# Stochastic Analysis of Two-Tier HetNets Employing LTE and WiFi

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***Abstract*—In order to increase the capacity of mobile networks, operators are investigating the use of unlicensed spectrum in addition to their licensed spectrum. Already today it is possible to integrate WiFi networks into the core networks and perform offloading of traffic to a WiFi network. In future releases of 3GPP a tighter interaction of WiFi and LTE is foreseen that will even enable the aggregation of these two technologies. The design and performance analysis of different offloading and aggregation strategies is a difficult task, because of the different radio access technologies (RAT) involved. One widely used tool for the analysis of such systems is stochastic geometry, but most existing works do not take the heterogeneity of the different RATs into account. This work models the LTE and WiFi networks as well as the users using a probabilistic approach based on stochastic geometry and takes the particular physical layer properties of the two RATs into account. The proposed model allows for the performance analysis of those different networks using closed form expressions. Using these newly developed tools, we show that the max-throughput criterion, which takes the different characteristics of the two RATs into account, performs better than simple offloading and max-SINR association criteria.**

***Index Terms*—Stochastic Geometry; Heterogeneous Networks; LTE; WiFi; Aggregation; Offloading;**

## I. Introduction

Due to the overload of the macro-cell network, heterogeneous networks (HetNet) are one of the key aspects of next generation telecommunication systems. To offload the primary (macro-cell) network through a secondary (small-cell) one is a HetNet functionality that is expected to alleviate, at least temporarily, the problem of congestion in macro-cells.

One of the most promising HetNet scenarios, is the one of orthogonal tiers using LTE macro-cell in a licensed band and WiFi small-cell in unlicensed band [1]–[3]. To achieve optimal gain from aggregating two different networks a proper analysis of those is obligatory. In [4] the authors provide an approach to analyze the interference of a cellular network, not with the traditionally and computationally intensive grid-based simulations, but by modeling BSs positions as Poisson Point Process (PPP) and using tools from stochastic geometry.

The same framework was followed in many other studies in order to model heterogeneous networks. [5] provides an analysis for k-tier HetNets, [6] analyze heterogeneous cellular networks with carrier aggregation capabilities, [7] tackles the problem of offloading in heterogeneous networks, [8] presents load-aware modeling of heterogeneous cellular networks, [9] models downlink coverage probability in MIMO HetNets, [10] studies the problem of fractional frequency reuse for heterogeneous cellular networks, and [11] analyzes the backhaul at the heterogenous networks.

In this work we are using the same framework in order to do a large scale statistical analysis of different offloading and aggregation methodologies in two-tier HetNets using LTE and WiFi. We focus on downlink and assume that users are saturated, i.e., they will use all the resources they can get. Our contributions are: a) An analytical formula that captures the distribution of users' cardinality in an arbitrary cell, that gives insight about the probabilistic performance of the network; b) We propose an analytical model for LTE/WiFi HetNets that captures physical layer performance, providing statistics for coverage maps and MCS distributions and c) We use our analytical framework to study the impact of popular user association policies like *offload* (all users within range of a WiFi AP are associated to the WiFi network), *max SINR* (a user is associated with the BS offering the best SINR, among any tier), and *max RATE* (taking into consideration that different RAT achieving different rates for common SINR). Our preliminary results provide some interesting qualitative and quantitative insights.

Our model is described in section II. In section III we present the basic results of stochastic geometry, in order to calculate the probability of each modulation and coding scheme (MCS). In section IV, we provide a closed form solution for the distribution of the number of users (cardinality) at an arbitrary cell. In section V we model the PHY layer of our networks in order to compute the average bit rate for LTE and for WiFi users. Section VI provides the results of our analysis about the rate and the blocking probability for macro-cells and small-cells separately, as well as, the performance of aggregate those networks with different user's association criteria. We give conclusions in section VII.

## II. Our Model

The proposed model consists of three types of objects (eNodeBs, WiFi APs and Users), which are all modeled as homogeneous PPP ($\Phi_{\text{LTE}}$, $\Phi_{\text{WiFi}}$ and $\Phi_{\text{u}}$) with different densities ($\lambda_{\text{LTE}}$, $\lambda_{\text{WiFi}}$ and $\lambda_{\text{u}}$). So, assuming a density $\lambda$ for a given point set at a certain area $S$, the number points $N$ is a random variable following a Poisson distribution

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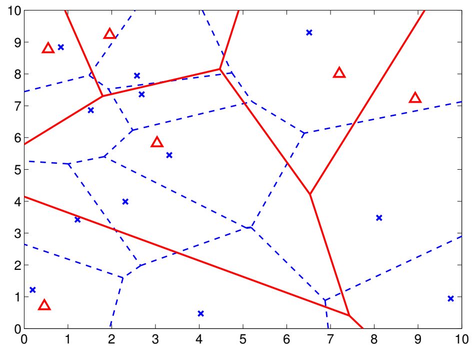

Fig. 1. Voronoi Tessellation example, with $\lambda_{\text{LTE}} = \frac{6}{100m^2}$ (red $\triangle$) and $\lambda_{\text{WiFi}} = \frac{12}{100m^2}$ (blue $\times$)

$$P(N = k \mid S) = \frac{(\lambda S)^k e^{-\lambda S}}{k!}, \quad k = 0, 1, \dots \quad . \tag{1}$$

Homogeneous means that after selecting the cardinality of points, their positions follows a uniform distribution at $2D$ space. To assume that users or WiFis are randomly placed at a certain area is a reasonable assumption, in contrast to the random placement of the eNodeB, which should be commented. The hexagonal or grid eNodeB topology is ideal but not representative of real networks topologies. On the other hand, fully random placement also has an unrealistic consequence that eNodeBs have a probability to end up asymptotically close. We can consider the aforementioned two topologies as ideal and worst case scenarios respectively in terms to interference. As shown in [4], the coverage probability in terms of the SINR threshold, in real BS deployments, lies in most cases roughly midway between the coverage probability in the two extreme cases above.

LTE and WiFi operate at different frequencies, thus they form two orthogonal networks. In other words, interference at each BS originates only from BSs of the same network. We assume different transmit powers for each network but the same for all BSs in the network. Moreover, the users association criterion is maximum SINR. Under the assumption that, on average, the received power is monotonic with respect to distance, our criterion is simplified to the closest distance criterion, so the BSs's coverage areas can be represented by Voronoi Regions (Tessellations) as shown in Fig. 1, where solid lines correspond to tessellations with respect to $\triangle$ network and dash lines to $\times$ network.

We assume that users are saturated, meaning that they always have load that has to be served and want the highest possible rate. The PPP captures this real life phenomenon, where the number of active users is not constant but a random variable. Regarding scheduling, we assume that BSs always give the total bandwidth to users and on average all associated users at specific BS will allocate the same amount of time-frequency resources.

Using this model we can derive two main performance indicators of our heterogeneous network:

*1) Average User Rate:* The average rate of an arbitrary user can be expressed as

$$\overline{Rate} = \sum_{mcs=0}^{M} \sum_{n=1}^{N} P(mcs)P(n)Rate(n, mcs, RAT) \ , \tag{2}$$

where

- $P(mcs)$ is the probability an arbitrary user at a random call to operates with each MCS; this pmf will be calculated at section III,
- $P(n)$ is the pmf of the users' cardinality $n$ at an arbitrary cell; this probability will be derived in section IV,
- $Rate(n, mcs, RAT)$ is the rate for each RAT for a given $mcs$ and $n$; this entity will be modeled in section V,
- $N$ is the maximum number of associated users at the same cell and $M$ is the highest $mcs$ order.

*2) Congestion Probability:* In order to define our second metric, we need to set a minimum threshold at user rate $TP_{min}$, below which users will not be associated with the BS. We can assume that the minimum throughput demand is the same for all users or a random variable with average equal to $TP_{min}$. The congestion probability can be expressed in two ways: a) User perspective, as the probability that the achievable user rate $TP_u$ is less than the minimum threshold

$$P^u_{cong} = P(TP_u < TP_{min}) \ . \tag{3}$$

b) operator perspective, as the percentage of the network that is congested. The maximum affordable number of associated users at a BS is $n_{max} = Rate_{BS}/TP_{min}$, where $Rate_{BS}$ is total rate of the BS, $Rate_{BS} = \sum \overline{Rate} \cdot n \cdot P(n)$. Finally, using the pmf of users' cardinality, congestion probability is

$$P^{BS}_{cong} = P(n > n_{max}) \ . \tag{4}$$

It can further be shown that $P^{BS}_{cong} = P^u_{cong}$.

## III. Distribution of MCS

This section defines the way to calculate the probability distribution of $mcs$. The steps to achieve that, are (1) to define the propagation model and the SINR, (2) to compute the coverage probability w.r.t. a power threshold and, (3) to obtain the $mcs$ distribution.

### A. Propagation Model

The standard power loss propagation model is used. We assume a path loss exponent $\alpha > 2$, only a Rayleigh fading at the channel with mean 1 and constant transmit power of $P_{\text{tx}} = \mu$. So, the received power at distance $d$ from a BS is given by $P_{\text{rx}} = P_{\text{tx}} h d^{-\alpha}$ where $h$ follows an exponential distribution, $h \sim \exp(1)$. So SINR, for a user that is associated with the $i$-th BS is given by

$$SINR_i = \frac{P_{\text{rx}_i}}{\sum\limits_{n \neq i} P_{\text{rx}_n} + \sigma^2} \ , \tag{5}$$

where sigma is calculated w.r.t. bandwidth $(BW)$ from $\sigma^2_{\text{dBm}} = -174 + 10\log_{10}(BW)$ [12].

### B. Coverage Probability

In [4] authors present an approach for coverage probability of randomly located users, if the BSs are arranged according to homogeneous PPP. For completeness, we will rewrite those results which are applicable to the problem. $T$ is the SINR threshold for the coverage, $\lambda$ the density of the BSs and $\alpha$ is the path loss constant. The coverage probability is given by

$$p_c(T,\lambda,\alpha) \triangleq \mathbb{P}[SINR > T]$$
$$p_c(T,\lambda,\alpha) = \pi\lambda \int_0^{\infty} e^{-\pi\lambda u(1+\beta(T,\alpha)) - \frac{1}{\mu}T\sigma^2 u^{\alpha/2}} du \; , \tag{6}$$

where $\beta(T,\alpha) = T^{2/\alpha} \int_{T^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}} du$.

If we assume that additive noise is negligible w.r.t. interference, the equation (6) can be significantly simplified as $p_c(T,\lambda,\alpha) = 1/(1+\beta(T,\alpha))$. Furthermore, if we assume that $\alpha = 4$, we obtain a closed form solution

$$p_c(T,\lambda,4) = \frac{1}{1+\sqrt{T}\left(\pi/2 - \arctan(1/\sqrt{T})\right)} \; . \tag{7}$$

### C. Probability distribution of MCS

For a given RAT, each $mcs$ corresponds to an SNR threshold (for a minimum error rate), lets denote it as $\tau$. So $\tau_i$ is the the SNR threshold to operate with $mcs_i$ for a target Block Error Rate or Packet Error Rate (BLER, PER). We model the interference as Gaussian noise, so SNR and SINR thresholds are the same. Finally the probability for each $mcs$ is given by

$$p(mcs_i,\lambda,\alpha) = p_c(\tau_i,\lambda,\alpha) - p_c(\tau_{(i+1)},\lambda,\alpha) \; . \tag{8}$$

## IV. Cardinality Distribution of Poisson Point Process at Poisson Voronoi Tessellations

In this section we study the following problem: Let two independent sets (one for the BSs, for eNodeB or WiFi, and one for users), $\Phi_{\text{BS}}$ and $\Phi_{\text{u}}$ follow an homogeneous PPP having different densities ($\lambda_{\text{BS}}$, $\lambda_{\text{u}}$) in two dimensional space, assuming that Voronoi Tessellations (cells) are generated w.r.t. $\Phi_{\text{BS}}$. The probability distribution of $\Phi_{\text{u}}$, cardinality $N_{\text{u}}$, lying at same arbitrary cell is given by

$$P(N_{\text{u}} = k) = \frac{343}{k!15}\sqrt{\frac{7}{2\pi}} \frac{\rho^k}{(\rho+\frac{7}{2})^{k+\frac{7}{2}}}\Gamma(k+\frac{7}{2}) \; , \tag{9}$$

where $\rho = \frac{\lambda_{\text{u}}}{\lambda_{\text{BS}}}$ and $\Gamma$ is the gamma distribution. For paper's brevity and clarity, proof is given in our technical report [13]. Figure 2 depict the PDF for different values of ratio $\rho$.

First and second moments of the distribution are

$$\langle k \rangle = \rho \qquad \text{and} \qquad \text{var}_k = \rho + \frac{2}{7}\rho^2 \; . \tag{10}$$

From equation (10), we observe that the mean of the number of users within a cell drops linearly but the variance drops quadratically w.r.t. the density of deployed BSs. The coefficient variation is greater than 1. So the relative variance of points in a Voronoi cell is not decreasing by the rising of BS density.

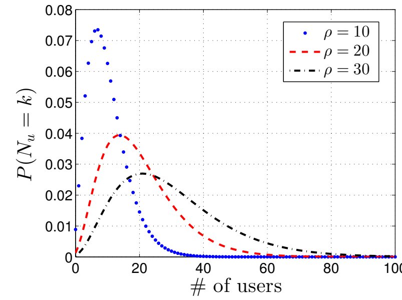


Fig. 2. PDF for different values of ratio $\rho = \frac{\lambda_{\text{u}}}{\lambda_{\text{BS}}}$

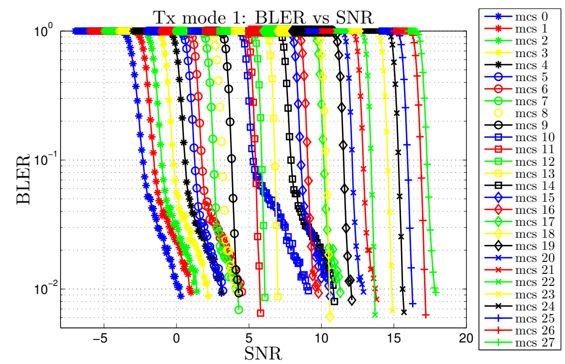


Fig. 3. BLER vs SINR

## V. Bit Rate for each RAT

This Section models LTE and WiFi RATs and computes the average data Rate for given $mcs$ and $n$ associated users. This work suppose 20 MHz eNodeB with a single antenna and a 802.11$n$ single stream with 20 MHz bandwidth WiFi.

### A. LTE

From the OpenAirInterface LTE downlink simulator [14], Block Error Rate (BLER) vs SNR, for LTE Tx mode 1 (downlink use Single-antenna port, the port 0) [15], is generated for each $mcs$ and shown in Fig. 3. So, for a given BLER threshold (commonly at $10^{-1}$) the SINR threshold ($\tau$) for each $mcs$ can be specified and therefore, from equation (8) the probability of each $mcs$ can be calculated.

LTE uses OFDM on the DL and divides the total frequency and time resources into resource blocks (RB). The size of a RB is 180 kHz in the frequency domain and 0.5 ms in the time domain.

In the 20 MHz bandwidth configuration there are 100 RB (also plus some white spaces for system's robustness to intra-cell interference). Each two RBs are grouped into one subframe with period one Transmission Time Interval (TTI), 1ms. We assume that all users will be allocated the same amount of subframes on average, so for a given number of associated users ($n$) at an eNodeB, each of them will be allocated $\frac{10^5}{n}$ subframes per second.

For a given $mcs$, LTE PHY specification 36.213, section 7.1.7.1 maps the index of the MCS to the index of the Transfer

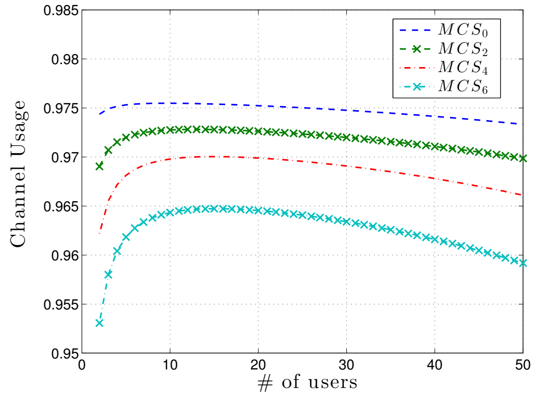


Fig. 4. MAC performance of 802.11n/ac Frame Aggregation

Block Size (I-TBS) for Downlink, which, together with the number of RBs, defines the amount of transferred bits per TTI (section 7.1.7.2.1).

### B. WiFi

For WiFi 802.11n and ac, for each $mcs$ we can extract the SINR threshold ($\tau$) from [16] and the physical data rate $PhyRate(msc)$ from [17]. Collisions, unused periods, overhead, etc are taken into account by using an expansion of Bianchi's model [18], which was extended in [19] to include newer techniques of 802.11n and ac (frame aggregation, block of ACKs, RTS/CTS, etc) that raise the utility of the MAC layer for high throughput cases. The percentage of the successful channel usage / normalized system throughput (% channel usage) w.r.t. number of users $n$ and different rates shown at Fig. 4. As we can see, for a reasonable number of connected users the performance of the MAC layer is roughly the same for a given MCS.

So the average user throughput is given by

$$Rate(msc, n) = \frac{P_s(n, Rate)}{n} PhyRate(msc) \; . \qquad (11)$$

Where $P_s$ is percentage of successful transmission, as it shown at Fig. 4.We can take into account the frequency reuse of the WiFi network by modeling it as 4 orthogonal WiFi networks with the same density, and calculate the new MCS distribution $f'_{\text{MCS}}$ combing the pmf $f_{\text{MCS}}$ and the assumption that each user will be connected at the WiFi network that provides the higher MCS.

## VI. Results

Our results present: (a) rate vs SINR for each RAT as output PHY modeling procedure, (b) the coverage probability of each RAT and (c) how the networks are scaling w.r.t. users density, for two cases. First, each network separately and second, different aggregation schemes of those networks. Table I shows the model parameters which where used at this section.

TABLE I
Model Parameters

| | |
|---|---|
| LTE $P_{\text{tx}}$ | 43 dBm |
| WiFi $P_{\text{tx}}$ | 20 dBm |
| $\alpha_{\text{LTE}} = \alpha_{\text{WiFi}}$ | 4 |
| $BW_{\text{LTE}} = BW_{\text{WiFi}}$ | 20 MHz |
| $\sigma^2$ | -100 dBm |
| # of antennas per eNodeB | 1 |
| # of spatial streams per WiFi | 1 |

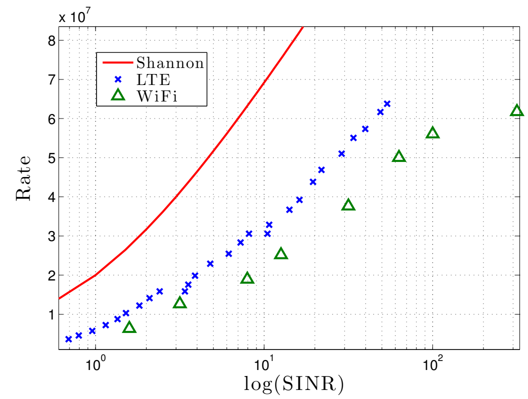


Fig. 5. Comparison between RATs rates and Shannon's limit

### A. Rate and MCS

Actual RATs do not provide an elegant way to calculate the user's rate, so it is common, when analyzing wireless networks to use the Shannon's theorem, as it constitutes a more simplified approach. As we mention at the introduction, when a single network been analyzed, this assumption does not affect the validity of the qualitative results. However, in the case of HetNets, and especially when HetNets operating with different RAT, this assumption does not hold. The user's rate of different RATs does not scale with the same way, with respect to SINR.

For instance, Fig. 5 presents our first result that is the output of the PHY layer modeling procedure that presented at section V, every marker corresponds to an MCS and the $x$-coordinates of them are the SINR threshold $\tau_i$ and the $y$-coordinates are the corresponding rates $rate_i$.

LTE performs 37%, on average, closer to Shannon than the WiFi, at their common operating SINR range. Thus, for those HetNets, if the rates of both networks modeled according to Shannon's theorem, WiFi will be overestimated compared to LTE.

### B. Coverage Probability

At this point we should discuss about coverage probability. Taking into account the SINR thresholds, as we obtained them at the aforementioned PHY layer modeling procedure, we are able to specify the percentage of users that are not capable ($SINR < \tau_0$) to associate at each RAT.

So, an arbitrary user is able to be connected at LTE and WiFi network respectively with probability $P_{cov_{\text{LTE}}} \simeq 0.67$

and $P_{cov_{\text{WiFi}}} \simeq 0.47$ respectively. The coverage probability for LTE is unacceptable low, but this originated from our worst case assumptions which are: a) random topology of BSs, so there is a probability that a interfering BS to be asymptotically close to the one that the user is associated; and b) saturated BSs, which means that all BSs are radiating continuously so they cause the maximum interference. We should comment that this unrealistic consequence of the homogeneous PPP topology was not so clear due to Shannon's formula; where a large amount of users are operating with low rate. Finally, the coverage probability is independent from the network density when the additive noise is negligible with respect to the interference [4].

### *C. Scaling*

In the sequel, we investigate network performance for two different perspectives, the "user" perspective where the performance metric is the average user rate and how it scales w.r.t. user density; and the "operator" perspective where the performance metric is the percentage of network that is congested w.r.t. user's average demand. Firstly, we present the the performance of each network separately. Then, we consider the case that LTE and WiFi networks can be combined, more specific, the aggregation scenarios are: (a) offload association scenario, where if the user is able to establish connection with the WiFi network, he does it without any further criterion, (b) users are associated with the tier that provides the best SINR and (c) users are associated with the tier that provides the higher throughput.

We made two basic assumptions about resources allocation for all scenarios, one about the demand of users and one for the BSs scheduler. First, we assume that each connected user is saturated, other words, he has infinite demand and allocates as much resources is possible. Second, we assume that the schedulers of all RATs on the long run, could represented as a resource fair scheduler. For the case of WiFi this assumption is not so accurate taking into account that WiFi currently operates worst than resource fair, [20]. But there are works which show that WiFi with simple modifications can upgrade its performance resource fair scheduler [21], [22].

*1) Separate LTE and WiFi Network:* Assuming that eNodeBs and WiFi networks density is equal to $1/\text{km}^2$, Fig. 6 depicts how the average user rate (Mbit/s) scales w.r.t. users density for LTE, WiFi and WiFi with frequency reuse. It can be seen that LTE performs better than WiFi w.r.t. average user rate. Additionally, WiFi with frequency reuse performed slightly better than LTE, but this gain is not so much if we take into account that operates with 4 times more bandwidth. The same conclusions for congestion probability w.r.t. user demand can be obtained by Fig. 7, where we suppose constant density of users $\lambda_u = 40$.

*2) Aggregation case:* In this subsection we examine the cooperation of LTE and WiFi network. As previous mentioned, three basic association criteria will be investigated, offload, maxSINR and maxThroughput criteria. In Fig. 8 we have set the densities of both networks equal to $\lambda_{LTE} = \lambda_{WiFi} = 1$

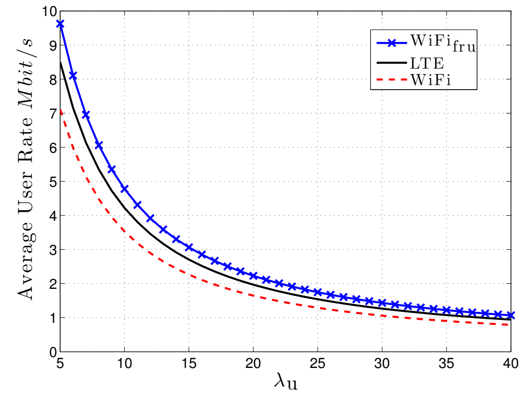


Fig. 6. LTE, WiFi and WiFi with frequency reuse performance w.r.t. users density

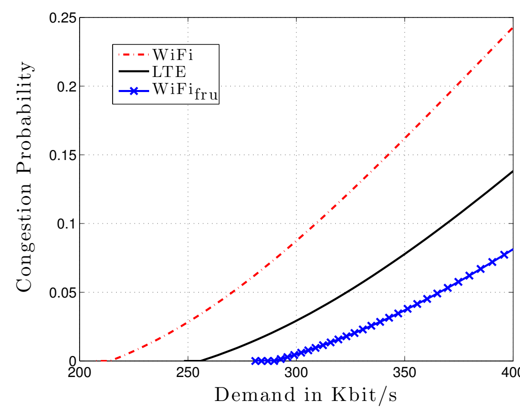


Fig. 7. Congestion probability of LTE, WiFi and WiFi with frequency reuse w.r.t. mean user demand

and shows the comparison between those different association criteria w.r.t. users density. The gain of maxSINR association compared with the offload association is mentionable. Additionally, there is a slight gain of max Throughput compared to maxSINR, due to different RATs the providing rate of a tier could be higher than the other at the same SINR. For comparison we have also plotted and the performance of single LTE network in Fig. 8.

In Fig. 9, we set the users density equal to $\lambda_u = 80$, the one of the LTE BS equal to $\lambda_{BS} = 1$, and we show the performance of each association criteria w.r.t. the WiFi density. The conclusions are the same as the previous figure, furthermore, the average user rate is scales linearly w.r.t. WiFi density.

Fig. 10 depicts the congestion probability for the aggregated network w.r.t. average user demand, for $\lambda_{LTE} = \lambda_{WiFi} = 1$ and $\lambda_u = 40$. Again, the offload scenario performs worse compared to the more complex ones.

## VII. Conclusions and Future Work

This work models the topology and the RAT for LTE and WiFi networks for the case of saturated users and presents

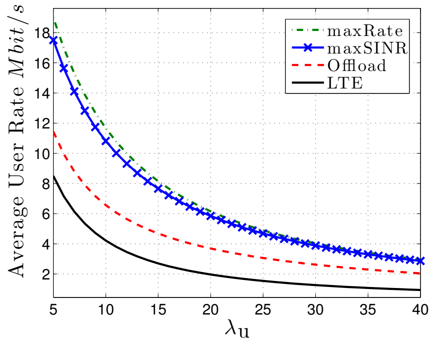


Fig. 8. Indoor scenario for different shadow fading values

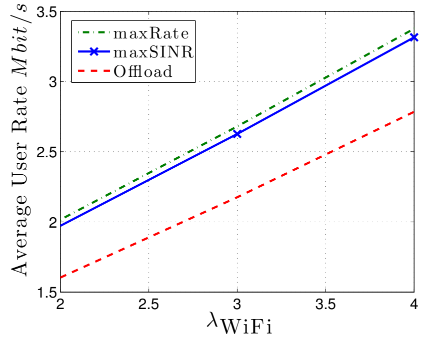


Fig. 9. Outdoor scenario for different shadow fading values

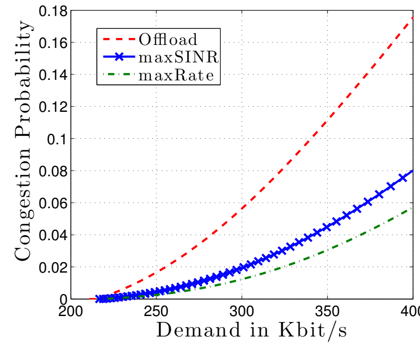


Fig. 10. Indoor scenario for various pilot placement cases and shadow fading equal to 8dB

a frame work to analyze network performance w.r.t. users and BS density. An analysis for widely used cooperative schemes presented as well. Between the cooperative schemes, maxSINR association could be more complex compared with the offload criterion but offers a large gain at average user rate and even more at congestion probability of the network, this gain could be extend if we take into account the different RAT and to associate with max Throughput criterion. In future work we will model the schedulers of each RAT as a queuing model, and to investigate the performance of different association criteria with dynamic flows and not for saturated users. Additionally, we have to examine the performance gain if we take into account the carrier aggregation capabilities.

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