On a Mutual Coupling Agnostic Maximum Likelihood Angle of Arrival Estimator by Alternating Projection

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***Abstract*—The problem of Angle of Arrival estimation of multiple sources in the presence of mutual coupling is addressed. The presence of unknown mutual coupling between antenna array elements is known to degrade the performance of super resolution direction-finding algorithms. In this paper, we propose an iterative algorithm based on Alternating Projection in order to optimise the Maximum Likelihood cost function, which takes mutual coupling into account. Simulation results demonstrate the potential of the proposed algorithm, as it is compared to the Cramer-Rao bound of joint mutual coupling and Angle of Arrival estimation.**

***Index Terms*—Maximum Likelihood Estimation, Angle-of-Arrival, Mutual Coupling Agnostic, Alternating Projection**

I. INTRODUCTION

Mutual coupling between antennas is a popular problem in array signal processing. This phenomenon arises when antennas are close to each other [1], and thus the current developed in an antenna element depends on its own excitation and on the contributions from adjacent antennas. As a consequence, an ideal model is no longer valid, and therefore the performance of the high resolution algorithms that perform Angle-of-Arrival (AoA) estimation, such as MUSIC [2], ESPRIT [3], etc., deteriorate significantly. It is also worth mentioning other phenomena that perturb an ideal model, when not taken into account, such as different gain/phases [4] across antennas, synchronization and jitter effect [5], local scattering [6], etc.

Methods that aim on solving the mutual coupling problem are sometimes referred to as calibration methods, which are of two types: Offline and Online. In an offline calibration approach, one estimates the mutual coupling parameters using known locations, such as the techniques in [7]–[9]. In contrast, online calibration consists of jointly estimating the coupling and AoA parameters. In this paper, our main focus is on the latter.

In the literature, several techniques deal with the online calibration problem, such as those found in [10]–[14], [18]–[23] and references within. The authors in [10] jointly estimate the coupling parameters and AoAs by alternating minimisation steps between the former and the latter using the MUSIC cost function. In [11], the algorithm is iterative and the sources are assumed to be totally uncorrelated. Moreover, the method in [11] estimates the coupling parameters in order to utilise it in the MUSIC cost function. In [12], [13], the array elements are assumed to be partly calibrated. In other words, one has access to a few coupling parameters. Moreover, methods in [19]–[21] propose to use the *"middle sub-array"*. In particular, let N be the number of antennas placed in a uniform and linear fashion, and p be the number of coupling parameters. If $p \leq \frac{N}{2}$, then there exists a *"middle sub-array"*, of size $N-2p+2$, over which the effect of the mutual coupling preserves the *Vandermonde* structure of the array response [19]–[21]. As a matter of fact, one could mathematically show that the array response on the *"middle sub-array"* is a known functional *Vandermonde* vector multiplied by an unknown scalar, and therefore high resolution techniques could be applied to estimate the AoAs. Also, spatial smoothing [27] could be employed only on the *"middle sub-array"* to "de-correlate" the signals so as subspace techniques work properly [19]. However, this is highly suboptimal because only an effective number of antennas, i.e. $N-2p+2$, are utilised for parameter estimation. To address this major loss of antennas, the authors in [23] suggest to add *"Guard"* antennas on the edges of the array, in particular $p-1$ antennas on the left edge and the other $p-1$ on the right edge. Therefore, the main array of size N plays the role of a *"middle sub-array"*. The aforementioned arguement on the *"middle sub-array"* holds on the main array. Unfortunately, appending antennas is not possible in some applications, such as Wi-Fi.

In this paper, we are motivated by the classical Alternating Projection (AP) method, which was utilised by Ziskind and Wax in [29] to obtain a Maximum Likelihood (ML) estimate of the AoAs, in the absence of mutual coupling. The method is computationally efficient, as it requires multiple 1-dimensional searches to optimise the ML cost function. It is well-known that ML algorithms, such as [25], [29], can estimate the AoAs of coherent sources [24], which is the case of multipath propagation. The contribution of this paper is the derivation of a suitable AP algorithm that could estimate the AoAs of different sources in the presence of mutual coupling, thus *"Mutual Coupling Agnostic"*. Furthermore, the proposed AP algorithm could also deal with coherent sources, whereas the algorithms in [10]–[14], [18] can not.

The paper is divided as follows: Section II presents the system model, assumptions, and problem statement. Section III reviews the Deterministic Maximum Likelihood (ML) of joint Angle and mutual coupling estimation. The proposed mutual coupling agnostic ML Angle of Arrival (AoA) estimator by Alternating Projection is presented in Section IV. Simulation results of the proposed algorithm for different scenarios are demonstrated in Section V, with comparison to the Cramer-Rao bound that takes into account joint AoA and mutual coupling parameter estimation. We conclude the paper in Section VI.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ represent the transpose and the transpose-conjugate operators. The matrix $\mathbf{I}$ is the identity matrix with suitable dimensions. For any matrix $\mathbf{B}$, the $(i,j)^{th}$ entry of $\mathbf{B}$ is represented as $[\mathbf{B}]_{(i,j)}$. For any two variables $\mathbf{A}$ and $\mathbf{B}$, the notation $\mathbf{A} \leftarrow \mathbf{B}$ means that the value of $\mathbf{B}$ is stored in $\mathbf{A}$. Furthermore, the symbol $\varnothing$ indicates an empty matrix. For any matrix $\mathbf{B}$, the operators $\|\mathbf{B}\|$, $\det\{\mathbf{B}\}$, and $\mathrm{vec}(\mathbf{B})$ denote the *Frobenius* norm, determinant, and vectorisation, respectively.

II. System Model

A. Mathematical Formulation

Assume a planar arbitrary array of N antennas. Furthermore, consider $q < N$ narrowband sources attacking the array from different angles, i.e. $\mathbf{\Theta} = [\theta_1 \dots \theta_q]$. Collecting L time snapshots and following [26], we can write

$$\mathbf{X} = \tilde{\mathbf{A}}(\mathbf{\Theta})\mathbf{S} + \mathbf{N} \tag{1}$$

where $\mathbf{X} \in \mathbb{C}^{N \times L}$ is the data matrix with l^{th} time snapshot, $\mathbf{x}(t_l)$, stacked in the l^{th} column of $\mathbf{X}$. The matrix $\mathbf{S} \in \mathbb{C}^{q \times L}$ is the source matrix. Similar to $\mathbf{X}$, matrix $\mathbf{S}$ contains the l^{th} transmitted source vector $s(t_l)$ in its l^{th} column. Moreover, rhe steering matrix $\tilde{\mathbf{A}}(\mathbf{\Theta}) \in \mathbb{C}^{N \times q}$ is composed of q steering vectors, i.e. $\tilde{\mathbf{A}}(\mathbf{\Theta}) = [\tilde{\mathbf{a}}(\theta_1) \dots \tilde{\mathbf{a}}(\theta_q)]$. Each vector $\tilde{\mathbf{a}}(\theta_i)$ is the response of the array to a source impinging the array from direction θ_i. Note that we shall use the notations $\tilde{\mathbf{a}}(\theta)$ and $\mathbf{a}(\theta)$ to denote an array response due to angle θ in the presence and absence of mutual coupling, respectively. The matrix $\mathbf{N} \in \mathbb{C}^{N \times L}$ is background noise.

In this paper, we restrict ourselves with Uniform Linear Arrays (ULAs). Furthermore, it is well known that the response of a ULA in the absence of mutual coupling is given as

$$\mathbf{a}(\theta) = [1, z_\theta, \dots z_\theta^{N-1}]^{\mathrm{T}} \tag{2}$$

where $z_\theta = e^{-j2\pi\frac{d}{\lambda}\sin(\theta)}$, d is the inter-element spacing and λ is the signal's wavelength. Following [14], the response $\tilde{\mathbf{a}}(\theta)$ could be modelled as

$$\tilde{\mathbf{a}}(\theta) = \mathcal{T}_p(\mathbf{m})\mathbf{a}(\theta) \tag{3}$$

where $\mathcal{T}_p(\mathbf{m}) \in \mathbb{C}^{N \times N}$ is a banded symmetric Toeplitz matrix defined as follows

$$\mathcal{T}_p(\mathbf{m}) = \begin{bmatrix} 1 & m_1 & m_2 & \cdots & m_{p-1} & 0 & \cdots & 0 \\ m_1 & 1 & m_1 & \cdots & m_{p-2} & m_{p-1} & \cdots & 0 \\ \vdots & & & \ddots & \ddots & & & \vdots \\ 0 & \cdots & m_{p-1} & m_{p-2} & \cdots & m_1 & 1 & m_1 \\ 0 & \cdots & 0 & m_{p-1} & \cdots & m_2 & m_1 & 1 \end{bmatrix} \tag{4}$$

Note that the matrix $\mathcal{T}_p(\mathbf{m})$ is independent from the AoAs. The model in equations (3) and (4) suggest that antennas that are placed at least p inter-element spacings apart do not interfere, i.e. $m_i = 0$ for all $i \geq p$. This is due to the fact that the mutual coupling is inversely proportional to the distance between antennas.

B. Assumptions and Problem Statement

Throughout the paper, we assume the following:

- **A1**: $\tilde{\mathbf{A}}(\mathbf{\Theta})$ is full column rank.
- **A2**: The transmitted signals $\mathbf{s}(t_l)$ are fixed within a snapshot. The signals are allowed to be highly or fully correlated.
- **A3**: The number of sources, i.e. q, is known.
- **A4**: The vector $\mathbf{n}(t_l)$ is Gaussian noise with zero mean and covariance $\sigma^2\mathbf{I}$ and independent from the sources.

For assumption **A3**, algorithms exist for estimating the number of sources, such as Minimum Description Length [15], Modified MDL [16], Benjamin Hochberg procedure [17], and so forth. We are now ready to address our problem:

"Given $\{\mathbf{x}(l)\}_{l=1}^{L}$, p and q, estimate the AoAs $\mathbf{\Theta}$ in the presence of mutual coupling $\mathcal{T}_p(\mathbf{m})$."

III. The Deterministic Maximum Likelihood (ML) Estimator

This section is a review of the Deterministic ML estimators of the angles of arrival of the transmitting sources. In a *deterministic* approach [28], these parameters are modelled as unknown deterministic sequences, i.e. the signal parameters are assumed to be nonrandom and unknown. Under assumption **A4**, we can say that all snapshots $\{\mathbf{x}_l(t)\}_{l=1}^{L}$ are independent and

$$\mathbf{x}(t_l) \sim \mathcal{N}\Big(\tilde{\mathbf{A}}(\mathbf{\Theta})\mathbf{s}(t_l), \sigma^2\mathbf{I}_n\Big), \quad l = 1 \dots L \tag{5}$$

To simplify notation, we stack all signal and noise parameters into one vector, say

$$\Omega = \Big[\mathbf{\Theta}^{\mathrm{T}}, \mathbf{m}^{\mathrm{T}}, \mathrm{vec}(\mathbf{S}), \sigma^2\Big]^{\mathrm{T}} \tag{6}$$

We can now express the joint probability distibution function of all the snapshots $\mathbf{X}$, given the unknown signal and noise parameters Ω as

$$\begin{aligned} &f\Big(\mathbf{X}|\Omega\Big) \\ &= \prod_{l=1}^{L} \frac{1}{\pi\det\big\{\sigma^2\mathbf{I}_N\big\}}\exp\Big\{-\frac{1}{\sigma^2}\big\|\mathbf{x} - \tilde{\mathbf{A}}(\mathbf{\Theta})\mathbf{s}(t_l)\big\|^2\Big\} \end{aligned} \tag{7}$$

The Deterministic ML estimates of the noise and signal parameters, i.e. $\hat{\Omega}^{\mathrm{ML}}$, are obtained through the following criterion

$$\hat{\Omega}^{\mathrm{ML}} = \arg\max_{\Omega} f\Big(\mathbf{X}|\Omega\Big) \tag{8}$$

Finally, $\hat{\Omega}^{\mathrm{ML}}$ is given by the following

$$\hat{\Omega}^{\mathrm{ML}} = \Big[\hat{\mathbf{\Theta}}^{\mathrm{T}}, \hat{\mathbf{m}}^{\mathrm{T}}, \mathrm{vec}(\hat{\mathbf{S}}), \hat{\sigma}^2\Big]^{\mathrm{T}} \tag{9a}$$

$$\hat{\sigma}^2 = \frac{1}{NL}\big\|\mathbf{X} - \tilde{\mathbf{A}}(\hat{\mathbf{\Theta}})\hat{\mathbf{S}}\big\|^2 \tag{9b}$$

$$\hat{\mathbf{S}} = \Big(\tilde{\mathbf{A}}^{\mathrm{H}}(\hat{\mathbf{\Theta}})\tilde{\mathbf{A}}(\hat{\mathbf{\Theta}})\Big)^{-1}\tilde{\mathbf{A}}^{\mathrm{H}}(\hat{\mathbf{\Theta}})\mathbf{X} \tag{9c}$$

$$[\hat{\mathbf{\Theta}}, \hat{\mathbf{m}}] = \arg\max_{\mathbf{\Theta}, \mathbf{m}} \mathrm{tr}\Big\{\mathscr{P}_{\tilde{\mathbf{A}}(\mathbf{\Theta})}\hat{\mathbf{R}}\Big\} \tag{9d}$$

where $\mathscr{P}_{\tilde{\mathbf{A}}(\mathbf{\Theta})}$ is the projector onto the signal subspace, i.e. the space spanned by columns of $\mathscr{P}_{\tilde{\mathbf{A}}(\mathbf{\Theta})}$

$$\mathscr{P}_{\tilde{\mathbf{A}}(\mathbf{\Theta})} = \tilde{\mathbf{A}}(\mathbf{\Theta})\Big(\tilde{\mathbf{A}}^{\mathrm{H}}(\mathbf{\Theta})\tilde{\mathbf{A}}(\mathbf{\Theta})\Big)^{-1}\tilde{\mathbf{A}}^{\mathrm{H}}(\mathbf{\Theta}) \tag{9e}$$

and $\hat{\mathbf{R}}$ is the sample covariance matrix of the snapshots

$$\hat{\mathbf{R}} = \frac{1}{L}\mathbf{X}\mathbf{X}^{\mathrm{H}} \tag{9f}$$

Note that once the estimate $[\hat{\mathbf{\Theta}}, \hat{\mathbf{m}}]$ is obtained by solving (9d), then one could plug $[\hat{\mathbf{\Theta}}, \hat{\mathbf{m}}]$ in (9b) and (9c) to obtain the ML estimate of the noise variance and signal matrix, respectively. It turns out that the optimisation problem in (9d) is highly nonlinear, as it requires a $\big(q+2(p-1)\big)$-dimensional search[1], and its direct optimisation would require cumbersome optimisation techniques. It is worth stressing a point here: *The rest of the paper focuses on solving* (9d) *in order to estimate $\mathbf{\Theta}$, by treating $\mathbf{m}$ as a nuissance parameter. The problem of estimating the coupling parameters $\mathbf{m}$ is beyond the scope of this paper. Once again, our aim is to estimate the AoAs of multiple sources in the presence of mutual coupling, thus the term "Mutual Coupling Agnostic".*

[1] It is a $\big(q + 2(p-1)\big)$-dimensional search: q is due to the number of parameters in $\mathbf{\Theta}$ and $2(p-1)$ is the number of real and imaginary unknown parameters in $\mathbf{m}$.

IV. Mutual Coupling Agnostic ML Angle of Arrival Estimator by Alternating Projection

In the absence of mutual coupling, i.e. $p = 1$ and $\mathbf{m} = 1$, Ziskind and Wax have proposed to optimise (9d) in order to estimate $\boldsymbol{\Theta}$ via Alternating Projection [29]. That is, the value of θ_i at the k^{th} iteration is obtained by solving the following 1-dimensional optimisation problem

$$\hat{\theta}_i^{(k)} = \arg\max_{\theta_i} \frac{\mathbf{a}^{\mathrm{H}}(\theta_i)\mathscr{P}^{\perp}_{\mathbf{A}_{\bar{i}}}\hat{\mathbf{R}}\mathscr{P}^{\perp}_{\mathbf{A}_{\bar{i}}}\mathbf{a}(\theta_i)}{\mathbf{a}^{\mathrm{H}}(\theta_i)\mathscr{P}^{\perp}_{\mathbf{A}_{\bar{i}}}\mathbf{a}(\theta_i)} \tag{10}$$

where $\mathscr{P}^{\perp}_{\mathbf{Y}} = \mathbf{I} - \mathscr{P}_{\mathbf{Y}}$ and $\mathbf{A}_{\bar{i}}$ is obtained by omitting the i^{th} column from matrix $\mathbf{A}(\boldsymbol{\Theta}^{(k)})$. The vector $\boldsymbol{\Theta}^{(k)}$ represents the estimated AoAs at iteration k, in an attempt of estimating the i^{th} AoA. In other words, at iteration k and sub-iteration i, vector $\boldsymbol{\Theta}^{(k)}$ could be expressed as

$$\hat{\boldsymbol{\Theta}}^{(k)} = [\hat{\theta}_1^{(k)}, \hat{\theta}_2^{(k)} \dots \hat{\theta}_{i-1}^{(k)}, \hat{\theta}_i^{(k-1)}, \hat{\theta}_{i+1}^{(k-1)} \dots \hat{\theta}_q^{(k-1)}]^{\mathrm{T}} \tag{11}$$

Naturally, the algorithm is iterative. At each iteration, the 1-dimensional search in (10) is done per AoA ($i = 1 \dots q$) in a successive manner until the vector $\hat{\boldsymbol{\Theta}}^{(k)}$ converges.

Notice that, in the presence of mutual coupling, we could follow similar steps as in [29] to get

$$[\hat{\theta}_i^{(k)}\ ,\ \hat{\mathbf{m}}_i^{(k)}] = \arg\max_{[\theta_i,\mathbf{m}]} \frac{\tilde{\mathbf{a}}^{\mathrm{H}}(\theta_i)\mathscr{P}^{\perp}_{\tilde{\mathbf{A}}_{\bar{i}}}\hat{\mathbf{R}}\mathscr{P}^{\perp}_{\tilde{\mathbf{A}}_{\bar{i}}}\tilde{\mathbf{a}}(\theta_i)}{\tilde{\mathbf{a}}^{\mathrm{H}}(\theta_i)\mathscr{P}^{\perp}_{\tilde{\mathbf{A}}_{\bar{i}}}\tilde{\mathbf{a}}(\theta_i)} \tag{12}$$

where the maximisation is also done over the coupling parameters $\mathbf{m}$. To proceed, we find the following definition and theorem useful:

Definition 1: For any vector $\mathbf{a} \in \mathbb{C}^{N\times 1}$ and matrix $\mathbf{B}_p \in \mathbb{C}^{N\times p}$. We say that "$\mathbf{a}$ *generates* $\mathbf{B}_p$", or $\mathbf{B}_p \triangleq \mathcal{G}_p(\mathbf{a})$, if

$$\mathbf{B}_p = \left[\ \mathbf{a}\ \middle|\ \mathbf{S}_1\mathbf{a}\ \middle|\ \dots\ \middle|\ \mathbf{S}_{p-1}\mathbf{a}\ \right] \tag{13}$$

where $\mathbf{S}_k \in \mathbb{C}^{N\times N}$ is an all-zero matrix except at the k^{th} sub- and super-diagonals, which are set to 1.

Theorem 1: *(Commutativity of Symmetric Toeplitz Matrices) Let* $\mathbf{m} = [m_0, m_1 \dots m_{p-1}]^T \in \mathbb{C}^{p\times 1}$ *and* $\mathbf{a} \in \mathbb{C}^{N\times 1}$. *Define the corresponding matrix* $\mathcal{T}_p(\mathbf{m})$ *as in equation* (4). *Then the following is true for any* $1 \le p \le N$

$$\mathcal{T}_p(\mathbf{m})\mathbf{a} = \mathbf{B}_p\mathbf{m} \tag{14}$$

where $\mathbf{B}_p = \mathcal{G}_p(\mathbf{a})$. (See **Definition 1**)

Proof: See [10], [30].

Using **Theorem 1**, we can say that

$$\tilde{\mathbf{a}}(\theta) = \mathcal{T}_p(\mathbf{m})\mathbf{a}(\theta) = \mathbf{B}(\theta)\mathbf{m} \tag{15}$$

where $\mathbf{B}(\theta) = \mathcal{G}_p(\mathbf{a}(\theta))$. Equation (12) is re-written as

$$[\hat{\theta}_i^{(k)}\ ,\ \hat{\mathbf{m}}_i^{(k)}] = \arg\max_{[\theta_i,\mathbf{m}]} \frac{\mathbf{m}^{\mathrm{H}}\mathbf{Q}(\theta_i)\mathbf{m}}{\mathbf{m}^{\mathrm{H}}\mathbf{S}(\theta_i)\mathbf{m}} \tag{16a}$$

where

$$\mathbf{Q}(\theta_i) = \mathbf{B}^{\mathrm{H}}(\theta_i)\mathscr{P}^{\perp}_{\tilde{\mathbf{A}}_{\bar{i}}}\hat{\mathbf{R}}\mathscr{P}^{\perp}_{\tilde{\mathbf{A}}_{\bar{i}}}\mathbf{B}(\theta_i) \tag{16b}$$

$$\mathbf{S}(\theta_i) = \mathbf{B}^{\mathrm{H}}(\theta_i)\mathscr{P}^{\perp}_{\tilde{\mathbf{A}}_{\bar{i}}}\mathbf{B}(\theta_i) \tag{16c}$$

Maximising first with respect to $\mathbf{m}$ according to the following criterion

$$\begin{cases} \underset{\mathbf{m}\in\mathbb{C}^{p\times 1}}{\text{maximise}} & \mathbf{m}^{\mathrm{H}}\mathbf{Q}(\theta_i)\mathbf{m} \\ \text{subject to} & \mathbf{m}^{\mathrm{H}}\mathbf{S}(\theta_i)\mathbf{m} = 1 \end{cases} \tag{17}$$

gives rise to the following cost function

$$\hat{\theta}_i^{(k)} = \arg\max_{\theta_i} \lambda_{\max}\Big(\mathbf{Q}(\theta_i); \mathbf{S}(\theta_i)\Big) \tag{18a}$$

where $\lambda_{\max}(\mathbf{Y};\mathbf{Z})$ is the maximum generalised eigenvalue of the matrix pencil (or matrix pair) $(\mathbf{Y};\mathbf{Z})$. Then, the vector $\hat{\mathbf{m}}_i^{(k)}$ is estimated after maximising (18a) and obtaining $\hat{\theta}_i^{(k)}$, viz.

$$\hat{\mathbf{m}}_i^{(k)} = \mathbf{v}_{\max}\Big(\mathbf{Q}(\hat{\theta}_i^{(k)}); \mathbf{S}(\hat{\theta}_i^{(k)})\Big) \tag{18b}$$

where $\mathbf{v}_{\max}(\mathbf{Y};\mathbf{Z})$ is the generalised eigenvector corresponding to the maximum generalised eigenvalue of the matrix pencil $(\mathbf{Y};\mathbf{Z})$. The vector $\hat{\mathbf{m}}_i^{(k)}$ is also normalised with respect to its first element. Then, an update is done on the vector $\hat{\boldsymbol{\Theta}}^{(k)}$ by replacing $\theta_i^{(k-1)}$ with $\theta_i^{(k)}$.

After estimating $\hat{\theta}_i^{(k)}$ and $\hat{\mathbf{m}}_i^{(k)}$, an update should be done on the corresponding column of $\tilde{\mathbf{A}}$ according to equation (15) as follows

$$\left[\tilde{\mathbf{A}}\right]_{(:,i)} \leftarrow \mathcal{T}_p(\hat{\mathbf{m}}_i^{(k)})\mathbf{a}(\hat{\theta}_i^{(k)}) \tag{19}$$

Then, increment $i \leftarrow i+1$ and do the same procedure to estimate the next AoA. If $i > q$, then $i \leftarrow 1$, and $k \leftarrow k+1$. The procedure is repeated until the vector $\hat{\boldsymbol{\Theta}}^{(k)}$ shows no satisfying improvement. The algorithm is summarised in the table **Algorithm 1**, below.

Algorithm 1: Implementation of the Proposed Agnostic Mutual Coupling ML AoA Estimator by Alternating Projection

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DATA: Collect $\mathbf{X}$ and compute $\hat{\mathbf{R}}$ according to equation (9f).

INITIALISATION:

$k \leftarrow 0$; $\quad \tilde{\mathbf{A}} \leftarrow \varnothing$; $\quad \mathscr{P}^{\perp} \leftarrow \mathbf{I}$; $\quad \hat{\boldsymbol{\Theta}}^{(k)} \leftarrow \varnothing$;

for i = 1 *to* q **do**

- **Step I.1:** Estimate $\hat{\theta}_i^{(k)}$ via 1D search using (18a), where:
  - $\mathbf{Q}(\theta) = \mathbf{B}^{\mathrm{H}}(\theta)\mathscr{P}^{\perp}\hat{\mathbf{R}}\mathscr{P}^{\perp}\mathbf{B}(\theta)$.
  - $\mathbf{S}(\theta) = \mathbf{B}^{\mathrm{H}}(\theta)\mathscr{P}^{\perp}\mathbf{B}(\theta)$.
- **Step I.2:** Obtain $\hat{\mathbf{m}}_i^{(k)}$ using equation (18b) and $\hat{\theta}_i^{(k)}$.
- **Step I.3:** Update the following quantities:
  - $[\tilde{\mathbf{A}}]_{(:,i)} \leftarrow \mathcal{T}_p(\hat{\mathbf{m}}_i^{(k)})\mathbf{a}(\hat{\theta}_i^{(k)})$.
  - $\mathscr{P}^{\perp} \leftarrow \mathbf{I} - \tilde{\mathbf{A}}(\tilde{\mathbf{A}}^{\mathrm{H}}\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{A}}^{\mathrm{H}}$.
  - $[\boldsymbol{\Theta}^{(k)}]_{(i,1)} \leftarrow \hat{\theta}_i^{(k)}$.

MAIN LOOP:

do

- $\hat{\boldsymbol{\Theta}}^{\text{old}} \leftarrow \hat{\boldsymbol{\Theta}}^{(k)}$
- $k \leftarrow k+1$

for i = 1 *to* q **do**

- Compute $\mathscr{P}^{\perp} \leftarrow \mathbf{I} - \tilde{\mathbf{A}}_{\bar{i}}(\tilde{\mathbf{A}}_{\bar{i}}^{\mathrm{H}}\tilde{\mathbf{A}}_{\bar{i}})^{-1}\tilde{\mathbf{A}}_{\bar{i}}^{\mathrm{H}}$, where $\tilde{\mathbf{A}}_{\bar{i}}$ is obtained by omitting the i^{th} column from matrix $\tilde{\mathbf{A}}$.
- Do **Step I.1** to estimate $\hat{\theta}_i^{(k)}$.
- Do **Step I.2** to obtain $\hat{\mathbf{m}}_i^{(k)}$.
- Update $[\tilde{\mathbf{A}}]_{(:,i)} \leftarrow \mathcal{T}_p(\hat{\mathbf{m}}_i^{(k)})\mathbf{a}(\hat{\theta}_i^{(k)})$ and $[\boldsymbol{\Theta}^{(k)}]_{(i,1)} \leftarrow \hat{\theta}_i^{(k)}$ as done in **Step I.3**.

while $\|\hat{\boldsymbol{\Theta}}^{(k)} - \hat{\boldsymbol{\Theta}}^{\text{old}}\| > \xi$

V. Simulation Results

This section presents some simulations to demonstrate the potential of the proposed algorithm. More simulation results should be found in [32]. We have conducted three experiments by fixing the following simulation parameters: $N = 7$ antennas, $q = 2$ sources, and the

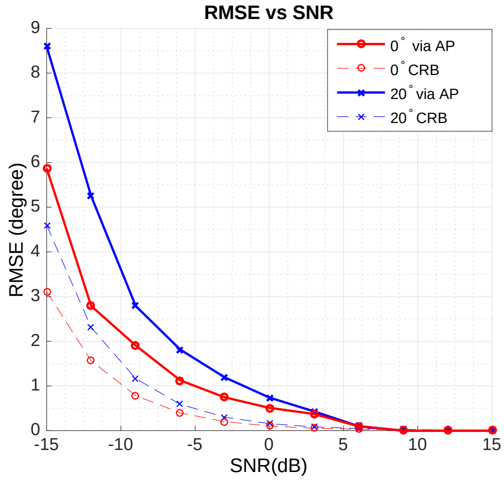

Fig. 1: RMSE of AoAs on a log-scale vs. SNR of the 1^{st} experiment.

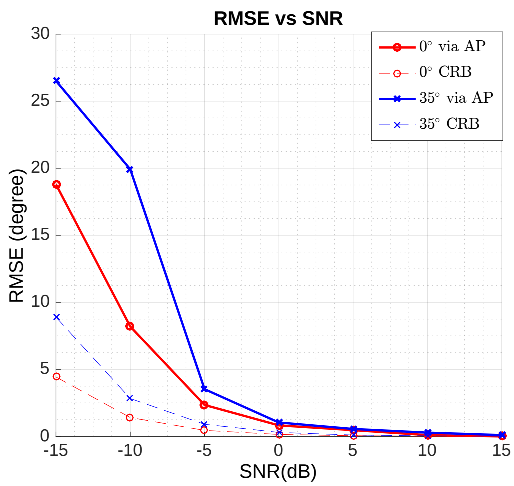

Fig. 2: RMSE of AoAs on a log-scale vs. SNR of the 2^{nd} experiment.

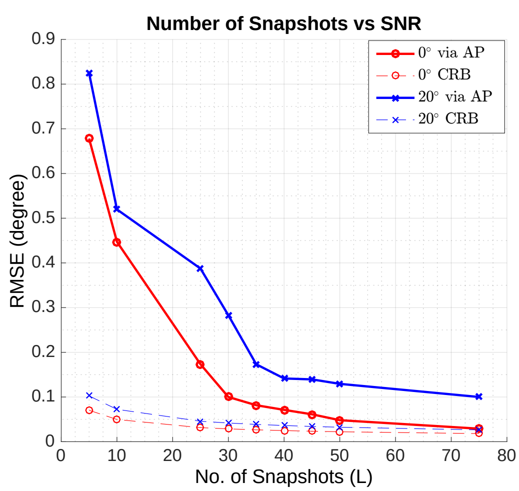

Fig. 3: RMSE of AoAs on a log-scale vs. Number of Snapshots of the 3^{rd} experiment.

RMSE is averaged over 200 trials. In all the experiments, we compare the RMSE of the AoA estimates with the Cramer-Rao bound that takes into account joint estimation of AoAs and coupling parameters [31]. This threshold indeed depends on several factors, such as separation, correlation, and number of sources.

In the 1^{st} experiment (Fig. 1), we have fixed the AoAs to $\theta_1 = 0°$ and $\theta_2 = 20°$. The sources are uncorrelated and are generated as independent and identically distributed (i.i.d) according to a Gaussian distibution. The number of snapshots is $L = 100$. Also, the number of coupling parameters are $p = 3$ with $\mathbf{m} = [1, 0.3115+0.3911j, -0.3063-0.1314j]$. We can see that the RMSE of both AoA estimates via the proposed method are close to their corresponding CRBs when SNR exceeds 5 dB.

In the 2^{nd} experiment (Fig. 2), the two sources are coherent. the AoAs are now more separated compared to the 1^{st} experiment, namely $\theta_1 = 0°$ and $\theta_2 = 35°$. The number of snapshots is $L = 100$. Moreover, the number of coupling parameters are $p = 2$, with $\mathbf{m} = [1, 0.1563 - 0.475j]$. We can see that the RMSE per SNR is higher than those of the 1^{st} experiment, even though we have less coupling parameters and AoAs being more separated. This is due to coherency of the sources. However, we see that the RMSE of both AoA estimates are close to their corresponding CRBs, when SNR exceeds 15 dB.

In the 3^{rd} experiment (Fig. 3), we plot the RMSE v.s. number of snapshots L, with SNR fixed to 5 dB. The sources are uncorrelated and are generated as i.i.d according to a Gaussian distibution. The AoAs are brought back to the values of experiment 1, i.e. $\theta_1 = 0°$ and $\theta_2 = 20°$, but with less coupling parameters, i.e. $p = 2$ with $\mathbf{m} = [1, 0.3561 - 0.22j]$. The RMSE of $\hat{\theta}_1$ is close to its corresponding CRB, when L exceeds 75, however, $\hat{\theta}_2$ still shows some error of about $0.1°$.

VI. Conclusion

In this paper, we have proposed an iterative algorithm based on Alternating Projection in order to optimise the Deterministic Maximum Likelihood cost function that takes into account mutual coupling. Throughout the operation of the algorithm, mutual coupling parameters were treated as nuissance parameters, thus the name *"Mutual Coupling Agnostic"*. Furthermore, the sources are allowed to be coherent. Simulation results demonstrate the potential of the proposed algorithm, as it does attain the Cramer-Rao bound, when the SNR or number of snapshots exceed a certain threshold.

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