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PHY and MAC layer Modeling of LTE and WiFi RATs

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Abstract

We consider LTE and WiFi networks in order to model both PHY and MAC layers. Our physical layer abstraction consists a mapping between user's SINR and their corresponding rate. On the other hand, we will propose a proper queue model to represent the schedulers of each radio access technique (RAT), at the MAC layer.

1 Introduction

The scope of this technical report is the proper modeling of both PHY and MAC layer of two widely used radio access techniques (RATs), LTE and WiFI. So, we have two main tasks, one for each layer (one for PHY and one for MAC).

- To match the user's SINR with the corresponding data rate. This tasks needs two internal steps: i) specify the SINR threshold of each MCS, ii) specify the rate of each MCS.
- To answer, how the resources are allocated with the presence of other users, and how the overall performance of the systems depends on the users' rate distribution. In other words, we should model each RAT scheduler with a proper queueing system in order to be able to analyze the dynamic behavior of a base station in terms of incoming load.

This work supposes 20MHz eNodeB with a single antenna and a $802.11n$ single stream access point, both of them operate with 20MHz bandwidth, but easily could be extended to other cases.

2 PHY modeling

The supported MCS modes are RAT depended and always defined at the corresponding protocol description documents [1], [2]. The operation threshold for each mode is not always defined in the protocol since it heavily depends on the receiver implementation characteristics. So, we will need one SINR table for the LTE modes and one for the WiFi. The reference receivers of the LTE will be the open air interface platforms. For the WiFi we used a generic SINR threshold that is presented at [3].

2.1 LTE

From the OpenAirInterface LTE downlink simulator [4], Block Error Rate (BLER) vs SNR, for LTE Tx mode 1 (downlink use Single-antenna port, the port

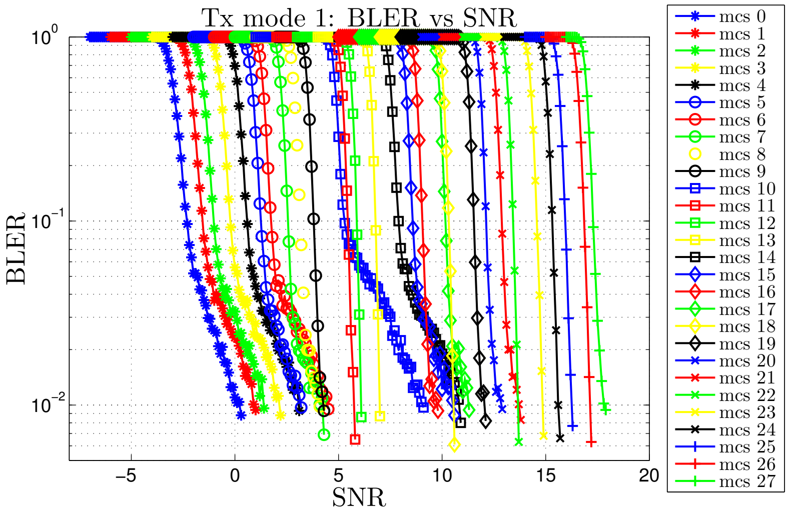


Figure 1: BLER vs SINR

0) [5], is generated for each MCS and shown in Fig. 1. So, for a given BLER threshold (commonly at 10^{-1}) the SINR threshold (τ) for each MCS can be specified. Additionally, if we are interested for theoretical analysis, we can combine the knowledge for SINR distribution (or ”coverage probability”) with MCS threshold τ in order to end with MCS distribution.

LTE uses OFDM on the DL and divides the total frequency and time resources into resource blocks (RB) [5]. The size of a RB is 180kHz in the frequency domain and 0.5ms in the time domain,Fig. 2.

In the 20MHz bandwidth configuration there are 100 RB (also plus some white spaces for system’s robustness to intra-cell interference). Each two RBs are grouped into one subframe with period one Transmission Time Interval (TTI), 1ms. We assume that all users will be allocated the same amount of subframes on average, so for a given number of associated users (n) at an eNodeB, each of them will be allocated $\frac{10^5}{n}$ subframes per second.

For a given mcs, LTE PHY specification 36.213, section 7.1.7.1 maps the index of the MCS to the index of the Transfer Block Size (I-TBS) for Downlink (part of this matrix is shown at table 1), which, together with the number of RBs, defines the amount of transferred bits per TTI, section 7.1.7.2.1 (part of this matrix is shown at table 2). Thus, by combining all the previous, for a given SINR value we are able

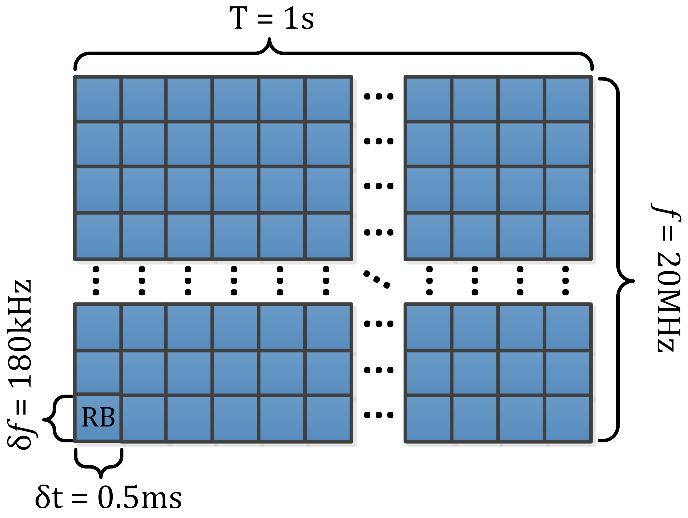


Figure 2: LTE Resource Blocks

to calculate the bit rate.

Table 1: mcs index vs tbs index

I_{mcs}	modulation order	I_{TBS}
1	2	1
2	2	2
3	2	3
4	2	4
5	2	5
6	2	6
7	2	7
8	2	8
9	2	9
10	4	9
11	4	10
	$\vdots$	

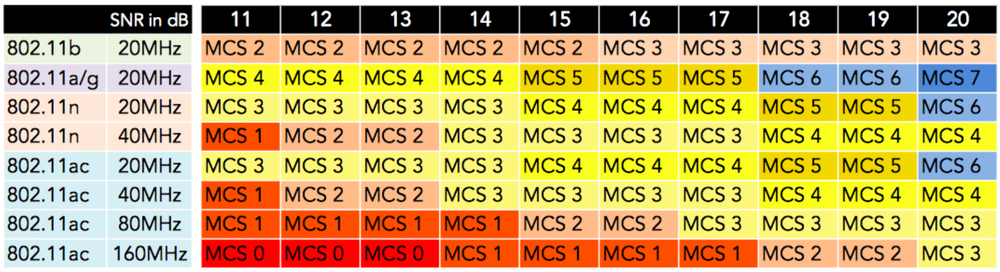
Table 2: Transmitted Bits w.r.t. to TBS index and the Number of RB

I_{TBS}	N_{RB}						
	1	2	3	4	5	6	
1	16	32	56	88	120	156	
2	24	56	88	144	176	208	$\dots$
3	32	72	144	176	208	256	
4	40	104	176	208	256	328	
5	56	120	208	256	328	424	

$\vdots$

2.2 WiFi

For WiFi 802.11n and ac, for each mcs we can extract the SINR threshold (τ) for each MCS from [3]. Part of this matrix could be seen at Fig. 3 and the physical data rate $PhyRate(msc)$ could be obtained from [6].



SNR in dB		11	12	13	14	15	16	17	18	19	20
802.11b	20MHz	MCS 2	MCS 2	MCS 2	MCS 2	MCS 2	MCS 3	MCS 3	MCS 3	MCS 3	MCS 3
802.11a/g	20MHz	MCS 4	MCS 4	MCS 4	MCS 4	MCS 5	MCS 5	MCS 5	MCS 6	MCS 6	MCS 7
802.11n	20MHz	MCS 3	MCS 3	MCS 3	MCS 3	MCS 4	MCS 4	MCS 4	MCS 5	MCS 5	MCS 6
802.11n	40MHz	MCS 1	MCS 2	MCS 2	MCS 3	MCS 3	MCS 3	MCS 3	MCS 4	MCS 4	MCS 4
802.11ac	20MHz	MCS 3	MCS 3	MCS 3	MCS 3	MCS 4	MCS 4	MCS 4	MCS 5	MCS 5	MCS 6
802.11ac	40MHz	MCS 1	MCS 2	MCS 2	MCS 3	MCS 3	MCS 3	MCS 3	MCS 4	MCS 4	MCS 4
802.11ac	80MHz	MCS 1	MCS 1	MCS 1	MCS 1	MCS 2	MCS 2	MCS 3	MCS 3	MCS 3	MCS 3
802.11ac	160MHz	MCS 0	MCS 0	MCS 0	MCS 1	MCS 1	MCS 1	MCS 1	MCS 2	MCS 2	MCS 3

Figure 3: WiFi's SINR thresholds for each MCS

Collisions, unused periods, overhead, etc are taken into account by using an expansion of Bianchi's model [7], which was extended in [8] to include newer techniques of 802.11n and ac (frame aggregation, block of ACKs, RTS/CTS, etc) that raise the utility of the MAC layer for high throughput cases. The percentage of the successful channel usage / normalized system throughput (% channel usage, P_s) w.r.t. the number of users n_u and different rates shown at Fig. 4. As we can see, for a reasonable number of connected users the performance of the MAC layer is roughly the same for a given MCS.

So the average user throughput of a user with a given MCS in the present of n other users is given by

$$Rate(msc, n) = \frac{P_s(n, Rate)}{n} PhyRate(msc) \ . \tag{1}$$

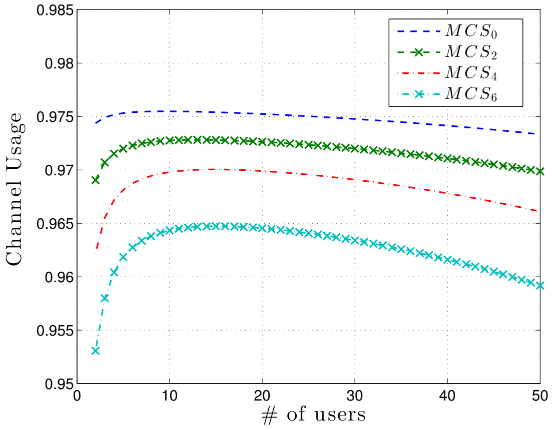


Figure 4: MAC performance of 802.11n/ac Frame Aggregation

2.3 Summary

Actual RATs do not provide an elegant way to calculate the user's rate, so it is common, when analyzing wireless networks to use the Shannon's theorem, as it constitutes a more simplified approach. When a single network is being analyzed, this assumption does not affect the validity of the qualitative results. However, in the case of modern heterogeneous networks (HetNets), and especially when HetNets operate with different RAT, this assumption does not hold. The user's rate of different RATs does not scale with the same way, with respect to SINR.

For instance, Fig. 5 presents the output of the PHY layer modeling procedure that is presented at this section, every marker corresponds to an MCS and the x-coordinates of them are the SINR threshold τ_i and the y-coordinates are the corresponding rates $rate_i$.

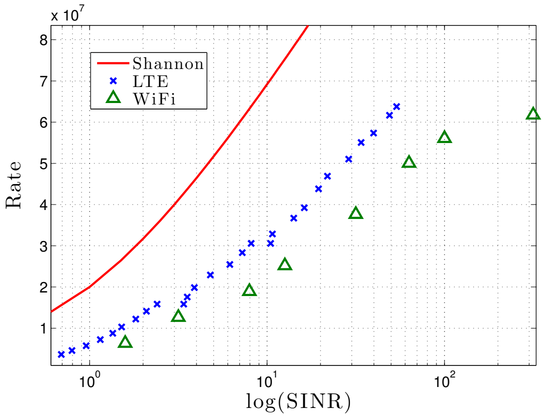


Figure 5: Comparison between RATs rates and Shannon's limit

LTE performs 37%, on average, closer to Shannon than the WiFi, at their common operating SINR range. Thus, for those HetNets, if the rates of both networks modeled according to Shannon's theorem, WiFi will be overestimated compared to LTE.

3 MAC modeling

When more than one users are served in parallel by a BS, the BS operates as a *queueing system*. The service rate depends on the number and SINR of associated users (BS load), and also on the centralized scheduler (e.g., in the case of 3G/4G) or distributed media access control (MAC) protocol (in the case of WiFi) which decide how the available resources will be distributed between users. While a number of different scheduling algorithms exist, the majority of them try to allocate the available resources between competing flows (e.g. LTE resource blocks, WiFi channel) in a fair or proportionally fair manner.

3.1 LTE (Resource Fair Scheduler)

Assume all flows are allocated by the BS the same amount of resources and they are served simultaneously, e.g., with a round robin, TDMA-like algorithm. If the service time slot is small (e.g., of packet size) compared to the total size of a flow, the flow level performance at that BS can be approximated by a multi-class M/G/1 Processor Sharing (PS) system, as shown in Fig. 6. This model has already been used to analyzed 3G/3G+ BS performance [9, 10]. While each flow shares the channel for the same amount of time (hence "resource fair") during that time it might transmit at a different rate, depending on its SINR and resulting MCS (hence the "multi-class" service).

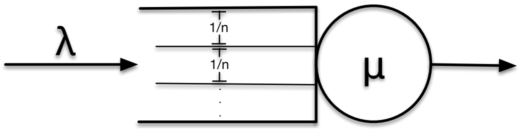


Figure 6: M/G/1/PS Resource Fair

LTE schedulers are significantly more complex, allocating competing flows both time and frequency resources (Resource Blocks), possibly taking into account the queue backlog of each flow and flow priority, and also attempting to take advantage of instantaneous SINR variations in time and frequency to achieve further multi-user diversity [5]. While a large number of algorithms have been proposed (see e.g., [11] for an extensive survey), in the lack of special priority traffic, most implemented schedulers lead to a proportionally fair throughput allocation between flows [5] and can also be approximated by a similar multi-class M/G/1 PS queue.

The following is a direct application of the multi-class M/G/1/PS result [12].

lemma: For a BS with n users generating flows of mean size $\langle s \rangle$, with instantaneous transmission rates drawn from distribution $f_{\mathrm{R}}(r)$, and allocated resources by a resource fair scheduler, the effective service rate of the cell is

$$\langle \mu \rangle_{rf} = \left(\sum_{r} \frac{f_R(r) \cdot \langle s \rangle}{r} \right)^{-1} \text{ flows/sec,} \tag{2}$$

and the mean flow delay is given by

$$E[T]_{rf} = \frac{1}{\langle \mu \rangle_{rf} - n\lambda_f} \quad , \tag{3}$$

when the system is stable $(n\lambda_f < \langle \mu \rangle)$

We further define the BS's load as $\rho = \frac{\text{input job rate}}{\text{service job rate}} = \frac{n\lambda_f}{\langle \mu \rangle}$ when the system is stable $\rho < 1$.

Performance gains from opportunistic scheduling can be included in the above equation as a multiplicative factor in front of $\langle \mu \rangle$.

3.2 WiFi (Throughput Fair Scheduler)

Some schedulers attempt to achieve fairness more aggressively, by trying to equalize per flow throughput for all nodes. For example, if two concurrent flows experience different channel conditions (say one being "far" and one being "near" the BS) a throughput fair scheduler will attempt to give more resources to the flow with the worse channel (e.g., more resource blocks in the case of LTE, or schedule the far flow more often in the case of 3G). This can be seen as a Generalized or Discriminatory Processor Sharing system (a generalized version of the M/G/1/PS) [13], with different weights per flow that, for throughput-fair systems, can be taken as inversely proportional to the average rate experienced by that flow.

It is known that throughput fair schedulers perform poorly compared to proportionally fair ones, and thus are not often considered [14]. Nevertheless, throughput fair scheduling turns out to be a good approximation of how the 802.11 WiFi MAC allocates resources between flows [15]. In WiFi, all nodes compete for the channel and when they do get access, in the basic implementation, they send a single frame and then have to retry. WiFi like LTE supports rate adaptation, therefore each frame might be transmitted at a different rate, depending on the maximum MCS that can be offered to the respective node. Nevertheless, due to the random access MAC, each node gets access with equal chance, regardless of their distance from the AP. If each flow corresponds to a large number of frames (usually a good assumption given the small max size of a frame), this essentially equalizes the long-term throughput of each flow, regardless of its MCS. Hence, the WiFi scheduler for a single BS could be seen as throughput-fair, and can be modeled as a Discriminatory Processor Sharing (DPS) queue. The following lemma derives the mean service rate (μ) for such a throughput-fair scheduler in a system with rate adaptation.

Lemma: The mean service rate for a throughput fair scheduler with rate adaptation, where a random user/frame is transmitted with an instantaneous rate r with probability $f_R(r)$, is given by

$$\langle\mu\rangle_{\text{tf}} = \left(\sum_r \frac{f_R(r)\cdot\langle s\rangle}{r}\right)^{-1}. \tag{4}$$

Proof: Consider a long time interval during which N packets get transmitted, corresponding to different flows. Assume each packet is of equal size S (e.g., the max WiFi frame size) but is transmitted with a possibly different rate r drawn from pmf $f_R(r)$ with K discrete values, depending on the MCS used for transmitting that packet. Assume that out of these N packets, N_i are transmitted with rate r_i, ($\sum_i N_i = N$). Hence, the *average* transmission rate in terms of bits/sec for these N packets is

$$\frac{\text{bits in } N \text{ pkts}}{\text{transmission time for } N \text{ pkts}} = \frac{N\cdot S}{N_1\frac{S}{r_1} + N_2\frac{S}{r_2} + \cdots + N_K\frac{S}{r_K}} \tag{5}$$

However, as N goes to infinity, the N_i converges to its mean value $f_R(r_i) \cdot N$ by the law of large numbers, hence the denominator of Eq.(5) converges to

$$\lim_{N \to \infty} (N_1 \frac{S}{r_1} + N_2 \frac{S}{r_2} + \cdots + N_K \frac{S}{r_K}) = \sum f_R(r_i) \cdot N \cdot \frac{S}{r_i}. \tag{6}$$

Since $\frac{1}{x}$ is continuous and all $r_i > 0$, we can use the Continuous Mapping Theorem [16](Th. 5.23) to show that Eq. (5) converges to

$$\frac{1}{\sum f_R(r_i) \cdot \frac{1}{r_i}}, \tag{7}$$

where N and S cancel out. Eq. (7) thus gives the average transmission rate of the scheduler over a sufficiently long sample path of packets for the scheduler. Since the system is ergodic, we can divide with the mean flow size $\langle s \rangle$ to get the mean service rate $\langle \mu \rangle_{\text{tf}}$.

Note that the above analysis, when applied to 802.11, ignores the impact of collisions and RTS/CTS frames, analyzed in [7], and thus is an upper bound. Nevertheless, in light of the high speeds and features of 802.11n\ac, such as frame aggregation or block of ACK transmissions (by a single node), implies that the impact of such overhead can be safely ignored Fig. 4.

It is interesting to observe that the above result implies that *the mean service rate, in the long rung, for a WiFi system with rate adaptation, turns out to be the same as that of a resource-fair system (Eq.* (2)). Nevertheless, this does *not* imply that the mean flow delay is also the same, as the scheduling discipline is different (DPS instead of PS). Unfortunately, there does not exist a closed form solution for the mean flow delay of a throughput fair system.

Except the resource fair as the lower bound of a throughput fair system, For general loads, to our best knowledge, the approximation from Avrachenkov *et al.* [17] for DPS systems, provides the most accurate solution, for large enough flow sizes. Specifically, the expected delay for flows of class k having size x, denoted as $E[T_k(x)]$, asymptotically converges as

$$\lim_{x \to \infty} \left(E[T_k(x)] - \frac{x}{1-\rho} \right) = \frac{\sum_j \lambda_j (1 - \frac{w_k}{w_j}) E[X_j^2]}{2(1-\rho)^2} \; . \tag{8}$$

This can be applied to our system, by having classes corresponding to different MCS. Furthermore, ρ is the load of the system (can be computed using the previews lemma and the incoming job rate λ), x is the service requirement (normalized in seconds) for a flow of class k, λ_j is the incoming job rate of class j (assuming that the probability of an incoming job to be at class j is π_j we define $\lambda_j = \pi_j \lambda$), $w_j = 1/r_j$ is the weight of each class (which, as explained earlier, is inversely proportional to the rate for that class), and $E[X_j^2]$ is the second moment of service requirement (flow sizes normalized in seconds) for flows of class j. Based on this, we can model the mean per flow delay in our system as

$$E[T]_{\text{tf}} = \sum_k \pi_k \left(\frac{S/r_k}{1-\rho} + \frac{\sum_j \pi_j \lambda (1 - \frac{r_j}{r_k})(S/r_j)^2]}{2(1-\rho)^2} \right) . \tag{9}$$

This result is an asymptotic as the service rate (flow size) is going to infinity, but even for small flow sizes our simulation results show that the approximation is decent. Roughly, for small flow size the performance of a throughput fair system is equally spaced between the two (resource fair and Avrachenkov's approximation), Fig. 7 presents the comparison between Approximation, simulation results and resource fair scheduler for average 1Mb flow size. The size is rising the performance of the system is approaching the approximation, Fig. 8 shows resource fair, approximation, and simulation results for flow size of $\langle s \rangle = 12.5$ MBytes.

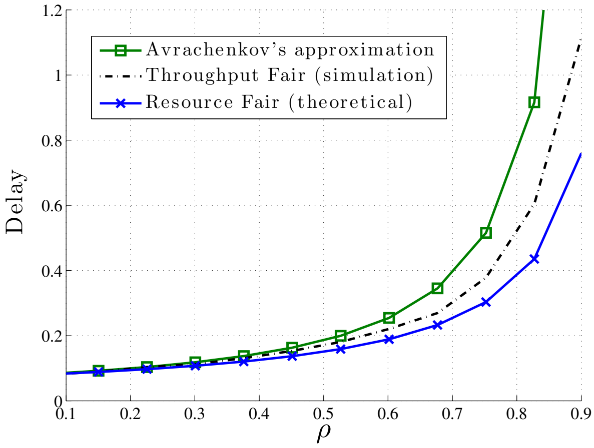


Figure 7: Delay vs load for a WiFi throughput fair system, for average flow size 1 Mb

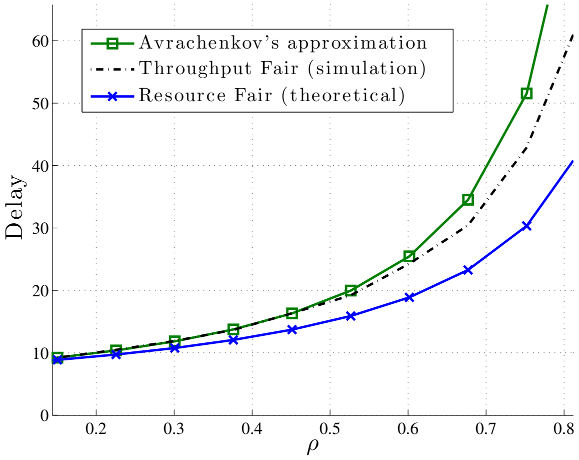


Figure 8: Delay vs load for a WiFi throughput fair system, for average flow size 12.5 Mb

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