathbf{H}) = \max_{\mathbf{f}} \sum_k u_k (-\ln(e_k(\mathbf{f}_k, \mathbf{g})))$.

Max EWSR by Stochastic Approximation

- In [Luo:spawc13] a stochastic approximation approach for maximizing the EWSR was introduced: replace statistical average by sample average (samples of $\mathbf{H}$ get generated according to its Gaussian CSIT distribution in a Monte Carlo fashion), and one iteration of the min WSMSE approach gets executed per term added in the sample average.
- Some issues: in this case the number of iterations may get dictated by a sufficient size for the sample average rather than by a convergence requirement for the iterative approach.
- Another issue is that this approach converges to a local maximum of the EWSR. It is not immediately clear how to combine this stochastic approximation approach with deterministic annealing.
- Below: various deterministic approximations and bounds for the EWSR, which can then be optimized as in the full CSI case.

EWSR Lower Bound: EWSMSE

- $EWSR(\mathbf{g})$: difficult to compute and to maximize directly. [Negro:iswcs12] much more attractive to consider $\mathrm{E}_{\mathbf{H}} e_k(\mathbf{f}_k, \mathbf{g}, \mathbf{H})$ since $e_k(\mathbf{f}_k, \mathbf{g}, \mathbf{H})$ is quadratic in $\mathbf{H}$. Hence optimizing $\mathrm{E}_{\mathbf{H}} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H})$.

$$\begin{aligned} &\min_{\mathbf{f},w} \mathrm{E}_{\mathbf{H}} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H}) \\ &\geq \mathrm{E}_{\mathbf{H}} \min_{\mathbf{f},w} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H}) = -EWSR(\mathbf{g}) \end{aligned}$$

  or hence $EWSR(\mathbf{g}) \geq -\min_{\mathbf{f},w} \mathrm{E}_{\mathbf{H}} WSMSE(\mathbf{g}, \mathbf{f}, w, \mathbf{H})$.
- So now only a lower bound to the EWSR gets maximized, which corresponds in fact to the CSIR being equally partial as the CSIT.

$$\begin{aligned} &\mathrm{E}_{\mathbf{H}} e_k = 1 - 2\Re\{\mathbf{f}_k^H \overline{\mathbf{H}}_k \mathbf{g}_k\} + \sum_{i=1}^K \mathbf{f}_k^H \overline{\mathbf{H}}_k \mathbf{g}_i \mathbf{g}_i^H \overline{\mathbf{H}}_k^H \mathbf{f}_k \\ &+ \mathbf{f}_k^H R_{r,k} \mathbf{f}_k \sum_{i=1}^K \mathbf{g}_i^H R_{t,k} \mathbf{g}_i + ||\mathbf{f}_k||^2. \end{aligned}$$

  $\Rightarrow$ signal term disappears if $\overline{\mathbf{H}}_k = 0$! Hence the EWSMSE lower bound is (very) loose unless the Rice factor is high, and is useless in the absence of mean CSIT.

EWSR Upper Bound: WSESINR

- Using the concavity of ln(.), we get

$$EWSR(\mathbf{g}) \leq \sum_{k=1}^{K} u_k \ln(1 + \mathrm{E}_{\mathbf{H}_k} \mathrm{SINR}_k(\mathbf{g}, \mathbf{H}_k)) \, .$$

Massive MIMO Limit

- We get a convergence for any term of the form

$$\mathbf{H}Q\mathbf{H}^H \stackrel{M\to\infty}{\longrightarrow} \mathrm{E}_{\mathbf{H}}\,\mathbf{H}Q\mathbf{H}^H = \overline{\mathbf{H}}Q\overline{\mathbf{H}}^H + \mathrm{tr}\{QC_t\}\,C_r\,.$$

Go one step further in separable channel correlation model: $C_{r,k,b_i} = C_{r,k},\ \forall b_i$. This leads us to introduce

$$\mathbf{H}_k = [\mathbf{H}_{k,1}\cdots\mathbf{H}_{k,C}] = \overline{\mathbf{H}}_k + C_{r,k}^{1/2}\widetilde{\mathbf{H}}_k C_{t,k}^{1/2}$$

$$Q = \begin{bmatrix} \sum_{i:b_i=1} Q_i & & \\ & \ddots & \\ & & \sum_{i:b_i=C} Q_i \end{bmatrix} = \sum_{j=1}^{C}\sum_{i:b_i=j} I_j Q_i I_j^H$$

$$Q_{\overline{k}} = Q - I_{b_i} Q_i I_{b_i}^H$$

where $C_{t,k} = \mathrm{blockdiag}\{C_{t,k,1},\ldots,C_{t,k,C}\}$, and I_j is an all zero block vector except for an identity matrix in block j. Then we get for the *WSR* (= *EWSR*),

$$WSR = \sum_{k=1}^{K} u_k \ln\det(\breve{R}_{\overline{k}}^{-1}\breve{R}_k)$$

where

$$\breve{R}_k = I_{N_k} + \overline{\mathbf{H}}_k Q \overline{\mathbf{H}}_k^H + \mathrm{tr}\{QC_{t,k}\}\,C_{r,k}$$
$$\breve{R}_{\overline{k}} = I_{N_k} + \overline{\mathbf{H}}_k Q_{\overline{k}} \overline{\mathbf{H}}_k^H + \mathrm{tr}\{Q_{\overline{k}}C_{t,k}\}\,C_{r,k}$$

Massive MIMO Limit (2)

- This leads to

$$WSR = u_k \ln\det(I + \breve{R}_{\overline{k}}^{-1}\left(\overline{\mathbf{H}}_{k,b_k}\mathbf{g}_k\mathbf{g}_k^H\overline{\mathbf{H}}_{k,b_k}^H + \mathrm{tr}\{\mathbf{g}_k\mathbf{g}_k^H C_{t,k,b_k}\}C_{r,k}\right)) + WSR_{\overline{k}}\ .$$

- Consider simplified case: "Ricean factor" $\mu \sim$ SNR, for the direct links $\mathbf{H}_{k,b_k}$ (only) (properly organized (intracell) channel estimation and feedback) $\Rightarrow$ approximation

$$WSR = u_k \ln\det(I + \mathbf{g}_k^H\breve{\mathbf{B}}_k\mathbf{g}_k) + WSR_{\overline{k}} \quad \text{with}$$

$$\breve{\mathbf{B}}_k = \overline{\mathbf{H}}_{k,b_k}^H\breve{R}_{\overline{k}}^{-1}\overline{\mathbf{H}}_{k,b_k} + \mathrm{tr}\{C_{r,k}\breve{R}_{\overline{k}}^{-1}\}C_{t,k,b_k}$$

The linearization of $WSR_{\overline{k}}$ w.r.t. Q_k now involves

$$\breve{A}_k = \sum_{i\neq k}^{K} u_i \left[\overline{\mathbf{H}}_{i,b_k}^H\left(\breve{R}_{\overline{i}}^{-1} - \breve{R}_i^{-1}\right)\overline{\mathbf{H}}_{i,b_k} + \mathrm{tr}\{\left(\breve{R}_{\overline{i}}^{-1} - \breve{R}_i^{-1}\right)C_{r,i}\}C_{t,i,b_k}\right]\ .$$

The rest of the development is now completely analogous to the case of perfect CSIT.

Large MIMO Asymptotics Refinement

- SU MIMO asymptotics from [Loubaton:IT0310],[Taricco:IT0808] (in which both $M, N \to \infty$, which tends to give more precise approximations when M is not so large) for a term of the form $\ln\det(Q\mathbf{H}^H\mathbf{H} + I)$ correspond to replacing $\mathbf{H}_k^H\mathbf{H}_k$ in the $\widetilde{R}_k$ and $\widetilde{R}_{\overline{k}}$ with a kind of $R_{t,k}$ with a different weighting of the $\overline{\mathbf{H}}_k^H\overline{\mathbf{H}}_k$ and $C_{t,k}$ portions, of the form $R'_{t,k} = a_k C_{t,k} + \overline{\mathbf{H}}_k^H \mathbf{B}_k \overline{\mathbf{H}}_k$ for some scalar a_k and matrix $\mathbf{B}_k$ that depends on $C_{r,k}$.
- For the general case of Gaussian CSIT with separable (Kronecker) covariance, get

$$
\begin{aligned}
&\mathrm{E}_{\,\mathbf{H}} \ln\det(I + \mathbf{H}Q\mathbf{H}^H) \\
&= \max_{z,w} \left\{ \ln\det \begin{bmatrix} I + wC_r & \overline{\mathbf{H}} \\ -Q\overline{\mathbf{H}}^H & I + zQC_t \end{bmatrix} - zw \right\}.
\end{aligned}
$$

$\max_{z,w}$ interpretation is new.

Large MIMO Asymptotics Refinement (2)

- Simpler case: zero channel means $\overline{\mathbf{H}}_k = 0$ and no Rx side correlations $C_r = I$, and with per user Tx side correlations $C_t \leftarrow C_k$, the EWSR w large MIMO asymptotics:

$$
\begin{aligned}
&EWSR = \\
&\sum_{k=1}^{K} \Big\{ u_k \max_{z_k, w_k} \left[\ln\det(I + z_k G G^H C_k) + N_k \ln(1 + w_k) - z_k w_k \right] \\
&- u_k \max_{z_{\overline{k}}, w_{\overline{k}}} \left[\ln\det(I + z_{\overline{k}} G_{\overline{k}} G_{\overline{k}}^H C_k) + N_k \ln(1 + w_{\overline{k}}) - z_{\overline{k}} w_{\overline{k}} \right] \Big\}
\end{aligned}
$$

where $G = [\mathbf{g}_1 \cdots \mathbf{g}_K]$ and $G_{\overline{k}}$ is the same as G except for column $\mathbf{g}_k$. Can be maximized by alternating optimization.

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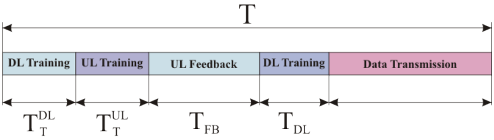
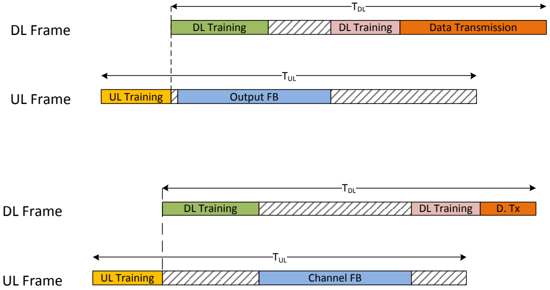
IC CSI Acquisition Outline

- Centralized CSIT Acquisition
- Distributed CSIT Acquisition
- Channel Feedback & Output Feedback
- DoF optimization as a function of coherence time



- We consider a block fading channel model with Coherence time interval T
- The general channel matrix $\mathbf{H}_{ik} \sim \mathcal{N}(0, \mathbf{I})$
- To acquire the necessary CSI at BS and MU side several training and feedback phases are necessary
- Hence a total overhead of T_{ovrhd} channel usage is dedicated to BS-MU signaling
- Only part of the time $T_{data} = T - T_{ovrhd}$ is dedicated to real data transmission

Output Feedback



- Output FB allows us to reduce the overhead due to CSI exchange
- In channel FB each MU has to wait the end of the DL training phase before being able to FB DL channel estimates
- For easy of exposition we consider $M_i = N_t \; \forall i$, $N_i = N_r \; \forall i$ where $N_t \geq N_r$

Importance of CSIR

- CSIR is usually neglected
- Some schemes for arbitrary time-varying channels assume that Rxs know all channel matrices at all time: impossible to realize in practice
- An additional DL training phase is required to build the Rx filters

TDD vs FDD

- Usually TDD transmission scheme is used to simplify the DL CSI acquisition at the BS side
- BS_k learns the DL channel $\mathbf{H}_{ik}$, $\forall i$ through reciprocity
- MU_i do not need to feedback $\mathbf{H}_{ik}$ to BS_k but this channel is required at $BS_{j\neq k}$
- In Distributed Processing reciprocity does NOT help in reducing channel feedback overhead $\Longrightarrow$ TDD almost equivalent to FDD
- In Centralized Processing reciprocity makes channel feedback NOT required

Concluding Remarks

- DoF in multi-user systems accounting for (channel) feedback are extremely sensitive to channel model.
- All this argues for shrinking the Feedback delay as much as possible: in FDD, feedback delay can be shrunk to roundtrip delay! **Immediate Feedback**.

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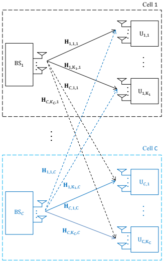
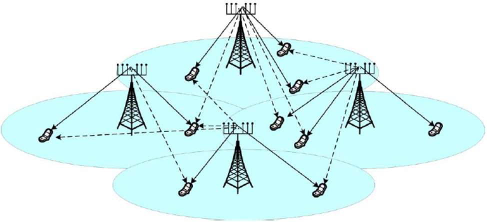
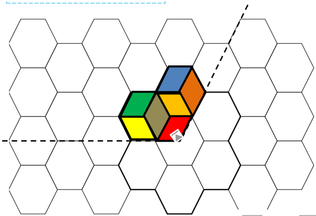
Reduced CSIT and Decoupled Tx/Rx Design

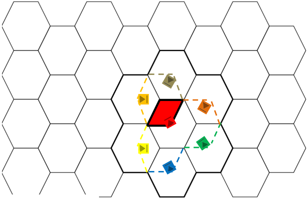
- for IA to apply to cellular: overall Tx/Rx design has to decompose so that the CSIT required is no longer global and remains bounded regardless of the network size.
- simplest case : local CSIT : a BS only needs to know the channels from itself to all terminals. In the TDD case : reciprocity. The local CSIT case arises when all ZF work needs to be done by the Tx: $d_{c,k} = N_{c,k}, \forall c, k$. The most straightforward such case is of course the MISO case: $d_{c,k} = N_{c,k} = 1$. It extends to cases of $N_{c,k} > d_{c,k}$ if less than optimal DoF are accepted. One of these cases is that of reduced rank MIMO channels.
- reduced CSIT [Lau:SP0913]: variety of approaches w reduced CSIT FB in exchange for DoF reductions.
- incomplete CSIT [deKerretGesbert:TWC13]: min some MIMO IC optimal DoF can be attained with less than global CSIT. Only occurs when M and/or N vary substantially so that subnetworks of a subgroup of BS and another subgroup of terminals arise in which the numbers of antennas available are just enough to handle the interference within the subnetwork.
- Massive MIMO leads to exploiting covariance CSIT, which will tend to have reduced rank and allows decoupled approaches.

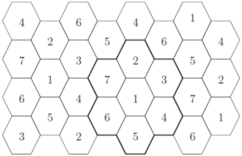
Massive MIMO: topological aspects











Clustered Topological MIMO IBC

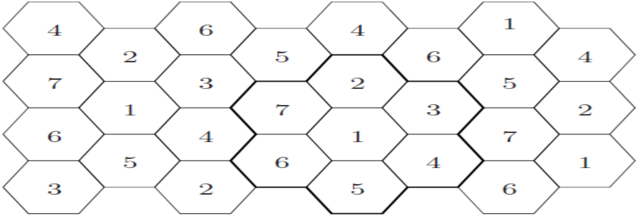


Figure : Hexagonal cellular system with cluster size $C = 7$.

We propose here an approach to an infinite IBC network by exploiting topology, enforcing CSI to be local to clusters, and reverse engineering the numbers of antennas required.

Consider partitioning an infinite IBC into finite IBC clusters. Within a finite IBC cluster, CSI acquistion can be performed in a distributed fashion. Then antennas get added to the BS in order to perform ZF of the finite inter-cluster links (due to topology, longer links can be neglected). Traditional cellular system, with interference limited to the first tier (6 cells). Hence we get a cluster size of $C = 7$.

As for GSM frequency reuse, the whole area can be covered by contiguous repetition of the cluster pattern. However, here cell numbering in a cluster has nothing to do with freq. reuse (=1).

Apart from these additional constraints for the cluster edge cell Txs, the Tx/Rx design within a cluster may seem like that of a standard MIMO IBC. However, the topology also affects within a cluster (alternatively, this could be not exploited). Hence if we consider the channel blocks between the 7 cells, we get an overall channel matrix of the form

$$\mathbf{H} = \begin{bmatrix} * & * & * & * & * & * & * \\ * & * & * & 0 & 0 & 0 & * \\ * & * & * & * & 0 & 0 & 0 \\ * & 0 & * & * & * & 0 & 0 \\ * & 0 & 0 & * & * & * & 0 \\ * & 0 & 0 & 0 & * & * & * \\ * & * & 0 & 0 & 0 & * & * \end{bmatrix}$$

where the "$*$" entries denote non-zero blocks. The $d_{c,k}$ streams for user (c, k) get extracted from its Rx signal $y_{c,k}$ by a $d_{c,k} \times N_{c,k}$ Rx filter $F_{c,k}$. To get the DoF, we need to count the number of streams that can pass through the Tx/Rx filters in parallel without suffering interference.

Sectored Cells

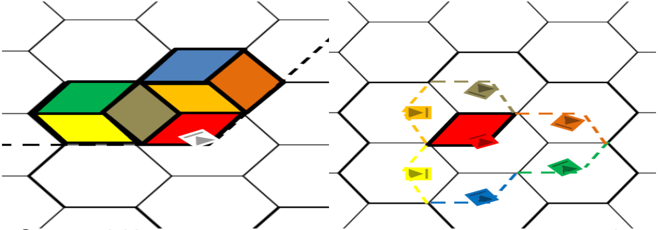


Figure : Sectored Hexagonal cellular system with 3 sectors and cluster size $C = 7$. The left figure indicates the sectors in which MTs receive a certain sector BS, as for data Tx or training. The right figure indicates the BS sectors that Rx FB from MTs in a certain sector.

The topological (distance) aspect introduces a certain "banded" character in the overall channel matrix $\mathbf{H}$: the number of non-zero blocks in any block row or block column remains finite (of cluster size C) regardless of the overall matrix size. Sectoring furthermore adds a certain spatial causality. Indeed a certain sector BS will only affect a portion $\frac{1}{3}$ of the MTs in the case of 3 sectors. This leads to a "triangular" $\mathbf{H}$. Nevertheless, as only the BS Tx/Rx are sectored, and not the MTs, with interference up to the first tier, again a cluster size of $C = 7$ (figure)

Interfering HetNets

- The design can be applied to the case of HetNets (heterogeneous networks), with multiple small cells per macro cell.
- In the topological case, we can e.g. assume that the small cell MTs Rx macro interference just like the macro MTs, but the small cell BS only interfere to the (all) MTs within the cell.
- In case of K_m macro cell MTs per cell and K_s small cell MTs, the design for the Tx filters at the macro BS remains unchanged, after replacing $K = K_m + K_s$. The small cell BS only needs Tx antennas to ZF to local users within the macro cell.

Local Receiver Design for Interfering HetNets

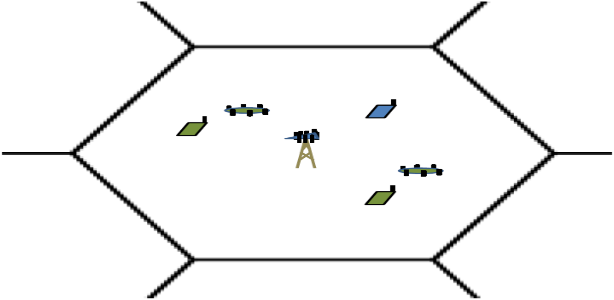


Figure : Zoom on a HetNet macro cell with two small cells in the macro cell and one MT for each of the three BS.

In the HetNet scenario, it may be of interest to adapt the Rx with interference that is only aligned for a subset of the interferers. For the remaining interferers, the Rx then appears as fixed and the ZF work has to be done by the corresponding Txs. As the concept of incomplete CSIT [deKerretGesbert:TWC13] shows, this may be not that suboptimal, depending on the antenna configurations. For the HetNet scenario, consider an IBC design per macro cell (as cluster)

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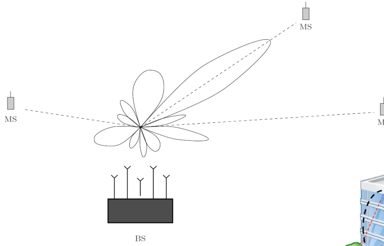
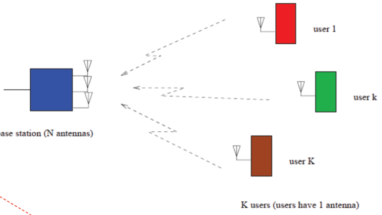
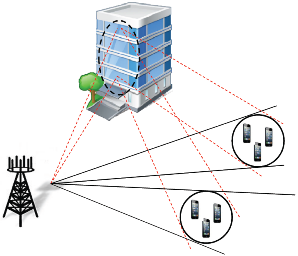
Massive MIMO: from spatial to spatiotemporal and back

- spatial: to null a user, need to null all paths of that user
  spatiotemporal: # antennas > # users
  spatial: # antennas > # paths ≫ # users
- but: paths are slowly fading, user channels are fast fading







- Keysight ieee comsoc M-MIMO tutorial, mmWave

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Specular Wireless (Massive) MIMO Channel Model

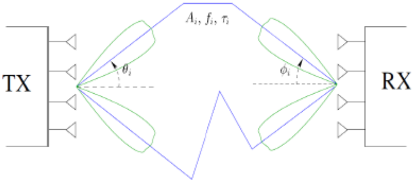

Figure 1 : MIMO transmission with M transmit and N receive antennas.

The antenna array responses are just functions of angles AoD, AoA in the case of standard antenna arrays with scatterers in the far field. In the case of distributed antenna systems, the array responses become a function of all position parameters of the path scatterers. The fast variation of the phases ψ_i (due to Doppler) and possibly the variation of the A_i correspond to the fast fading. All the other parameters vary on a slower time scale and correspond to slow fading.

MIMO channel transfer matrix at any particular subcarrier of a given OFDM symbol

$$\mathbf{H} = \sum_{i=1}^{N_p} A_i \, e^{j\psi_i} \, \mathbf{h}_r(\phi_i) \, \mathbf{h}_t^T(\theta_i) = \mathbf{B} \, \mathbf{A}^H \qquad (1)$$

where there are N_p (specular) pathwise contributions with

- $A_i > 0$: path amplitude
- θ_i: direction of departure (AoD)
- ϕ_i: direction of arrival (AoA)
- $\mathbf{h}_t(.)$, $\mathbf{h}_r(.)$: $M/N \times 1$ Tx/Rx antenna array response

and

$$\mathbf{B} = [\mathbf{h}_r(\phi_1) \, \mathbf{h}_r(\phi_1) \cdots] \begin{bmatrix} e^{j\psi_1} & & \\ & e^{j\psi_2} & \\ & & \ddots \end{bmatrix}$$
$$\mathbf{A}^H = \begin{bmatrix} A_1 & & \\ & A_2 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{h}_t^T(\theta_1) \\ \mathbf{h}_t^T(\theta_2) \\ \vdots \end{bmatrix} \qquad (2)$$

Mean and Covariance Gaussian CSIT
Dominant Paths Partial CSIT Channel Model

Given only mean and (separable) covariance information, the fitting maximum entropy distribution is Gaussian. Hence consider $\text{vec}(\mathbf{H}) \sim \mathcal{CN}(\text{vec}(\overline{\mathbf{H}}), \mathbf{C}_t^T \otimes \mathbf{C}_r)$ which can be rewritten as

$$\mathbf{H} = \overline{\mathbf{H}} + \mathbf{C}_r^{1/2}\,\widetilde{\mathbf{H}}\,\mathbf{C}_t^{1/2} \qquad (3)$$

where $\mathbf{C}_r^{1/2}$, $\mathbf{C}_t^{1/2}$ are Hermitian square-roots of the Rx and Tx side covariance matrices

$$\begin{aligned} \mathrm{E}(\mathbf{H}-\overline{\mathbf{H}})(\mathbf{H}-\overline{\mathbf{H}})^H &= \mathrm{tr}\{\mathbf{C}_t\}\ \mathbf{C}_r \\ \mathrm{E}(\mathbf{H}-\overline{\mathbf{H}})^H(\mathbf{H}-\overline{\mathbf{H}}) &= \mathrm{tr}\{\mathbf{C}_r\}\ \mathbf{C}_t \end{aligned} \qquad (4)$$

and the elements of $\widetilde{\mathbf{H}}$ are i.i.d. $\sim \mathcal{CN}(0,1)$. In what follows, it will also be of interest to consider the total Tx side correlation matrix

$$\mathbf{R}_t = \mathrm{E}\,\mathbf{H}^H\mathbf{H} = \overline{\mathbf{H}}^H\overline{\mathbf{H}} + \mathrm{tr}\{\mathbf{C}_r\}\,\mathbf{C}_t\ . \qquad (5)$$

Note that the Gaussian CSIT model could be considered an instance of Ricean fading in which the ratio $\mathrm{tr}\{\overline{\mathbf{H}}^H\overline{\mathbf{H}}\}/(\mathrm{tr}\{\mathbf{C}_r\}\mathrm{tr}\{\mathbf{C}_t\})$ could be considered the Ricean factor.

This Taylor series modeling of clusters is in contrast to the uniform DoA profile used in [Caire:mmWave], [Gesbert:arxiv1013].

Assuming the Tx disposes of not much more than the information about r dominant path AoDs, we shall consider the following MIMO (Ricean) channel model

$$\mathbf{H} = \mathbf{B}\,\mathbf{A}^H(\theta) + \sqrt{\beta}\,\widetilde{\mathbf{H}}' \qquad (6)$$

which follows from (1), (2) except restricted to the r strongest paths, with the rest modeled by $\sqrt{\beta}\,\widetilde{\mathbf{H}}'$ (elements i.i.d. $\sim \mathcal{CN}(0,\beta)$, independent of the ψ_i). Averaging over path phases $\psi_i \Rightarrow$ Tx side covariance matrix

$$\mathbf{C}_t = \mathbf{A}\mathbf{A}^H + N\beta\,I_M \qquad (7)$$

since due to the normalization of the antenna array responses, $\mathrm{E}\,\mathbf{B}^H\mathbf{B} = \mathbf{I}$. Note that $\mu = \mathrm{tr}\{\mathbf{A}\mathbf{A}^H\}/\beta NM$ could be considered a Ricean factor. When needed, we may also consider the $\mathbf{h}_r$, the columns of $\mathbf{B}$, to be isotropically distributed. Note that the rank of $\mathbf{A}\mathbf{A}^H$ can be substantially less than the number of paths. Consider e.g. a cluster of paths with narrow AoD spread, then we have $\theta_i = \theta + \Delta\theta_i$ where θ is the nominal AoD and $\Delta\theta_i$ is small $\Rightarrow$ $\mathbf{h}_t(\theta_i) \approx \mathbf{h}_t(\theta) + \Delta\theta_i\,\dot{\mathbf{h}}_t(\theta)$: rank 2 contribution to $\mathbf{A}\mathbf{A}^H$.

IA Feasibility Reduced Rank MIMO IBC

- The ZF from BS j to MT (i,k) requires

$$F_{i,k}^H \mathbf{H}_{i,k,j} G_{j,n} = F_{i,k}^H \mathbf{B}_{i,k,j} A_{i,k,j}^H G_{j,n} = 0$$

  which involves $\min(d_{i,k}d_{j,n}, d_{i,k}r_{i,k,j}, r_{i,k,}d_{j,n})$ constraints to be satisfied by the $(N_{i,k} - d_{i,k})d_{i,k}/(M_j - d_{j,n})d_{j,n}$ variables parameterizing the column subspaces of $F_{i,k}/G_{j,n}$.

- **IA feasibility singular MIMO IC with Tx/Rx decoupling**

$$F_{i,k}^H \mathbf{B}_{i,k,j} = 0 \text{ or } A_{i,k,j}^H G_{j,n} = 0 \ .$$

  This leads to a possibly increased number of ZF constraints $r_{i,k,j} \min(d_{i,k}, d_{j,n})$ and hence to possibly reduced IA feasibility. ZF of every cross link now needs to be partitioned between all Txs and Rxs, taking into account the limited number of variables each Tx or Rx has. The main goal of this approach however is that it leads to Tx/Rx decoupling.

Massive MIMO & Covariance CSIT

In massive MIMO, the Tx side channel covariance matrix is very likely to be (very) singular even though the channel response **H** may not be singular:

$$\text{rank}(C^t_{i,k,j} = A_{i,k,j}A^H_{i,k,j}) = r_{i,k,j}\,,\; A_{i,k,j} : M_j \times r_{i,k,j}$$

Let $P_{\mathbf{X}} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{\#}\mathbf{X}^H$ and $P^{\perp}_{\mathbf{X}}$ be the projection matrices on the column space of **X** and its orthogonal complement resp. Consider now a massive MIMO IBC with C cells containing K_i users each to be served by a single stream. The following result states when this will be possible.

Theorem

Sufficiency of Covariance CSIT for Massive MIMO IBC *In the MIMO IBC with (local) covariance CSIT, all BS will be able to perform ZF BF if the following holds*

$$||P^{\perp}_{A_{\overline{i,k},j}} A_{i,k,j}|| > 0\,,\; \forall i,k,j$$

where $A_{\overline{i,k},j} = \{A_{n,m,j},\, (n,m) \neq (i,k)\}$.

Massive MIMO & Covariance CSIT (2)

These conditions will be satisfied w.p. 1 if $\sum_{i=1}^{C}\sum_{k=1}^{K_i} r_{i,k,j} \leq M_j\,,\; j=1,\ldots,C$. In that case all the column spaces of the $A_{i,k,j}$ will tend to be non-overlapping. However, the conditions could very well be satisfied even if these column spaces are overlapping, in contrast to what [Gesbert:arxiv1013],[Caire:arxiv0912] appear to require.
In Theorem 1, we assume that all ZF work is done by the BS. However, if the MT have multiple antennas, they can help to a certain extent.

Theorem

Role of Receive Antennas in Massive MIMO IBC *If MT (i,k) disposes of $N_{i,k}$ antennas to receive a stream, it can perform rank reduction of a total amount of $N_{i,k}-1$ to be distributed over $\{r_{i,k,j}\,,\; j=1,\ldots,C\}$.*

Such rank reduction (by ZF of certain path contributions) facilitates the satisfaction of the conditions in Theorem 1.

Outline

- interference single cell: Broadcast Channel (BC)
  - utility functions: SINR balancing, (weighted) sum rate (WSR)
  - BC with user selection: DPC vs BF, MIMO: role of Rx antennas
- interference multi-cell/HetNets: Interference Channel (IC)
  - Degrees of Freedom (DoF) and Interference Alignment (IA)
  - multi-cell multi-user: Interfering Broadcast Channel (IBC)
  - Weighted Sum Rate (WSR) maximization and UL/DL duality
  - Deterministic Annealing to find global max WSR
- Max WSR with Partial CSIT
  - CSIT: perfect, partial, LoS
  - EWSMSE, Massive MIMO limit, large MIMO asymptotics
- CSIT acquisition and distributed designs
  - distributed CSIT acquisition, netDoF
  - topology, rank reduced, decoupled Tx/Rx design, local CSIT
  - Massive MIMO and mmWave aspects, covariance CSIT
  - distributed designs

Distributed Sum Utility Optimization

- weighted sum rate (WSR) inspired interference pricing:

$$\text{WSR} = \sum_k u_k \ \ln(1+\text{SINR}_k) = \sum_k u_k \ \ln(1+\frac{\text{S}_k}{\text{I}_k+\text{N}_k})$$

sensitivity rate user k to beamformer for user i

$$\frac{\partial}{\partial \mathbf{g}_i} \ln(1+\text{SINR}_k) = \underbrace{-\frac{\text{SINR}_k}{1+\text{SINR}_k}\frac{1}{\text{I}_k+\text{N}_k}}_{\text{interference price}} \frac{\partial}{\partial \mathbf{g}_i}\text{I}_k$$

Feed back interference prices instead of CSIT.

- UE initiated BS zero-forcing: UE decides which cross links (paths) need to be zero-forced by the corresponding BS
- Cognitive Radio approaches for distributed design: interference temperatures for intercell or intercluster links
- distributed utility optimization may call for new paradigms

Distributed IBC Design

- Global intracluster CSIT can also be gathered but it takes an overhead that evolves with C^2 [Negro:isccsp12]. Hence, high quality (high Ricean factor) intracell CSIT and Tx covariance only intercell CSIT may be a more appropriate setting. For what follows we shall assume the LoS Tx intercell CSIT. We shall focus on a MaMIMO setting.
- The approach considered here is non-iterative, or could be taken as initialization for further iterations.

Distributed IBC Design: Initialization

- Start with a per cell design.
- To simplify design, assume Rx antennas are used to handle intracell interference. Hence all intercell interference needs to be handled by Tx (BS) antennas.
- In that case, the crosslinks (cascades of channel and Rx) can be considered as independent from the intracell channels.
- In a MaMIMO setting, the ZF by BS j towards $K - K_j$ crosslink channels (or LoS components in fact) will tend to have a deterministic effect of reducing the effective number of Tx antennas by this amount and hence of reducing the Tx power by a factor $\frac{M_j}{M_j-(K-K_j)}$. Hence a per BS design can be carried out with (partial) intracell CSIT, with BS Tx power P_j replaced by $\frac{M_j}{M_j-(K-K_j)} P_j$, and with all intercell links $\mathbf{H}_{k,b_i} = 0$, $b_i \neq b_k$.
- This first step (which is itself an iterative design for the scenario considered with reduced Tx power and no intercell links) leads to BFs $\mathbf{g}^{(0)}$ which lead to

$$\breve{R}_k = (1 + \text{tr}\{Q^{(0)}_{b_k} C_{t,k,b_k}\}) I_{N_k} + \overline{\mathbf{H}}_{k,b_k} Q^{(0)}_{b_k} \overline{\mathbf{H}}^H_{k,b_k} \,,$$

$$\text{where} \quad Q^{(0)}_{b_k} = \sum_{i:b_i=b_k} \mathbf{g}^{(0)}_i \mathbf{g}^{(0)H}_i$$

and similarly for $\breve{R}_{\overline{k}}$.

- Do one iteration in order to adjust the Tx filters for the intercell interference.
- With the initial BFs $\mathbf{g}^{(0)}$, the local intercell CSIT C_{t,i,b_k} also, the correct power constraints, and $\breve{R}_k$, $\breve{R}_{\overline{k}}$ as above, we get $\breve{\mathbf{B}}_k$ as before, and $\breve{A}_k$ becomes

$$\breve{A}_k = \sum_{i \neq k : b_i = b_k} u_i \left[\overline{\mathbf{H}}_{i,b_k}^H \left(\breve{R}_{\bar{i}}^{-1} - \breve{R}_i^{-1} \right) \overline{\mathbf{H}}_{i,b_k} + \mathrm{tr}\{ \left(\breve{R}_{\bar{i}}^{-1} - \breve{R}_i^{-1} \right) C_{r,i} \} C_{t,i,b_k} \right]$$
$$+ \sum_{i : b_i \neq b_k} \underbrace{u_i \mathrm{tr}\{ \breve{R}_{\bar{i}}^{-1} - \breve{R}_i^{-1} \}}_{= \mu_i} C_{t,i,b_k} \,.$$

  Hence the only information that needs to be fed back from user i in another cell is the positive scalar μ_i. This is related to the interference pricing in game theory [XuWang:JSAC1012].
- The normalized BFs are then computed as $\mathbf{g}_k^{'} = V_{max}(\breve{\mathbf{B}}_k, \breve{A}_k + \lambda_{b_k} I)$ where the λ_{b_k} are taken from the previous iteration.
- The stream powers are obtained from the interference-aware WF.

A Few References

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5G Perspectives

- multi-user multi-cell interference management: theoretical possibilities, but (global) CSIT required
  - FB delay $\Rightarrow$ channel prediction and channel Doppler models crucial
  - analog channel FB?
  - FDD: immediate channel FB
  - distributed : yes but watch for fast fading
  - full duplex radio: reciprocity since single frequency, immediate use of channel estimates (no FDD FB or TDD ping-pong delay)
- Massive MIMO simplifications:
  separating fast and slow fading channel components
  decoupling of cells?
- mmWave (beamforming, bandwidth), spectrum aggregation
- new waveforms: windowed OFDM ?
- beyond classical cellular:
  - HetNets (macro/small):
  - wireless/self backhauling
  - D2D, cloud, IoT (low rate), COM for control (low latency),

Panel: How can mathematics and mathematicians help solve ICT Algorithm Challenges?

- big data: big networks here
- large random matrices for simplified analysis and design
- analytical tools to describe transmission rates at finite SNR: beyond degrees of freedom
- convergence and tracking analysis of adaptive distributed updating algorithms
- sparse techniques, compressed sensing, machine learning, big data
- finding the global optimum