Optimally Bridging the Gap from Delayed to Perfect CSIT in the K-user MISO BC

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***Abstract*—This work[1] derives the optimal Degrees-of-Freedom (DoF) of the K-User MISO Broadcast Channel (BC) with delayed Channel-State Information at the Transmitter (CSIT) and with additional current noisy CSIT where the channel estimation error scales in $P^{-\alpha}$ for $\alpha \in [0,1]$. The optimal sum DoF takes the simple form $(1-\alpha)K/H_K + \alpha K$ where $H_K \triangleq \sum_{k=1}^{K} \frac{1}{k}$. This optimal performance is the result of a novel scheme which deviates from existing efforts as it digitally combines interference, decodes symbols of any order in the MAT alignment [1], and utilizes a hierarchical quantizer whose output is distributed across rounds in a way that minimizes unwanted interference. These jointly deliver, for the first time, the elusive DoF-optimal combining of MAT and ZF.**

I. Introduction

In the wireless BC, feedback accuracy and timeliness can crucially affect performance, but are also notoriously difficult to obtain. In terms of accuracy, it is well known that increasing feedback quality, can elevate performance, from that of TDMA (sum DoF of 1), to the maximum possible interference-free performance of K sum-DoF. For example, the recent result in [2] tells us that having imperfect instantaneous CSIT with an estimation error that scales as $P^{-\alpha}$ ($\alpha \in [0,1]$), can allow, using a combination of standard ZF and rate splitting, to achieve the optimal sum-DoF of $1+(K-1)\alpha$. Note that this result can be seen as a particular case of the broader notion of "signal space partitioning" [3].

On the other hand, when perfect CSIT is obtained in a delayed manner — in the sense that the CSI is fed back with a delay that exceeds the channel coherence period — then, using retrospective *MAT* space-time alignment [1], one can surprisingly get substantial sum-DoF gains of the form $\frac{K}{H_K}$, which scales with K approximately as $\frac{K}{\ln(K)}$.

This interplay between performance and feedback (timeliness and quality), has sparked many different works that considered a variety of feedback mechanisms with delayed and imperfect CSIT. Such works can be found in [4], and in [5] which, for the two-user MISO BC setting, studied the case where the CSIT relative to one user is alternatively perfect, completely outdated, or non-existent (see extension in [6]).

One of the most interesting approaches came with the work in [7] which first introduced a feedback mechanism that offered perfect delayed CSIT, as well as imperfect-quality current (instantaneous) CSIT, that is in practice typically obtained from prediction using the delayed CSIT. In this setting, the channel estimation error of the current channel state was assumed to scale as $P^{-\alpha}$, for some CSIT quality exponent $\alpha \geq 0$.

The above work was then improved to reach the maximal DoF in a two-user MISO scenario in [8], [9]. This approach has since been extended to imperfect delayed CSIT in [10], [11], to the broad setting of any-time any-quality feedback in [12], and to the two-user MIMO BC and IC in [13], [14]. In the K-user case, a general outer bound was provided in [15], and efforts to reach this bound can be found in [16].

Many of the above efforts aimed to optimally combine ZF and MAT schemes. To date, this has remained an elusive open problem, and any attempt was either limited to the 2-user case, or resulted in a DoF that saturated at 2 in the particular case where the current estimate is of very bad quality, as in [15]. This open problem is resolved here, by resorting to a new scheme, referred to as the Q-MAT scheme, that combines different new ingredients that jointly allow for MAT and ZF components to optimally coincide. Combined with the outer bound in [15], the achieved DoF establishes the surprisingly simple optimal sum-DoF, equal to $(1-\alpha)K/H_K + \alpha K$.

II. System Model

A. K-User MISO Broadcast Channel

This work considers a K-User MISO BC where the TX is equipped with M antennas and serves K single-antenna users with $M \geq K$. At any time t, the signal received at Receiver (RX) $k, \forall k \in \mathcal{K}$, where $\mathcal{K} \triangleq \{1,\dots,K\}$, can be written as

$$y_k[t] = \boldsymbol{h}_k^{\mathrm{H}}[t]\mathbf{x}[t] + n_k[t] \tag{1}$$

where $\boldsymbol{h}_k^{\mathrm{H}} \in \mathbb{C}^{1\times M}$ is the channel to user k at time t, $\mathbf{x}[t] \in \mathbb{C}^{M\times 1}$ is the transmitted signal, and $n_k[t] \in \mathbb{C}$ is the additive noise at RX k, independent of the channel and the transmitted signal and distributed as $\mathcal{N}_{\mathbb{C}}(0,1)$. Furthermore, the transmitted signal $\mathbf{x}[t]$ fulfills the average asymptotic power constraint $\mathbb{E}[\|\mathbf{x}[t]\|^2] \doteq P$ where we use $\doteq$ to denote exponential equality, i.e., we write $f(P) \doteq P^B$ to denote $\lim_{P\to\infty} \frac{\log f(P)}{\log P} = B$.

The channel is assumed to be drawn from a continuous ergodic distribution such that all the channel matrices and all their sub-matrices are almost surely full rank.

B. Perfect Delayed CSIT and Imperfect Current CSIT

The considered CSIT model builds on the delayed CSIT model introduced in [1] and generalized to account for the

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availability of an imperfect estimate of the current channel state in [7], possibly obtained from predicting the current channel state from the delayed feedback whenever the delay is within the coherence time of the channel. According to this model, the TX has access at time t to the delayed CSI of all previous channel realizations up to time $t-1$, where we have assumed unit coherence time.

In addition, the TX has imperfect knowledge of the current channel state relative to the channel to user $k, \forall k \in \mathcal{K}$, being defined such that

$$\boldsymbol{h}_k^{\mathrm{H}}[t] = \hat{\boldsymbol{h}}_k^{\mathrm{H}}[t] + \sqrt{P^{-\alpha}}\tilde{\boldsymbol{h}}_k^{\mathrm{H}}[t] \tag{2}$$

where $\hat{\boldsymbol{h}}_k^{\mathrm{H}}[t]$ is the channel estimate and $\tilde{\boldsymbol{h}}_k^{\mathrm{H}}[t]$ is the channel estimation error. The channel estimate and the channel estimate error are independent and distributed according to a generic distribution. $\alpha \in [0,1]$ is called the *CSIT quality exponent* and is used to parametrize the accuracy of the current CSIT. Note that from a DoF perspective, we can restrict ourselves to $\alpha \in [0,1]$ since an estimation/quantization error scaling with P as P^{-1} is essentially perfect while an estimation error scaling as P^0 is essentially useless in terms of DoF [2], [15].

C. Degrees-of-Freedom Analysis

Let us denote by $\mathcal{C}(P)$ the (unknown) sum-capacity [17] of the MISO BC considered. The optimal sum DoF in this MISO BC with delayed and imperfect current CSIT is denoted by

$$\mathrm{DoF}^{\star}(\alpha) \triangleq \lim_{P\to\infty} \frac{\mathcal{C}(P)}{\log_2(P)}. \tag{3}$$

III. Main Results

We provide in this section our main results.

Theorem 1. *In the K-user MISO BC ($M \geq K$) with delayed CSIT and imperfect current CSIT with quality exponent α, the optimal sum DoF is given by*

$$\mathrm{DoF}^{\star}(\alpha) = (1-\alpha)\frac{K}{\sum_{k=1}^{K}\frac{1}{k}} + \alpha K. \tag{4}$$

The converse is proved in [15] and the main contribution of this work is to provide a new scheme achieving this DoF outerbound. We present here the scheme for $K=3$ as it conveys the main intuition more easily than the general case, while being directly extendeable to an arbitrary number of users. The description of the general case is thus relegated to the extended journal version [18].

IV. $K = 3$ Users Setting: Encoding and Transmission

Our scheme follows the structure of the MAT scheme. However, it also requires to consider several MAT *rounds*, in order to reach the optimal performance. This follows from the fact that some information needed for the decoding of a given round is sent during the following MAT round, as a consequence of the delay in the CSIT acquisition.

We describe in the following the transmission for an arbitrary round N. Whenever another round is considered, this will be explicitly mentioned.

A. Phase 1

Phase 1 spans 3 Time Slots (TS) denoted by t_ℓ for $\ell \in \{1,2,3\}$. During this phase, 9 order-1 (where an order-j data symbol is transmitted to j users [1]) so-called Q-MAT data symbols of rate $(1-\alpha)\log_2(P)$ bits are transmitted, and 3 order-2 so-called Q-MAT data symbols of rate $(1-\alpha)\log_2(P)$ bits are generated from the received signals, as we will show later.

1) Transmission: In TS t_ℓ, the transmitted signal is

$$\mathbf{x}[t_\ell] = \mathbf{V}[t_\ell]\boldsymbol{m}[t_\ell] + \sum_{k=1,k\neq\ell}^{3} \boldsymbol{v}_k^{\mathrm{ZF}}[t_\ell]a_k[t_\ell] + \sum_{k=1}^{3}\boldsymbol{v}_k^{\mathrm{ZF}}[t_\ell]s_k[t_\ell] \tag{5}$$

where

- $\boldsymbol{m}[t_\ell] \in \mathbb{C}^3$ is a vector containing 3 Q-MAT data symbols destined for user ℓ, each having a data rate of $(1-\alpha)\log_2(P)$ bits and power $\mathbb{E}\left[|\{\boldsymbol{m}[t_\ell]\}_1|^2\right] \doteq P$ and $\mathbb{E}\left[|\{\boldsymbol{m}[t_\ell]\}_i|^2\right] \doteq P^{1-\alpha}, \forall i \in \{2,3\}$. Furthermore, $\mathbf{V}[t_\ell] \in \mathbb{C}^{M\times 3}$ is defined as
$$\mathbf{V}[t_\ell] \triangleq \begin{bmatrix}\boldsymbol{v}_\ell^{\mathrm{ZF}}[t_\ell] & \mathbf{U}_1\end{bmatrix} \tag{6}$$
where $\boldsymbol{v}_\ell^{\mathrm{ZF}}[t_\ell] \in \mathbb{C}^M$ is the unitary ZF beamformer in the direction of user ℓ, and canceling interferences using the instantaneous imperfect CSIT and $\mathbf{U}_1 \in \mathbb{C}^{M\times 2}$ is a random subunitary matrix that is isotropically distributed.
- $a_k[t_\ell] \in \mathbb{C}, \forall k \neq \ell$ is a so-called *auxiliary data symbol* destined for user k and having rate $\min(1-\alpha,\alpha)\log_2(P)$ bits and power $\mathbb{E}\left[|a_k[t_\ell]|^2\right] \doteq P$.
- $s_k[t_\ell], \forall k \in \mathcal{K}$ is a so-called *ZF data symbol* destined for user k, with rate $\alpha\log_2(P)$ bits and power $\mathbb{E}\left[|s_k[t_\ell]|^2\right] \doteq P^{\alpha}$.

Upon omitting the noise realizations, the received symbols during TS $\ell \in \{1,2,3\}$ of phase 1 can be written as

$$\begin{aligned} y_\ell[t_\ell] &= \underbrace{\boldsymbol{h}_\ell^{\mathrm{H}}[t_\ell]\mathbf{V}[t_\ell]\boldsymbol{m}[t_\ell]}_{\doteq P} + \underbrace{\sum_{i=1,i\neq\ell}^{3}\boldsymbol{h}_\ell^{\mathrm{H}}[t_\ell]\boldsymbol{v}_i^{\mathrm{ZF}}[t_\ell]a_i[t_\ell]}_{\doteq P^{1-\alpha}} + \underbrace{z_\ell[t_\ell]}_{\doteq P^{\alpha}} \\ y_k[t_\ell] &= \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t_\ell]\boldsymbol{v}_k^{\mathrm{ZF}}[t_\ell]a_k[t_\ell]}_{\doteq P} + \underbrace{i_k[t_\ell]}_{\doteq P^{1-\alpha}} + \underbrace{z_k[t_\ell]}_{\doteq P^{\alpha}}, \forall k \neq \ell \end{aligned} \tag{7}$$

where $\forall k \neq \ell, n \triangleq \mathcal{K}\setminus\{\ell,k\}$, and where we have introduced the shorthand notation

$$i_k[t_\ell] \triangleq \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t_\ell]\mathbf{V}[t_\ell]\boldsymbol{m}[t_\ell]}_{\doteq P^{1-\alpha}} + \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t_\ell]\boldsymbol{v}_n^{\mathrm{ZF}}[t_\ell]a_n[t_\ell]}_{\doteq P^{1-\alpha}} \tag{8}$$

and where $z_k[t]$ is defined $\forall k \in \mathcal{K}$ and $\forall t \in \mathbb{N}$ as

$$z_k[t] \triangleq \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t]\boldsymbol{v}_k^{\mathrm{ZF}}[t]a_k[t]}_{\doteq P^{\alpha}} + \underbrace{\sum_{i=1,i\neq k}^{3}\boldsymbol{h}_k^{\mathrm{H}}[t]\boldsymbol{v}_i^{\mathrm{ZF}}[t]s_i[t]}_{\doteq P^{0}}. \tag{9}$$

2) Generation of Order-2 Data Symbols for Phase 2: At the end of phase 1, the TX can use its delayed CSIT to compute $i_k[t_\ell], \forall k \neq \ell, \forall \ell \in \{1,2,3\}$. We now consider separately the cases $\alpha \leq \frac{1}{2}$ and $\alpha \geq \frac{1}{2}$.

For the case of $\alpha \leq \frac{1}{2}$, to quantize the interference, we will use a specifically designed quantizer, and the following lemma.

Lemma 1. *Let y be a random variable with density, and with variance P^{β_1}, $\beta_1 \geq 0$, Then, there exists a properly-scaled integer quantizer of rate $(\beta_1 - \beta_2)\log_2(P)$ bits for any positive $\beta_2 \leq \beta_1$, denoted by Q_{β_1,β_2}, such that, for any random variable n with density and variance that does not scale above P^{β_2},*

$$\lim_{P\to\infty} \Pr\{Q_{\beta_1,\beta_2}(y+n) = Q_{\beta_1,\beta_2}(y)\} = 1 \tag{10}$$

and

$$\mathbb{E}\left[|Q_{\beta_1,\beta_2}(y) - y|^2\right] \dot{\leq} P^{\beta_2}. \tag{11}$$

For details on the design of quantizer Q_{β_1,β_2}, please see the extended version [18].

Using this quantizer, each of the interference terms $i_k[t_\ell], \forall k \neq \ell, \forall \ell \in \{1,2,3\}$ having a power scaling in $P^{1-\alpha}$ is quantized using $(1-2\alpha)\log_2(P)$ bits into $Q_{1-\alpha,\alpha}(i_k[t_\ell])$ with a quantization noise $n_k[t_\ell]$ having a power scaling in P^{α}.

In a second step, the aforementioned quantization noise $n_k[t_\ell]$ is itself quantized using any standard optimal quantizer with $\alpha \log_2(P)$ bits, which is known [17] to guarantee quantization noise that scales as P^0. For $\hat{n}_k[t_\ell]$ denoting the quantized version of $n_k[t_\ell^{(R_N)}]$, we get the final combined estimate carrying $(1-\alpha)\log_2(P)$ bits in the form

$$\hat{i}_k[t_\ell] \triangleq Q_{1-\alpha,\alpha}(i_k[t_\ell]) + \hat{n}_k[t_\ell]. \tag{12}$$

If $\alpha \geq \frac{1}{2}$: The interference term $i_k[t_\ell]$ is quantized using $(1-\alpha)\log_2(P)$ bits with a standard quantizer, guaranteed (cf. [17]) to have quantization noise that scales in P^0. The quantized signal obtained is also denoted by $\hat{i}_k[t_\ell]$.

Using the delayed CSIT, the TX then generates the following order-2 Q-MAT data symbols

$$m_{1,2} \triangleq (\hat{i}_2[t_1] \oplus \hat{i}_1[t_2]) \tag{13}$$
$$m_{2,3} \triangleq (\hat{i}_2[t_3] \oplus \hat{i}_3[t_2]) \tag{14}$$
$$m_{1,3} \triangleq (\hat{i}_3[t_1] \oplus \hat{i}_1[t_3]) \tag{15}$$

where $\oplus$ designates the exclusive or (XOR) and we use the same notation for the quantized signal and its representation in bits. We will see when studying the decoding that if these data symbols are transmitted to the destined users, i.e., $m_{1,2}$ to users 1 and 2, and so forth, it becomes possible for all users to decode their destined order-1 data symbols.

Remark 1. Note that we slightly abuse the notations because the order-2 data symbols generated and the ones transmitted during phase 2 are not written in the same way. This comes from the need to repeat phase 1 twice to generate the number of order-2 data symbols transmitted during phase 2. □

B. Phase 2 of Round N

Phase 2 spans 3 TS, denoted by $t_{\mathcal{S}}$ for $\mathcal{S} \in \{\{1,2\},\{2,3\},\{3,1\}\}$. During this phase, 6 order-2 Q-MAT data symbols of rate $(1-\alpha)\log_2(P)$ bits are transmitted and 2 order-3 Q-MAT data symbols of rate $(1-\alpha)\log_2(P)$ bits are generated from the received signals.

1) Transmission: In TS $t_{\mathcal{S}}$, the transmitted signal is

$$\mathbf{x}[t_{\mathcal{S}}] = \mathbf{V}[t_{\mathcal{S}}]\boldsymbol{m}[t_{\mathcal{S}}] + \boldsymbol{v}_{\bar{s}}^{\mathrm{ZF}}[t_{\mathcal{S}}]a_{\bar{s}}[t_{\mathcal{S}}] + \sum_{k=1}^{3} \boldsymbol{v}_k^{\mathrm{ZF}}[t_{\mathcal{S}}]s_k[t_{\mathcal{S}}] \tag{16}$$

where we have defined $\bar{s} \triangleq \mathcal{K} \setminus \mathcal{S}$ and:

- $\boldsymbol{m}[t_{\mathcal{S}}] \in \mathbb{C}^2$ is a vector containing 2 order-2 Q-MAT data symbols destined for the users in $\mathcal{S}$, each having a data rate of $(1-\alpha)\log_2(P)$ bits and power $\mathbb{E}\left[|\{\boldsymbol{m}[t_{\mathcal{S}}]\}_1|^2\right] \doteq P$ and $\mathbb{E}\left[|\{\boldsymbol{m}[t_{\mathcal{S}}]\}_2|^2\right] \doteq P^{1-\alpha}$. $\mathbf{V}[t_{\mathcal{S}}] \in \mathbb{C}^{M\times 2}$ defined as
$$\mathbf{V}[t_{\mathcal{S}}] \triangleq \begin{bmatrix} \boldsymbol{v}_{\mathcal{S}}^{\mathrm{ZF}}[t_{\mathcal{S}}] & \boldsymbol{u}_2 \end{bmatrix} \tag{17}$$
where $\boldsymbol{v}_{\mathcal{S}}^{\mathrm{ZF}}[t_{\mathcal{S}}] \in \mathbb{C}^M$ is a unitary ZF beamformer in the direction of the users in $\mathcal{S}$ and which ZF interferences towards user $\bar{s}$ and $\boldsymbol{u}_2 \in \mathbb{C}^M$ is a unitary vector isotropically distributed.
- $a_{\bar{s}}[t_{\mathcal{S}}] \in \mathbb{C}$ is an auxiliary data symbol destined for user $\bar{s}$, with rate $\min(1-\alpha,\alpha)\log_2(P)$ bits and power $\mathbb{E}\left[|a_{\bar{s}}[t_{\mathcal{S}}]|^2\right] \doteq P$.
- $s_k[t_{\mathcal{S}}], \forall k \in \mathcal{K}$ is a ZF data symbol destined for user k, with rate $\alpha \log_2(P)$ bits and power $\mathbb{E}\left[|s_k[t_{\mathcal{S}}]|^2\right] \doteq P^{\alpha}$.

Upon omitting the noise realizations, the received symbols during TS $\mathcal{S} \in \{\{1,2\},\{2,3\},\{3,1\}\}$ of phase 2 can be written for $k \in \mathcal{S}$ as

$$y_k[t_{\mathcal{S}}] = \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t_{\mathcal{S}}]\mathbf{V}[t_{\mathcal{S}}]\boldsymbol{m}[t_{\mathcal{S}}]}_{\doteq P} + \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t_{\mathcal{S}}]\boldsymbol{v}_{\bar{s}}^{\mathrm{ZF}}[t_{\mathcal{S}}]a_{\bar{s}}[t_{\mathcal{S}}]}_{\doteq P^{1-\alpha}} + \underbrace{z_k[t_{\mathcal{S}}]}_{\doteq P^{\alpha}} \tag{18}$$

and for user $\bar{s}$ as

$$y_{\bar{s}}[t_{\mathcal{S}}] = \underbrace{\boldsymbol{h}_{\bar{s}}^{\mathrm{H}}[t_{\mathcal{S}}]\boldsymbol{v}_{\bar{s}}^{\mathrm{ZF}}[t_{\mathcal{S}}]a_{\bar{s}}[t_{\mathcal{S}}]}_{\doteq P} + \underbrace{i_{\bar{s}}[t_{\mathcal{S}}]}_{\doteq P^{1-\alpha}} + \underbrace{z_{\bar{s}}[t_{\mathcal{S}}]}_{\doteq P^{\alpha}} \tag{19}$$

where $i_{\bar{s}}[t_{\mathcal{S}}]$ is defined as

$$i_{\bar{s}}[t_{\mathcal{S}}] \triangleq \boldsymbol{h}_{\bar{s}}^{\mathrm{H}}[t_{\mathcal{S}}]\mathbf{V}[t_{\mathcal{S}}]\boldsymbol{m}[t_{\mathcal{S}}]. \tag{20}$$

2) Generation of Order-3 Data Symbols for Phase 3: At the end of Phase 2, the TX uses its delayed CSIT to compute $i_3[t_{1,2}]$, $i_1[t_{2,3}]$, and $i_2[t_{1,3}]$. These interference terms are then quantized following the same quantization process as in phase 1 to obtain $\hat{i}_3[t_{1,2}]$, $\hat{i}_1[t_{2,3}]$, and $\hat{i}_2[t_{1,3}]$.

Using the delayed CSIT, the TX generates the following order-3 Q-MAT data symbols

$$m_{1,2,3} \triangleq (\hat{i}_1[t_{2,3}] \oplus \hat{i}_2[t_{1,3}]), \quad m_{1,2,3}" \triangleq (\hat{i}_2[t_{1,3}] \oplus \hat{i}_3[t_{1,2}]). \tag{21}$$

C. Phase 3

Phase 3 consists of 1 TS denoted by TS $t_{1,2,3}$, during which 1 order-3 Q-MAT data symbol of rate $(1-\alpha)\log_2(P)$ bits is transmitted. More specifically, the transmitted signal during TS t_{123} is given by

$$\mathbf{x}[t_{1,2,3}] = \boldsymbol{u}_{1,2,3} m[t_{1,2,3}] + \sum_{k=1}^{3} \boldsymbol{v}_k^{\mathrm{ZF}}[t_{1,2,3}] s_k[t_{1,2,3}] \quad (22)$$

where

- $m[t_{1,2,3}] \in \mathbb{C}$ is an order-3 (i.e., destined for all users) Q-MAT data symbol with data rate of $(1-\alpha)\log_2(P)$ bits and with power $\mathbb{E}\left[|m[t_{1,2,3}]|^2\right] \doteq P$. $\boldsymbol{u}_{1,2,3} \in \mathbb{C}^M$ is a random unitary vector that is isotropically distributed.
- $s_k[t_{1,2,3}], \forall k \in \mathcal{K}$ is a ZF data symbol destined for user k, with rate $\alpha \log_2(P)$ bits and power $\mathbb{E}\left[|s_k[t_{1,2,3}]|^2\right] \doteq P^{\alpha}$.

Upon omitting the noise realizations, the received signals are

$$y_k[t_{1,2,3}] = \underbrace{\boldsymbol{h}_k^{\mathrm{H}}[t_{1,2,3}] \boldsymbol{u}_{123} m[t_{1,2,3}]}_{\doteq P} + \underbrace{z_k[t_{1,2,3}]}_{\doteq P^{\alpha}}. \quad (23)$$

D. Repetition of the Phases

Having presented the 3 phases, it should now be clear that the successful transmission of the order-1 Q-MAT data symbols requires to repeat each phase a number of times such that all higher-order data symbols generated could be transmitted. In the 3-user case, this means that phase 1 and phase 3 are repeated twice.

E. Generation of Auxiliary Data Symbols for Round $N+1$

We show here how the auxiliary data symbols of round $N+1$ are formed as a function of the interference generated during round N. To denote the TS relative to round $N+1$, we use the notation t' instead of t. It is important to note that the auxiliary data symbols transmitted during phase i of round $N+1$, are generated solely on the basis of the same phase i during round N. This ensures that, during each round, the number of transmitted auxiliary data symbols is equal to the number of generated auxiliary data symbols.

a) Generation of Auxiliary Data Symbols for Phase 1 of Round $N+1$: The auxiliary data symbol $a_k[t'_\ell]$ (i.e., at round $N+1$) is defined $\forall \ell \in \{1,2,3\}, \forall k \neq \ell$ as

$$\begin{cases} a_k[t'_\ell] \triangleq \hat{n}_k[t_\ell] & ,\alpha \leq \frac{1}{2} \\ a_k[t'_\ell] \triangleq \hat{i}_k[t_\ell] & ,\alpha \geq \frac{1}{2}. \end{cases} \quad (24)$$

It remains to initialize the auxiliary data symbols for the first round, which is simply done by setting them to zero.

b) Generation of Auxiliary Data Symbols for Phase 2 of Round $N+1$: The auxiliary data symbols for phase 2 of round $N+1$ are defined such that $\forall \mathcal{S} \in \{\{1,2\},\{2,3\},\{3,1\}\}$

$$\begin{cases} a_{\bar{s}}[t'_{\mathcal{S}}] \triangleq \hat{n}_{\bar{s}}[t_{\mathcal{S}}] & ,\alpha \leq \frac{1}{2} \\ a_{\bar{s}}[t'_{\mathcal{S}}] \triangleq \hat{i}_{\bar{s}}[t_{\mathcal{S}}] & ,\alpha \geq \frac{1}{2} \end{cases} \quad (25)$$

As in phase 1, the auxiliary data symbols of the first round are set equal to zero, for initialization purpose.

c) Generation of Auxiliary Data Symbols for Phase 3 of Round $N+1$: Phase 3 does not transmit, nor generate, auxiliary data symbols.

V. $K=3$ Users Setting: Decoding

A. Decoding: Proof by Induction

We now turn to the decoding part and we assume that the transmissions of all phases and all rounds have ended. We will show that it is possible for every user to decode all its desired data symbols during every round. The proof has to be done by induction due to the fact that the auxiliary data symbols contain information relative to the previous round.

For the sake of brevity, we present solely the decoding of the second phase. Indeed, the decoding of the first phase follows in a very similar way, while the decoding of the third phase requires no induction, as there is no auxiliary data symbols. We refer to the extended version [18] for the full proof.

Let us consider without loss of generality the decoding at user 1. Our induction statement is that if $a_2[t_{1,3}]$ and $a_3[t_{1,2}]$ are already decoded, user 1 can decode $a_2[t'_{1,3}]$ and $a_3[t'_{1,2}]$ (i.e., user 1 can decode the auxiliary data symbols of round $N+1$). Through this induction, it will be clear that each user can also decode all its destined data symbols during every round.

We start by proving that the result is true for an initial case before showing the inductive step.

1) Base Case: The initialization follows trivially from the fact that all auxiliary data symbols of the first round are set to zero, and can therefore be decoded at round zero.

2) Inductive Step: We now consider an arbitrary round N. Due to our induction statement, $a_2[t_{1,3}]$ and $a_3[t_{1,2}]$ are already decoded at user 1.

a) Decoding of Phase 3: Before decoding phase 2, phase 3 of round N is decoded using successive decoding. Indeed, it can be seen in (23) that the SINR of the Q-MAT data symbol scales in $P^{1-\alpha}$. Once this data symbol is decoded, it becomes possible to decode the ZF data symbols.

b) Decoding of the Interference: As a first step, user 1 uses the received signal of phase 2 of round $N+1$ to decode $a_1[t'_{2,3}]$. This is possible as it can be seen in (19) that the scaling of the SINR matches the scaling of the rate.

Going back to the decoding of round N, we need to differentiate the two cases $\alpha \geq \frac{1}{2}$ or $\alpha \leq \frac{1}{2}$.

- If $\alpha \geq \frac{1}{2}$, user 1 has obtained $\hat{i}_1[t_{2,3}]$ from $a_1[t'_{2,3}]$.
- If $\alpha \leq \frac{1}{2}$, user 1 has obtained $\hat{n}_1[t_{2,3}]$ from $a_1[t'_{2,3}]$. To recover the quantized interference $\hat{i}_1[t_{2,3}]$, it is necessary for user 1 to obtain $Q_{1-\alpha,\alpha}(i_1[t_{23}])$ (see (12)). This quantized interference can be obtained by quantization of the received signal $y_1[t_{2,3}]$ using the quantizer $Q_{1-\alpha,\alpha}$ used for the encoding. Indeed, following Lemma 1, it holds that

$$\lim_{P\to\infty} \Pr\left\{Q_{1-\alpha,\alpha}\left(y_1[t_{2,3}]\right) = Q_{1-\alpha,\alpha}\left(i_1[t_{2,3}]\right)\right\} = 1. \quad (26)$$

  Thus, user 1 can decode $\hat{i}_1[t_{2,3}] = Q_{1-\alpha,\alpha}\left(y_1[t_{2,3}]\right)$ and use it with $\hat{n}_1[t_{2,3}]$ to obtain $\hat{i}_1[t_{2,3}]$.

In both cases, user 1 has obtained $\hat{i}_1[t_{2,3}]$.

c) Decoding of the Q-MAT *Data Symbols of Phase* 2*:* From phase 3, user 1 has decoded

$$\begin{aligned} m_{1,2,3} &\triangleq (\hat{i}_1[t_{2,3}] \oplus \hat{i}_2[t_{1,3}]) \\ m_{1,2,3}" &\triangleq (\hat{i}_2[t_{1,3}] \oplus \hat{i}_3[t_{1,2}]) \end{aligned}. \quad (27)$$

Combining them with $\hat{i}_1[t_{2,3}]$, user 1 obtains $\hat{i}_3[t_{1,2}]$ and $\hat{i}_2[t_{1,3}]$. Let us consider first the decoding of $\boldsymbol{m}[t_{1,2}]$. User 1 has received

$$\underbrace{\boldsymbol{h}_1^{\mathrm{H}}[t_{1,2}]\mathbf{V}[t_{1,2}]\boldsymbol{m}[t_{1,2}] + \boldsymbol{h}_1^{\mathrm{H}}[t_{1,2}]\boldsymbol{v}_3^{\mathrm{ZF}}[t_{1,2}]a_3[t_{1,2}]}_{\doteq P} + \underbrace{z_1[t_{1,2}]}_{\doteq P^{\alpha}}$$
$$\underbrace{\boldsymbol{h}_3^{\mathrm{H}}[t_{1,2}]\mathbf{V}[t_{1,2}]\boldsymbol{m}[t_{1,2}]}_{\doteq P^{1-\alpha}} \quad (28)$$

where quantization noise at the noise floor has been neglected.

By induction, $a_3[t_{1,2}]$ is assumed to be already decoded at user 1, such that its contribution to the received signal in (28) can be removed. Consequently, user 1 has obtained two signals with a SINR scaling in $P^{1-\alpha}$ and can decode its two destined data symbols $\boldsymbol{m}[t_{1,2}]$.

The data symbols in $\boldsymbol{m}[t_{1,3}]$ are decoded similarly.

d) Decoding of the ZF Data Symbols at User k: If the Q-MAT data symbols have been decoded at the user, it is possible for the user to use successive decoding to decode the ZF data symbols. However, when the user was not among the destined user, he cannot decode the Q-MAT data symbols as he does not have sufficiently many observations. Yet, in that case, the user has obtained a quantized version of the interference with a quantization at the noise floor, thanks to the decoding of the auxiliary data symbols (See Section V-A2b). Therefore, it can remove the interference created by the Q-MAT data symbols up to the noise floor, which makes it possible to decode its desired ZF data symbol.

e) Decoding of the Auxiliary Data Symbols of Round $N+1$: In order to conclude the inductive step, it remains to prove that it is possible for user 1 to decode $a_2[t'_{1,3}]$ and $a_3[t'_{1,2}]$ (i.e., relative to round $N+1$). This result follows directly from the definition of the auxiliary data symbols in (25). Indeed, the auxiliary data symbol during TS $t'_{1,3}$ (resp. $t'_{1,2}$) depends only on the Q-MAT data symbols in $\boldsymbol{m}[t_{1,3}]$ (resp. $\boldsymbol{m}[t_{1,2}]$). These data symbols being already decoded at user 1, user 1 can decode these auxiliary data symbols relative to round $N+1$, which concludes the inductive step.

B. Calculation of the DoF

Adding the rates of the transmitted data symbols and dividing by the number of TS of one round, provides the DoF achieved by one arbitrary round. Letting the number of rounds become large allows to neglect the loss due to initialization and termination and provides the DoF expression of the theorem.

VI. Conclusion

In the K-user MISO BC with delayed and imperfect current CSIT, the novel Q-MAT scheme combines MAT alignment (which uses delayed CSIT) and ZF (which uses current CSIT) to achieve the optimal sum DoF. By exploiting both versions of the CSIT, the Q-MAT scheme is more robust with respect to both the delay and the estimation noise, and could hence be practically relevant in many settings. Several innovative solutions have here been introduced in order to achieve the optimal DoF, such as the digital combination of interference, the decoding of symbols of any order, and the use of a hierarchical quantizer whose output is distributed across rounds in a way that minimizes unwanted interference. These new methods can be useful in other wireless configurations with limited/delayed/distributed CSIT.

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