WEIGHTED SUM RATE MAXIMIZATION OF CORRELATED MISO INTERFERENCE BROADCAST CHANNELS UNDER LINEAR PRECODING: A LARGE SYSTEM ANALYSIS

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ABSTRACT

The weighted sum rate (WSR) maximizing linear precoder algorithm is studied in large correlated multiple-input single-output (MISO) interference broadcast channels (IBC). We consider an iterative WSR design which exploits the connection with Weighted sum Minimum Mean Squared Error (WMMSE) designs as in [1], [2] and [3], focusing on the version in [1]. We propose an asymptotic approximation of the signal-to-interference plus noise ratio (SINR) at every iteration. We also propose asymptotic approximations for Matched Filter (MF) precoders. Simulations show that the approximations are accurate, especially when the channels are correlated.

Index Terms— random matrix theory, beamforming, weighted sum rate maximization

1. INTRODUCTION

We consider the MISO IBC with linear precoding at the transmitter. In this case, we have C base stations (BS), each one of them is endowed with M antennas, whereas the K users of each cell c $\in 1, 2, ...C$ have single-antenna receivers. The precoding matrix that maximizes (local optimum) the WSR for IBC is obtained from an iterative algorithm proposed by Luo et al. and Slock et al. in [1] and [2] respectively which is called the IBC WMMSE algorithm.

In this contribution, we carry out a large system analysis of this latter. Herein, we extend the work based on [3] and presented in [4], that presents the deterministic equivalent expressions of the SINR of the WMMSE iterative algorithm for broadcast channels (BC) in [3], and we inspire from the works in [5] and [6] which present Massive MISO deterministic equivalents of the SINR corresponding to the sub-optimal zero-forcing (ZF) and regularized zero-forcing (RZF) precoders and , all for large M and K. Although our work will inspire from the works in [5] and [6] and will be an extension of the work in [4], however it is not straightforward and needs careful attention as concerning the impact of inter-cell interference. Other works on large systems exist, e.g. [7], [8], [9], [10] and [11], where a multi cell RZF denoted iaRZF is presented in [8], this latter maximizes the sum rate as our precoder does but is not optimal for all existing scenarios, e.g. the scenario where many users are located on the cell edges, in fact, it corresponds to an optimal beamforming only in the case of identical intra-cell channel attenuation and identical inter-cell channel attenuation. Now a footnote in Roman* Algorithms that minimize the total transmit power for large systems are presented in [9], [10] and [11], however, they are different than the WMMSE approach that maximizes the total sum rate instead of minimizing the total power. Furthermore, the deterministic limits of the SINRs corresponding to the iterative IBC WMMSE process leading to the optimal WSR are presented, which makes it possible to evaluate its performance more easily and compare with other algorithms and precoders. Simulations show that the proposed SINR approximation is close to the real performance, i.e. the performance of the IBC WMMSE algorithm. Notation: The operators $()^H, tr(.)\ and\ E[.]$ denote conjugate transpose, trace and expectation, respectively. The $M \times M$ identity matrix is denoted I_M and $\log(.)$ is the natural logarithm.

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2. SYSTEM MODEL

In the following, we analyze a cellular downlink IBC MISO scenario where C cells are presented, c=1...C, each of the C cells consists of one BS associated with a number K of single-antenna receivers. We assume transmission on a single narrow-band carrier. the received signal $y_{c,k}$ at the kth user in cell c reads

$$y_{c,k} = \sum_{m=1}^{C} \sum_{l=1}^{K} h_{m,c,k}^{H} g_{m,l} s_{m,l} + n_{c,k} \tag{1}$$

where the user symbols are chosen from a Gaussian codebook, i.e, $s_{m,l} \sim \mathcal{NC}(0, 1)$, are linearly precoded and form the transmit signal; $g_{m,l} \in \mathbb{C}^M$ is the precoding vector of user l of cell m, $h_{m,c,k}^H \in \mathbb{C}^{1\times M}$ is the channel vector from the mth transmitter to the kth user of cell c, and the $n_{c,k}$ are independent complex Gaussian noise terms with zero mean and variance σ^2. Moreover, the precoders are subject to an average power constraint and the channel $h_{i,c,k}^H$ is correlated as
$\mathbb{E}\left[h_{i,c,k} h_{i,c,k}^H\right] = \Theta_{i,c,k}$ thus

$$h_{i,c,k} = \sqrt{M}\Theta_{i,c,k}^{1/2} z_{i,c,k} \tag{2}$$

$$trG_cG_c^H \preceq P_c\ for\ c \in \mathcal{C} \tag{3}$$

where $\mathcal{C}$ is the set of all BSs, $z_{i,c,k}$ has i.i.d. complex entries of zero mean and variance $\frac{1}{M}$ and the $\Theta_{i,c,k}^{1/2}$ is the Hermitian square-root of $\Theta_{i,c,k}$. The correlation matrix $\Theta_{i,c,k}$ is non-negative Hermitian and of uniformly bounded spectral norm w.r.t. to M. For notational convenience, we denote $\Theta c, c, k$ as $\Theta c, k$.

$G_c = [g_{c,1}, g_{c,2}, ..., g_{c,K}] \in \mathbb{C}^{M\times K}$ is the precoding matrix and P_c is the total available transmit power of cell c.

Under the assumption of optimal single-user decoding and perfect Channel State Information (CSI) at the transmitters and receivers, the achievable rate of the kth user of cell c is given by

$$R_{c,k} = \log(1 + \gamma_{c,k}) \tag{4}$$

$$\gamma_{c,k} = \frac{|h_{c,c,k}^H g_{c,k}|^2}{\sum_{(m,l)\neq(c,k)} h_{m,c,k}^H g_{m,l} g_{m,l}^H h_{m,c,k} + \sigma^2} \quad (5)$$

where $\gamma_{c,k}$ is the SINR of the kth of cell c.
The precoders maximize the WSR of all users so we are facing an optimization problem which is the following

$$\begin{aligned} G^* =& arg \max_{G} \sum_{c=1}^{C} \sum_{k=1}^{K} u_{c,k} R_{c,k} \\ & s.t. \; trG_c G_c^H \leq P_c \; for \; c \in \mathcal{C} \end{aligned} \quad (6)$$

where G is the short notation for $\{G_c\}_{c\in\mathcal{C}}$ and where $u_{c,k} \geq 0$ is the weight of the k^{th} user of cell c. The optimization problem in (6) is hard to solve directly, since it is highly non convex in the precoding matrix G. To solve the problem in (6), we consider the virtual linear receive filters $a_{c,k} \in \mathbb{C}$, the error variance $e_{c,k}$ after the linear receive filtering, given in (8), and we introduce additional weighting scalars $w_{c,k}$, so that the utility function (6) can be modified and an equivalent optimization problem can be formulated as in [1] and [2]

$$\begin{aligned} &\{G^*, \{a_{c,k}^*\}, \{w_{c,k}^*\}\} = \\ &arg \min_{G,\{a_{c,k}\},\{w_{c,k}\}} \sum_{(c,k)} w_{c,k} e_{c,k} - u_{c,k} \log(u_{c,k}^{-1} w_{c,k}) \\ &s.t. \; trG_c G_c \leq P_c \; for \; c \in \mathcal{C} \end{aligned} \quad (7)$$

with

$$e_{c,k} = E[(a_{c,k} y_{c,k} - s_{c,k})(a_{c,k} y_{c,k} - s_{c,k})^H]. \quad (8)$$

Denote $\rho_c = \frac{P_c}{\sigma^2}$, the signal-to-noise ratio (SNR) in cell c. From (7), and after applying alternating optimization techniques, the precoders are obtained as the following

$$a_{c,k}^* = g_{c,k}^H h_{c,c,k} (\sigma^2 + \sum_{m=1}^{C} \sum_{l=1}^{K} h_{m,c,k}^H g_{m,l} g_{m,l}^H h_{m,c,k})^{-1} \quad (9)$$

$$e_{c,k}^* = (1 + \gamma_{c,k})^{-1} \quad (10)$$

$$w_{c,k}^* = u_{c,k} (e_{c,k}^*)^{-1} \quad (11)$$

$$\widetilde{g}_{c,k}^* = (H_c^H D H_c + \frac{trD_c}{\rho_c} I_M)^{-1} h_{c,c,k} a_{c,k}^H w_{c,k} \quad (12)$$

where $g_{c,k}^* = \xi_c \widetilde{g}_{c,k}^*$ with $\xi_c = \sqrt{\frac{P_c}{tr\widetilde{G_c}^* \widetilde{G_c}^{*H}}}$. Also we defined $W_c = diag(w_{c,1}^*, ..., w_{c,K}^*)$, $A_c = diag(a_{c,1}^*, ..., a_{c,K}^*)$, $D_c = A_c^H W_c A_c$, and
$A = diag(A_1, A_2, ...A_C)$, $D = diag(D_1, D_2, ..., D_C)$,
$H_c = [h_{c,1,1}, ..., h_{c,1,K}, h_{c,2,1}, \dots, h_{c,2,K}, \dots, h_{c,C,K}]^H \in \mathbb{C}^{KC\times M}$ is the compound channel. For notational convenience, we drop the superscript* in the sequel. Subsequently $a_{c,k}$ and $w_{c,k}$ are computed, which then constitute the new precoder $g_{c,k}$. This process is repeated until convergence to a local optimum.

3. LARGE SYSTEM ANALYSIS

In this section, performance analysis is conducted for the proposed precoder. The large-system limit is considered, where M and K go to infinity while keeping the ratio K/M finite such that $limsup_M K/M < \infty$ and $liminf_M K/M > 0$. The results should be understood in the way that, for each set of system dimension parameters M and K we provide an approximate expression for the SINR and the achieved sum rate, and the expression is tight as M and K grow large. Before we continue with our performance analysis of the above precoder, a deterministic equivalent of the SINR of the MF precoder is required. All vectors and matrices should be understood as sequences of vectors and matrices of growing dimensions.

3.1. Deterministic Equivalent of the SINR for the MF

Our precoder must me initialized in some way, so we have chosen the MF precoder to do the job.
Theorem 1: Let $\gamma_{c,k}^{MF}$ be the SINR of user k under MF precoding, i.e., $G_c = \frac{\xi_c}{M} H_{\hat{c}}^H$ then, $\gamma_{c,k}^{MF} - \overline{\gamma}_{c,k}^{MF} \xrightarrow{M\to\infty} 0$, almost surely, where $H_{\hat{c}} = [h_{c,c,1}, ..., h_{c,c,K}]^H$ and

$$\overline{\gamma}_{c,k}^{MF} = \frac{1}{\frac{1}{\beta_c \rho_c} + \frac{1}{M^2} \sum_{(l,i)\neq(c,k)} tr\Theta_{l,c,k}\Theta_{l,i}} \quad (13)$$

Proof: The normalization parameter is $\xi_c = \sqrt{\frac{P_c}{\frac{1}{M^2} trH_{\hat{c}}^H H_{\hat{c}}}}$, where and thus we have

$$\overline{\xi_c} = \sqrt{\frac{P_c}{\frac{1}{M^2}\sum_{k=1}^{K} tr\Theta_{c,k}}} = \sqrt{\beta_c P_c} \quad (14)$$

Denote $P_{c,k} = \|g_{c,k}^H h_{c,c,k}\|^2$ the signal power of the k^{th} user of cell c. Applying [6, Lemma 2.7] we have $\frac{1}{M} h_{c,c,k}^H h_{c,c,k} - 1 \xrightarrow{M\to\infty} 0$ and hence

$$\overline{P}_{c,k} = \overline{\xi_c^2} = \beta_c P_c \quad (15)$$

The interference is $\frac{\xi_c^2}{M} \sum_{m=1}^{C} z_{m,c,k}^H \Theta_{m,c,k}^{1/2} H_{\hat{m}}^H H_{\hat{m}} \Theta_{m,c,k}^{1/2} z_{m,c,k}$, where $H_{\hat{m},[k]} = [h_{m,m,1}, ..., h_{m,m,k-1}, h_{m,m,k+1}, ...h_{m,m,K}]^H$. Now we apply again [6,Lemma 2.7] since $\frac{1}{M}\Theta_{m,c,k}^{1/2} H_{\hat{m}}^H H_{\hat{m}} \Theta_{m,c,k}^{1/2}$ and $\frac{1}{M}\Theta_{c,k}^{1/2} H_{\hat{c},[k]}^H H_{\hat{c},[k]} \Theta_{c,k}^{1/2}$ have uniformly bounded spectral norm w.r.t M almost surely, and obtain

$$\begin{aligned} &[\frac{1}{M} \sum_{m=1,m\neq c}^{C} z_{m,c,k}^H \Theta_{m,c,k}^{1/2} H_{\hat{m}}^H H_{\hat{m}} \Theta_{m,c,k}^{1/2} z_{m,c,k} \\ &\qquad + \frac{1}{M} z_{c,c,k}^H \Theta_{c,k}^{1/2} H_{\hat{c},[k]}^H H_{\hat{c},[k]} \Theta_{c,k}^{1/2} z_{c,c,k}] \\ &-[\frac{1}{M^2} \sum_{m\neq c} \sum_{i=1}^{K} tr\Theta_{m,c,k}\Theta_{m,i} + \frac{1}{M^2} \sum_{i\neq k} tr\Theta_{c,k}\Theta_{c,i}] \to 0 \end{aligned} \quad (16)$$

almost surely. Substituting the terms in (5) by their respective deterministic equivalents yields (13), which completes the proof.

3.2. Deterministic equivalent of the SINR of proposed precoder for correlated channels

For the precoder (12), a deterministic equivalent of the SINR is provided in the following theorem
Theorem 2: Let $\gamma_{c,k}$ be the SINR of the kth user of cell c with the precoder defined in (12). Then, a deterministic equivalent $\overline{\gamma}_{c,k}^{(j)}$ at iteration $j > 0$ and under MF initialization, is given by $\overline{\gamma}_{c,k}^{(j)}$ is given by

$$\overline{\gamma}_{c,k}^{(j)} = \frac{\overline{w}_{c,k}^{(j)} (\overline{m}_{c,k}^{(j)})^2}{\overline{\Upsilon}_{c,k}^{(j)} + \overline{\hat{\Upsilon}}_{c,k}^{(j)} + \overline{d}_{c,k}^{(j)} \frac{\overline{\Psi}_c^{(j)}}{\rho_c} (1 + \overline{m}_{c,k}^{(j)})^2} \quad (17)$$

where

$$\overline{m}_{c,k}^{(j)} = \frac{1}{M} tr \overline{\Theta}_{c,k}^{(j)} V_c \tag{18}$$

$$\overline{\Psi}_c^{(j)} = \frac{1}{M} \sum_{i=1}^{K} \frac{\overline{w}_{c,i}^{(j)} e_{c,i}^{'}}{(1+e_{c,i})^2} \tag{19}$$

$$\overline{\Upsilon}_{c,k}^{(j)} = \frac{1}{M} \sum_{l=1,l\neq k}^{K} \frac{1}{(1+\overline{m}_{c,l}^{(j)})^2} e_{c,c,k,c,l}^{'} \tag{20}$$

$$\overline{\hat{\Upsilon}}_{c,k}^{(j)} = \frac{1}{M} \sum_{m=1,m\neq c}^{C} \frac{(1+\overline{m}_{c,k}^{(j)})^2}{(1+\overline{m}_{m,c,k}^{(j)})^2} \sum_{l=1}^{K} \frac{1}{(1+\overline{m}_{m,l}^{(j)})^2} e_{m,c,k,m,l}^{'} \tag{21}$$

with $\overline{\Theta}_{m,c,k} = d_{c,k}\Theta_{m,c,k}$, $\overline{m}_{m,c,k}^{(j)} = \frac{1}{M} tr \overline{\Theta}_{m,c,k}^{(j)} V_m$ and $\overline{a}_{c,k}^{(j)}, \overline{w}_{c,k}^{(j)}$ and $\overline{d}_{c,k}^{(j)}$ are given by

$$\overline{a}_{c,k}^{(j)} = \frac{1}{\sqrt{\overline{P}_{c,k}^{(j-1)}}} \frac{\overline{\gamma}_{c,k}^{(j-1)}}{1+\overline{\gamma}_{c,k}^{(j-1)}} \tag{22}$$

$$\sqrt{\overline{P}_{c,k}^{(j-1)}} = \frac{1}{\overline{a}_{c,k}^{(j-1)}} \sqrt{\frac{P}{\overline{\Psi}_c^{(j-1)}}} \frac{\overline{m}_{c,k}^{(j-1)}}{1+\overline{m}_{c,k}^{(j-1)}} \tag{23}$$

$$\overline{w}_{c,k}^{(j)} = (1+\overline{\gamma}_{c,k}^{(j-1)}) \tag{24}$$

$$\overline{d}_{c,k}^{(j)} = \overline{w}_{c,k}^{(j)} \overline{a}_{c,k}^{2,(j)}. \tag{25}$$

Denoting

$$V_c = (F_c + \overline{\alpha}_c I_M)^{-1} \tag{26}$$

with $\overline{\alpha}_c^{(j)} = \frac{tr \overline{D}_c^{(j)}}{M\rho_c}$, three systems of coupled equations have to be solved. First, we need to introduce $e_{m,c,k} \forall \{m,c,k\} \in \{\mathcal{C},\mathcal{C},\mathcal{K}_c\}$, where $\mathcal{K}_c$ is the set of all users of cell c, which form the unique positive solutions of

$$e_{m,c,k} = \frac{1}{M} tr \overline{\Theta}_{m,c,k} V_m, \tag{27}$$

$$F_m = \frac{1}{M} \sum_{j=1}^{C} \sum_{i=1}^{K} \frac{\overline{\Theta}_{m,j,i}}{1+e_{m,j,i}}. \tag{28}$$

$e_{c,c,k}$ and $m_{c,c,k}$ denote $e_{c,k}$ and $m_{c,k}$ respectively. Secondly,we give $e_{1,1}', ..., e_{1,K}', ...e_{C,1}', ..., e_{C,K}'$ which form the unique positive solutions of

$$e_{c,k}' = \frac{1}{M} tr \overline{\Theta}_{c,k} V_c (F_c' + I_M) V_c, \tag{29}$$

$$F_c' = \frac{1}{M} \sum_{j=1}^{C} \sum_{i=1}^{K} \frac{\overline{\Theta}_{c,j,i} e_{j,i}'}{(1+e_{c,j,i})^2}. \tag{30}$$

And finally, we provide $e_{m,c,k,m,l}' \forall \{m,c,k,l\} \in \{\mathcal{C},\mathcal{C},\mathcal{K}_c,\mathcal{K}_c\}$ which form the unique positive solutions of

$$e_{m,c,k,m,l}' = \frac{1}{M} tr \overline{\Theta}_{m,c,k} V_m (F_{m,m,l}' + \overline{\Theta}_{m,l}) V_m \tag{31}$$

$$F_{m,m,l}' = \frac{1}{M} \sum_{j=1}^{C} \sum_{i=1}^{K} \frac{\overline{\Theta}_{m,j,i} e_{m,j,i,m,l}'}{(1+e_{m,j,i})^2}. \tag{32}$$

For $j = 0, \overline{\gamma}_{c,k}^{(0)} = \overline{\gamma}_{c,k}^{MF}$, given by Theorem 1 and $\overline{P}_{c,k}^{(0)} = \beta_c P_c$, cf. (15).

Proof: For $j \geq 1$, define $\Gamma_c^{(j)} = \frac{1}{M} H_c^H \overline{D}^{(j)} H_c + \overline{\alpha}_c^{(j)} I_M$, the precoder at the end of iteration j is given by

$$\overline{g}_{c,k}^{(j)} = \frac{\xi_c^{(j)}}{M} (\Gamma_c^{(j)})^{-1} h_{c,c,k} \overline{a}_{c,k}^{H,(j)} \overline{w}_{c,k}^{(j)} \tag{33}$$

for each user k in the cell c,
where $\xi_c^{(j)}$ is

$$\xi_c^{(j)} = \sqrt{\frac{P_c}{\frac{1}{M^2} tr(\Gamma_c^{(j)})^{-2} H_{\hat{c}}^H \overline{A}_c^{H,(j)} \overline{W}_c^{2,(j)} \overline{A}_c^{(j)} H_{\hat{c}}}} \tag{34}$$

$$= \sqrt{\frac{P_c}{\Psi_c^{(j)}}}. \tag{35}$$

We derive the deterministic equivalents of the normalization term $\xi_c^{(j)}$, the signal power $|\overline{g}_{c,k}^{H,(j)} h_{c,c,k}|^2$ and the interference power $\sum_{m=1}^{C} \sum_{l\neq k \ if \ m=c}^{K} h_{m,c,k}^H \overline{g}_{m,l}^{(j)} \overline{g}_{m,l}^{H,(j)} h_{m,c,k}$ similarly to [4], [5] and [6], i.e, using the same logic and mathematical approach, but for a more complex problem. We will show that in the following.

a) Power normalization: The term $\Psi_c^{(j)}$ can be written as

$$\Psi_c^{(j)} = \frac{1}{M^2} \sum_{k=1}^{K} \overline{w}_{c,k}^{(j)} \overline{d}_{c,k}^{(j)} z_{c,c,k}^H \Theta_{c,k}^{(1/2)} (\Gamma_c^{(j)})^{-2} \Theta_{c,k}^{(1/2)} z_{c,c,k} \tag{36}$$

$$= \frac{1}{M^2} \sum_{k=1}^{K} \overline{w}_{c,k}^{(j)} z_{c,c,k}^H \overline{\Theta}_{c,k}^{(1/2)} (\Gamma_c^{(j)})^{-2} \overline{\Theta}_{c,k}^{(1/2)} z_{c,c,k}. \tag{37}$$

Similarly to [4], [5] and [6] a deterministic equivalent $\overline{\Psi}_c$ such that $\Psi_c - \overline{\Psi}_c \xrightarrow{M\to\infty} 0$, almost surely, is given by

$$\overline{\Psi}_c^{(j)} = \frac{1}{M} \sum_{k=1}^{K} \overline{w}_{c,k}^{(j)} \frac{\frac{1}{M} tr \overline{\Theta}_{c,k}^{(j)} (\Gamma_c^{(j)})^{-2}}{(1+\frac{1}{M} tr \overline{\Theta}_{c,k}^{(j)} (\Gamma_c^{(j)})^{-1})^2} \tag{38}$$

$$= \frac{1}{M} \sum_{k=1}^{K} \overline{w}_{c,k}^{(j)} \frac{\overline{m}_{c,k}^{',(j)}}{(1+\overline{m}_{c,k}^{(j)})^2} = \frac{1}{M} \sum_{k=1}^{K} \overline{w}_{c,k}^{(j)} \frac{e_{c,k}^{'}}{(1+e_{c,k})^2}, \tag{39}$$

where we denote $\overline{m}_{c,k}^{(j)} = \frac{1}{M} tr \overline{\Theta}_{c,k}^{(j)} (\Gamma_c^{(j)})^{-1}$ and $\overline{m}_{c,k}^{',(j)}$ the derivative w.r.t z at $z = -\overline{\alpha}_c^{(j)}$.

b) Signal power: The square-root of the signal power $P_{c,k}^{(j)} = |\overline{g}_{c,k}^{H,(j)} h_{c,c,k}|^2$ is

$$\sqrt{P_{c,k}^{(j)}} = \xi_c^{(j)} \overline{a}_{c,k}^{(j)} \overline{w}_{c,k}^{(j)} z_{c,c,k}^H \Theta_{c,k}^{\frac{1}{2}} (\Gamma_c^{(j)})^{-1} \Theta_{c,k}^{\frac{1}{2}} z_{c,c,k} \tag{40}$$

$$= \frac{\xi_c^{(j)}}{\overline{a}_{c,k}^{(j)}} z_{c,c,k}^H \overline{\Theta}_{c,k}^{1/2} (\Gamma_c^{(j)})^{-1} \overline{\Theta}_{c,k}^{\frac{1}{2},(j)} z_{c,c,k}. \tag{41}$$

Again, following [4],[5],[6] a deterministic equivalent $\sqrt{\overline{P}_{c,k}^{(j)}}$ of (41) is given by

$$\sqrt{\overline{P}_{c,k}^{(j)}} = \frac{\overline{\xi}_c^{(j)}}{\overline{a}_{c,k}^{(j)}} \frac{\overline{m}_{c,k}^{(j)}}{1+\overline{m}_{c,k}^{(j)}}, \tag{42}$$

where $\overline{\xi}_c^{(j)} = \sqrt{\frac{P_c}{\overline{\Psi}_c^{(j)}}}$.

c) Interference power: The interference power received by user k of cell c can be written as

$$\sum_{m=1}^{C}\sum_{l=1,l\neq k\ if\ m=c}^{K} h^H_{m,c,k}\overline{g}^{H,(j)}_{m,l}\overline{g}^{H,(j)}_{m,l}h_{m,c,k} \tag{43}$$

$$=\frac{\xi_c^{2,(j)}}{M^2}\sum_{m=1}^{C} h^H_{m,c,k}(\Gamma_m^{(j)})^{-1}\times \tag{44}$$

$$\sum_{l=1,l\neq k\ if\ m=c}^{K}\overline{a}^{2,(j)}_{m,l}\overline{w}^{2,(j)}_{m,l}h_{m,m,l}h^H_{m,m,l}(\Gamma^{(j)}_m)^{-1}h_{m,c,k}$$

$$=\frac{\xi_c^{2,(j)}}{\overline{d}^{(j)}_{c,k}}\sum_{m=1}^{C} z^H_{m,c,k}\overline{\Theta}^{\frac{1}{2},(j)}_{m,c,k}(\Gamma_m^{(j)})^{-1}\times\sum_{l\neq k\ if\ m=c}^{K}$$

$$\overline{w}^{(j)}_{m,l}\overline{\Theta}^{\frac{1}{2},(j)}_{m,l}z_{m,m,l}z^H_{m,m,l}\overline{\Theta}^{\frac{1}{2},(j)}_{m,l}(\Gamma^{(j)}_m)^{-1}\overline{\Theta}^{\frac{1}{2},(j)}_{m,c,k}z_{m,c,k} \tag{45}$$

The term in (45) is approximated as the following

$$\sum_{m=1}^{C}\sum_{l=1,(l,m)\neq(k,c)}^{K} h^H_{m,c,k}\overline{g}^{H,(j)}_{m,l}\overline{g}^{H,(j)}_{m,l}h_{m,c,k} - \frac{\overline{\xi}_c^{2,(j)}[\overline{\Upsilon}^{(j)}_{c,k}+\overline{\hat{\Upsilon}}^{(j)}_{c,k}]}{\overline{d}^{(j)}_{c,k}(1+\overline{m}^{(j)}_{c,k})^2}\xrightarrow{M\rightarrow\infty}0, \tag{46}$$

almost surely, where $\overline{\Upsilon}^{(j)}_{c,k}$ and $\overline{\hat{\Upsilon}}^{(j)}_{c,k}$ are given by the expressions (20) and (21) which represent the large system limits of the intra-cell and inter-cell interference respectively, the proof is omitted due to lack of space.

3.3. Numerical Results

In this section, results of simulations based on realistic settings with a finite number of transmit antennas corroborate the correctness of the proposed approximation.We use the IBC WMMSE algorithm with MF initialization and compare it to the large system approximation in Theorem 2. The channel correlation matrix is modeled as [10]

$$[\Theta_{m,c,k}]_{ij} = \frac{1}{\Theta_{m,c,k,max}-\Theta_{m,c,k,min}}\int_{\Theta_{m,c,k,min}}^{\Theta_{m,c,k,max}} e^{\mathbf{j}\frac{2\pi}{\lambda}\delta_{ij}cos(\Theta)}d\Theta \tag{47}$$

where $\mathbf{j}=\sqrt{-1}$, λ denotes the signal wavelength and δ_{ij} is the distance between antenna i and j.We choose the range of azimuth angle $\Theta_{m,c,k}$ of user k as $\Theta_{m,c,k,min}=-\pi$ and $\Theta_{m,c,k,max}=\phi_{m,c,k}-\pi$, where $\phi_{m,c,k}=2\pi\frac{c*k}{KC}$. The transmitter is endowed with a uniform linear array (ULA) of antennas. We assume that δ_{ij} is independent of M so that the spectral norm of $\Theta_{m,c,k}$ remains bounded as M grows large, let $\delta_{ij}=\frac{\lambda}{2}|j-i|$. figures 1 and 2 show the WMMSE precoder and its approximation for correlated channels ($\Theta_{m,c,k}\neq I_M$) and i.i.d. channels ($\Theta_{m,c,k}=I_M$) for $C=2$ and $C=3$ respectively. For the simulations of the IBC WMMSE algorithm, we have used 200 channel realizations. It can be observed that for i.i.d channels the approximation is accurate for low SNR, but less precise at high SNR. As the figures 1 and 2 suggest, this effect is diminished when the channel is correlated resulting in an increased accuracy of the approximation for high SNR. Or for i.i.d channels the inaccuracy effect at high SNR diminishes when the system load ($\frac{C*K}{M}$) decreases as shown in the figure 3 for load = 0.9. The reason of imprecision for full load $\frac{C*K}{M}=1$ is that the regularization term in (26) is going to be imprecise at high SNR. Moreover, we observed that the sum rate of our system stay unmodified for a same total number of users (Fig. 1 and Fig. 2) while keeping in mind the fact that we have more total power budget as the number of transmitters increases. Finally, we demonstrated also that our asymptotic sum rate follows the simulated one; which validates our asymptotic approach. Although the sum rate expression for the approximation approach (17) seems to be complex, however we need to calculate it only once per a given SNR, while we need to run the IBC WMMSE simulations as many times as the number of channel realizations, i.e. 200 times.

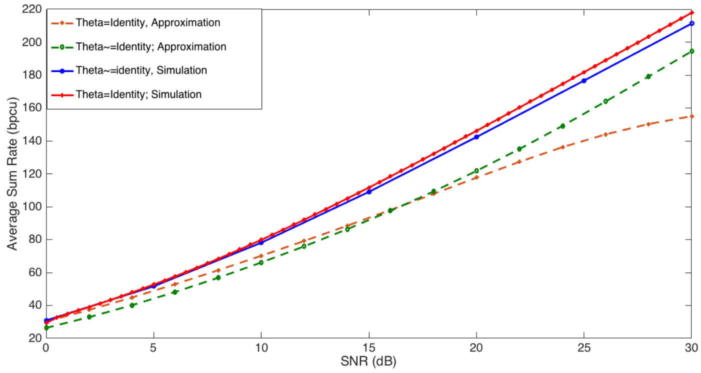

Fig. 1. Sum rate comparisons between the IBC WMMSE and our proposed approximation for C=2,K=15,M=30.

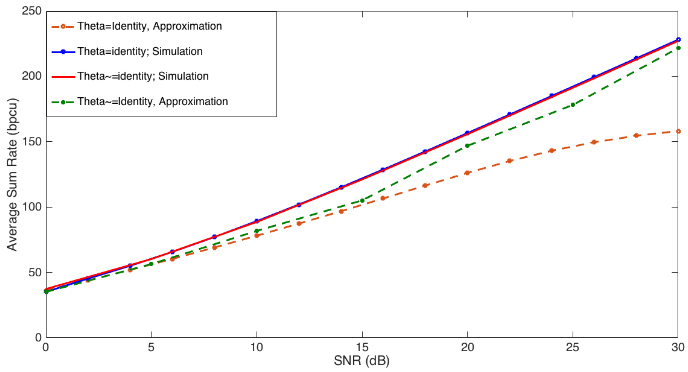

Fig. 2. Sum rate comparisons between the IBC WMMSE and our proposed approximation for C=3,K=10,M=30.

4. CONCLUSION

In this paper, we presented a consistent framework to study the optimal WMMSE precoding scheme based on the theory of large-dimensional random matrices. The tools from Random Matrix Theory (RMT) allowed us to consider a very realistic channel model accounting for per-user channel correlation as well as individual channel gains for each link. The system performance under this general type of channel is extremely difficult to study for finite dimensions but becomes feasible by assuming large system dimensions. Applied to practical optimization problems, the deterministic approximations lead to

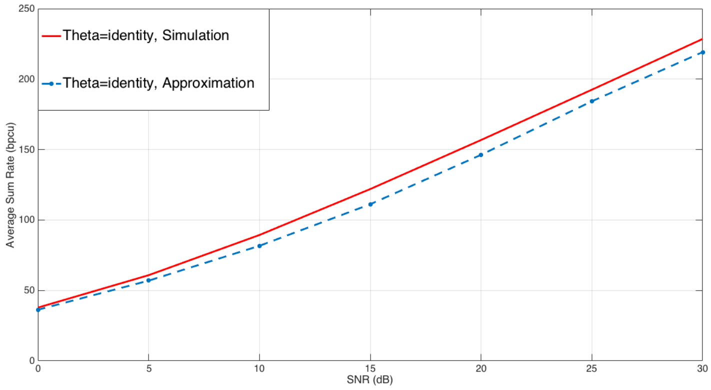

Fig. 3. Sum rate comparisons between the IBC WMMSE and our proposed approximation for C=3,K=9,M=30.

important insights into the system behavior, which are consistent with previous results, but go further and extend them to more realistic channel models and other linear precoding techniques.

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