Power Management for Cooperative Localization: A Game Theoretical Approach

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***Abstract*—Network cooperation among agents can significantly increase their position accuracy at the cost of power consumption. Current power management techniques aim at minimizing the total position estimation errors over all the agents subject to the power budgets. There are two main drawbacks for these approaches. First, the performance of a single agent may be sacrificed for the benefit of the whole network, and second, full power budget may be used for only marginal performance improvement on the position accuracy. This paper proposes a new type of power management strategies where each agent individually minimizes its square position error bound (SPEB) penalized by its power cost. The strategies are obtained as solutions to two power management games that are formulated under the knowledge of local information and global information, respectively. We show that agents are more likely to cooperate when global information is available or the channel quality is good. Analytical and numerical results show that the proposed strategies significantly reduce the energy consumption with only marginal performance loss in position accuracy.**

***Index Terms*—Localization, cooperative techniques, power allocation, game theory, Nash equilibrium**

I. Introduction

High-accuracy localization is promising for various applications and services, such as indoor navigation, social networking, logistics tracking, and emergency rescue [1]–[9]. However, high-accuracy localization may not be attainable by traditional localization techniques in many scenarios [10]–[15]. For example, the global positioning system (GPS) does not work well in harsh propagation environments, and received signal strength based localization does not provide a satisfactory precision in indoor applications.

There have been studies on improving position accuracy without upgrading infrastructure. In particular, the authors in [8] and [9] showed that agents (nodes with unknown positions) can improve their position accuracy by cooperating with each other, referred to as *cooperative localization*. For example, two agents can improve their position estimates by sharing their location information and by taking range measurements between each other. It was shown that more cooperation provides higher position accuracy. However, the gain of the position accuracy is achieved at the cost of additional range measurements, which induces power consumption. When nodes have poor channel qualities, cooperation can drain their batteries quickly without notable increase in their position accuracy. Since mobile devices are energy-limited [16]–[18], it is critical to balance position accuracy and energy consumption in cooperative localization networks.

Power-efficient network localization techniques have been studied for networks without agent cooperation in [19]–[24] and with agent cooperation in [25]–[27]. Most of the studies considered power optimization as a common global objective for all the agents. For example, the authors in [19] considered total power minimization subject to position accuracy requirements. Moreover, the power allocation techniques are mostly limited to synchronous networks, where agent nodes make one-way time-of-arrival (TOA) range measurement with each other based on perfect global clock synchronization [28]–[32]. Yet, it is highly challenging to achieve global clock synchronization up to the nano second timescale as required by high-accuracy localization.[1] Nevertheless, there is no trivial solution to extend the techniques in [25]–[27] to asynchronous networks. This is because, agents need to perform *round-trip TOA range* measurements in asynchronous networks [33], leading to power allocation strategies that are coupled among agents. As a result, a power optimization framework that incorporates individual objectives of the agents for both synchronous and asynchronous cooperative localization networks is needed.

The goal of this paper is to develop distributed power management strategies for cooperative network localization. The scenario of interest is a set of agents that have some

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[1]For example, in the LTE-Advanced network, the adjacent cell time synchronization is only up to micro seconds. Moreover, agents are not synchronized when they are associated with different Wi-Fi networks or bluetooth networks.

prior knowledge of their locations but want to improve the position accuracy by making inter-node range measurements. The power management discussed in this paper is different from the existing power allocation strategies (e.g., [25]–[27]), in the sense that, each agent not only allocates power over different cooperative links under a prescribed power budget, but also manages the total power budget for a better accuracy and power trade-off. In particular, agents are characterized as "selfish" distributed nodes that are unwilling to sacrifice their power for global performance gains. As a result, instead of formulating a common *global* objective optimization, a multi-agent optimization is considered, where each agent minimizes its own cost function. Such a scheme naturally falls into the scope of game theory [34]–[38]. It is worthy to emphasize that the traditional approaches may result in the scenario where some agents achieve much better performance than the other agents, whereas, using game theoretical approaches, mechanisms can be designed to balance the performance of all the agents.

Game theory has been applied for developing localization algorithms over recent years. In [39]–[44], the process of locating target nodes by a set of anchors was modeled as a coalitional game. Using the coalitional game approach, various algorithms have been developed to address problems such as sleep time allocation among anchors [40], [41], dynamic range measurement allocation [42], and node selection [43] or link selection [44] in forming a cooperative localization network. However, all these studies considered localizing target nodes by a set of anchors, and focused on the power and communication cost of the anchors, whereas little was known about the power management for agent cooperation.

The following two questions are essential for network localization with agent cooperation from a power management game perspective:

- **Which links to select for cooperation?** Cooperation requires considerable amount of power and communication overhead, while some links may only provide marginal performance improvement. Therefore, it is essential to select links that are more cost-effective for cooperation.
- **How much power to allocate?** The agents' power allocation strategies are correlated, even though they have separate objectives. In particular for asynchronous networks, the range quality between two agents depend on the power allocation of both agents, and hence it is non-trivial to find a good power allocation strategy that benefits all the agents.

In this paper, we propose two power management games for different application scenarios. The first game considers agents with only local information, such as the channel quality between themselves and their neighbors. They select the power allocation strategy as the *best response* to the knowledge of neighbors' tentative power allocations, and iterate among agents for a stable power allocation in the network. The Nash equilibrium (NE) is used as a solution concept. The conditions on the existence and uniqueness of the NE are investigated. The second game considers agents with global information and a *Pareto optimal* strategy via the Nash bargaining solution [45] is developed. This is of particular interest in a *small* network where the price of global information exchange is negligible. The main contributions of this paper are as follows:

- We propose a framework of power management strategies for cooperative localization in both synchronous and asynchronous networks.
- We determine the conditions for agent cooperation based on the channel quality. In particular, it is found that agents are more likely to cooperate when global information is available compared to when only local information is known.
- We demonstrate that the proposed strategies significantly reduce the energy consumption with only marginal degradation in position accuracy.

The rest of the paper is organized as follows. Section II presents the system model and defines the power management game for localization. Section III and IV study the competitive power management game and the coordinated power management game, respectively. Numerical results are demonstrated in Section V, and conclusions are given in Section VI.

Notations: The bold characters $\mathbf{a}$ and $\mathbf{A}$ denote a vector and a matrix, respectively. The notation $\mathbf{a} \succeq \mathbf{b}$ means $a_i \geq b_i$ for each i, and $\mathbf{A} \succeq \mathbf{0}$ means $\mathbf{A}$ is a positive-semidefinite matrix. For a function $f(x)$, $x \in \mathbb{R}$, $f^{'}(x) = \frac{d}{dx}f(x)$ and $f^{''}(x) = \frac{d^2}{dx^2}f(x)$. The function $y = \mathcal{O}(x^\alpha)$ means $\lim_{x\to\infty}\frac{y}{x^\alpha} = C < \infty$.

II. System Model

This section illustrates the model for cooperative localization and introduces the power management game for network localization.

A. Cooperative Localization

Consider a wireless network with K agents, which are nodes with unknown positions. The agents obtain initial estimates $\hat{\mathbf{p}}_k^{\mathrm{o}}$ of their positions $\mathbf{p}_k \in \mathbb{R}^2$ from the anchors, which are nodes with known positions. The associated accuracy of estimates $\hat{\mathbf{p}}_k^{\mathrm{o}}$ are captured by 2×2 equivalent Fisher information matrices (EFIMs) [8], denoted as $\mathbf{J}_k^{\mathrm{o}}$ and known to the agents. Fig. 1 illustrates an example topology for a $K = 3$ agent network, where the positions of anchors are not shown.

The agents aim at increasing their position accuracy through cooperation. Specifically, each agent k makes range measurements with its neighbors, and then obtains a new estimation $\hat{\mathbf{p}}_k$ from $\hat{\mathbf{p}}_k^{\mathrm{o}}$ based on the range measurements as well as the initial estimates $\hat{\mathbf{p}}_j^{\mathrm{o}}$ and $\mathbf{J}_j^{\mathrm{o}}$ from its neighbors. We focus on the performance after the agents have made one round of cooperation. It has been shown in [8] that the mean squared error (MSE) of the position estimation for agent k is bounded by the following *individual* SPEB[2] as

$$\mathbb{E}\left\{\|\hat{\mathbf{p}}_k - \mathbf{p}_k\|^2\right\} \geq \mathrm{tr}\{\mathbf{J}_k^{-1}\} \tag{1}$$

[2]The individual SPEB in (1) is used as the performance metric for position accuracy because it is tight in high signal-to-noise ratio (SNR) regimes, as demonstrated by numerical results in Section V.

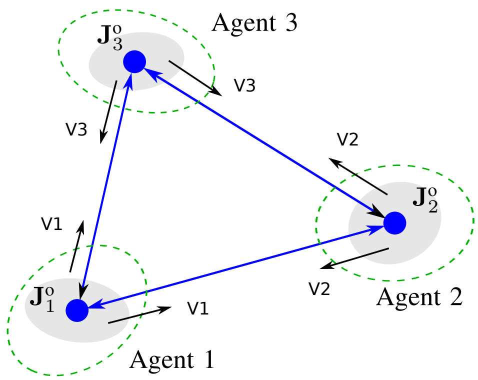


Fig. 1. A topology for a $K = 3$ agent network. As a geometric illustration, the shaded ellipses represent the initial location information [8] before cooperation and dash ellipses represent the expanded location information after cooperation.

where $\mathbf{J}_k$ is the 2×2 individual EFIM for agent k after cooperation. To specify $\mathbf{J}_k$, denote $\mathcal{N}(k)$ as the set of neighbors of agent k, let x_{kj} be the transmit power sent from agent k to agent j for range measurement, and let ξ_{kj} be the channel ranging quality [8] between agent k and agent j.[3] The individual EFIM $\mathbf{J}_k$ is given in the following lemma.

Lemma 1 (Individual EFIM): The individual EFIM $\mathbf{J}_k$ for agent k can be expressed as

$$\mathbf{J}_k = \mathbf{J}_k^{\text{o}} + \sum_{l \in \mathcal{N}(k)} g_{kl}(x_{kl}, x_{lk}) \mathbf{u}_{kl} \mathbf{u}_{kl}^{\text{T}} \tag{2}$$

where for round-trip TOA ranging in an asynchronous network

$$g_{kl}(x_{kl}, x_{lk}) \triangleq \frac{4 x_{kl} x_{lk} \xi_{kl}}{x_{kl} + x_{lk} + 4 x_{kl} x_{lk} \xi_{kl} \delta_{kl}} \tag{3}$$

and for one-way TOA ranging in a synchronous network

$$g_{kl}(x_{kl}, x_{lk}) \triangleq \frac{(x_{kl} + x_{lk}) \xi_{kl}}{1 + (x_{kl} + x_{lk}) \xi_{kl} \delta_{kl}} \tag{4}$$

in which

$$\delta_{kl} = \mathbf{u}_{kl}^{\text{T}} (\mathbf{J}_l^{\text{o}})^{-1} \mathbf{u}_{kl} \tag{5}$$

$\mathbf{u}_{kj} = [\cos\phi_{kj} \;\; \sin\phi_{kj}]^{\text{T}}$, and ϕ_{kj} denotes the angle between agent k and j as illustrated in Fig. 1.

Proof: (Sketch) The case for synchronous networks has been derived in [9, Appendix C]. For asynchronous networks, similar derivation can be applied with the modified equivalent ranging intensity $\lambda_{kj} = 4 x_{kj} x_{jk} \xi_{kj} / (x_{kj} + x_{jk})$ [20] for round-trip TOA ranging.[4] □

Throughout this paper, the analysis and insights mainly focus on asynchronous networks. The results for synchronous networks follow similarly, with details omitted due to page limit.

[3] In practice, the channel ranging quality ξ_{kj} can be calculated using the model in [8, Theorem 1]. Note that, in general, obtaining ξ_{kj} may require some communication overhead among agents.

[4] The equivalent ranging intensity λ_{kj} takes a form different from [46, equation (1)], because the constant 4 here was absorbed in the channel quality ξ_{kj} in [46, equation (1)]. Thus the gain function (3) is consistent to the term in [46, equation (8)] up to a scaled channel quality.

B. Power Management Game for Localization

Let $\mathbf{x}_k \triangleq \{x_{kj}\}_{j \neq k}$ be the collection of power allocation variables of agent k, and let $\mathbf{x}_{-k} \triangleq \{\mathbf{x}_j\}_{j \neq k}$ be the power allocation variables of all the other agents. Each agent k has its own objective (cost function) that minimizes the individual SPEB penalized by the power consumption, formulated as

$$f_k(\mathbf{x}_k, \mathbf{x}_{-k}) \triangleq \text{tr}\left\{\mathbf{J}_k(\mathbf{x}_k, \mathbf{x}_{-k})^{-1}\right\} + V_k \sum_{j \neq k} x_{kj} \tag{6}$$

where $V_k > 0$ is an agent-specific *power conservative level,* and the term $V_k \sum_{j \neq k} x_{kj}$ characterizes the power cost. Each agent k finds the power allocation $\mathbf{x}_k$ under the admissible set $\mathbf{x}_k \in \mathcal{P}_k = \{x_{kj} \geq 0, \forall j \neq k : \sum_{j \neq k} x_{kj} \leq P^{(k)}\}$.

The power management game can be written as a three-tuple $(\mathcal{K}, \boldsymbol{\mathcal{X}}, \mathbf{f})$, where $\mathcal{K}$ is the set of agents (players of the game); $\boldsymbol{\mathcal{X}} = \prod_{k=1}^{K} \mathcal{X}_k$ is the set of possible combinations of link selection and power allocation strategies (action), in which $\mathcal{X}_k$ denotes the strategy of agent k; and $\mathbf{f} = (f_1, f_2, \ldots, f_K)$ is the cost function vectors of all the agents under some power allocation $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K)$.

III. Competitive Power Allocation with Local Information

Consider that the agents have only local information (such as the power allocation and the EFIM) from their neighbors. In the absence of a central controller in the network, one reasonable choice of power allocation strategy is to follow the *best response* to the other agents' power allocation. Specifically, by observing the power allocation of all the other agents, agent k computes power allocation $\mathbf{x}_k$ to minimize its individual cost function $f_k(\mathbf{x}_k, \mathbf{x}_{-k})$. The *competitive* power management game is formulated as follows:

Competitive power management game:

$$(\mathcal{G}1): \quad \begin{array}{ll} \underset{\mathbf{x}_k \succeq \mathbf{0}}{\text{minimize}} & f_k(\mathbf{x}_k, \mathbf{x}_{-k}) \\ \text{subject to} & \sum_{j \neq k} x_{kj} \leq P^{(k)} \end{array}$$

for each agent k.

One important solution concept for the competitive power management game is the NE.

Definition 1 (Nash Equilibrium): A power allocation profile $\mathbf{x}^* = (\mathbf{x}_1^*, \mathbf{x}_2^*, \ldots \mathbf{x}_K^*)$ is called an NE (in pure strategies) if and only if the following holds:

$$f_k(\mathbf{x}_k^*, \mathbf{x}_{-k}^*) \leq f_k(\mathbf{x}_k, \mathbf{x}_{-k}^*) \qquad \forall \mathbf{x}_k \in \mathcal{P}_k \tag{7}$$

for all agents k.

The NE in the power management game is a power allocation profile for all the agents in the network that indicates that none of the agents can benefit more from changing its own power allocation unilaterally. The power management game can be interpreted as each agent "persuading" its neighbors to contribute more power for cooperation, but at the same time, trying to minimize its own power cost. The notion of the NE thus characterizes the situation where the agents reach an agreement on the power allocation.

A quick observation can be made from the following proposition.

Proposition 1 (Existence of the NE): In asynchronous networks, game $\mathcal{G}1$ always admits an NE $\mathbf{x}^* = \mathbf{0}$.

Proof: Let $\mathbf{x}^* = (\mathbf{x}_k^*, \mathbf{x}_{-k}^*) = \mathbf{0}$. Consider that agent k chooses power allocation $\mathbf{x}_k \neq \mathbf{0}$, where there exists $l \neq k$, such that $x_{kl} \neq 0$. However, since $x_{lk}^* = 0$, we have $g(x_{kl}, x_{lk}^*) = 0$ from (3) for all $l \in \mathcal{N}(k)$. As a result, $f_k(\mathbf{x}_k, \mathbf{x}_{-k}^*) = \text{tr}\left\{(\mathbf{J}_k^{\text{o}})^{-1}\right\} + V_k \sum_{l \neq k} x_{kl} > \text{tr}\left\{(\mathbf{J}_k^{\text{o}})^{-1}\right\} = f_k(\mathbf{x}_k^*, \mathbf{x}_{-k}^*)$, which suggests that $\mathbf{x}^* = \mathbf{0}$ is a NE from Definition 1. □

In asynchronous networks, not to cooperate is always an NE of game $\mathcal{G}1$. This is because, with round-trip TOA ranging, if either one of the agents allocates zero power on a round-trip TOA link, neither of them can achieve any improvement from this range measurement. Thus the concept of cooperating NE is introduced to characterize the scenario of cooperation.

Definition 2 (Cooperating NE): A *cooperating* NE of game $\mathcal{G}1$ is an NE $\mathbf{x}^*$ that satisfies $\mathbf{x}^* \neq \mathbf{0}$, whereas $\mathbf{x}^* = \mathbf{0}$ is a *non-cooperating* NE.

The cooperating NE (if exists) is more favorable than the non-cooperating NE. Therefore, we need to understand the condition on the existence of a cooperating NE, and once it exists, we need to know whether it is unique and how we find it. This section addresses these issues.

A. Best Response Strategy

Under the competition mechanism in game $\mathcal{G}1$, the *best response strategy* $\mathbf{T}_k(\mathbf{x}_{-k})$ for agent k given the knowledge of other agents' power allocation $\mathbf{x}_{-k}$ is the power allocation $\mathbf{x}_k^{\text{BR}}$ that minimizes the individual cost function f_k in (6), i.e.,

$$\mathbf{x}_k^{\text{BR}} = \mathbf{T}_k(\mathbf{x}_{-k}) \triangleq \arg \min_{\mathbf{x}_k \in \mathcal{P}_k} f_k(\mathbf{x}_k, \mathbf{x}_{-k}). \tag{8}$$

Note that given $\mathbf{x}_{-k}$, the problem in (8) is convex under both asynchronous and synchronous cases, and hence, there are efficient algorithms to find $\mathbf{x}_k^{\text{BR}}$ following the convex optimization framework [47], [48]. Moreover, as can be seen from cost function f_k in (6), computing $\mathbf{x}_k^{\text{BR}}$ only requires local information, such as channel quality $\{\xi_{kj}\}_{j \in \mathcal{N}(k)}$ and the power allocation $\{\mathbf{x}_j\}_{j \in \mathcal{N}(k)}$, from the neighbors of agent k.

Let $\widetilde{\mathbf{T}} = (\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_K) : \widetilde{\mathcal{P}} \mapsto \widetilde{\mathcal{P}}$ be the network best response mapping, where $\widetilde{\mathcal{P}} \triangleq \prod_{k=1}^K \mathcal{P}_k$ is the set of admissible power allocations in the network. The following dynamic equation characterizes the *best response iteration* $\mathbf{x}(n) \in \widetilde{\mathcal{P}}$:

$$\mathbf{x}(n+1) = \widetilde{\mathbf{T}}(\mathbf{x}(n)) \tag{9}$$

where n is the iteration index. The stationary points of (9) can be defined by the following *fixed point* equation:

$$\mathbf{x}^* = \widetilde{\mathbf{T}}(\mathbf{x}^*). \tag{10}$$

Note that the fixed point equation (10) also characterizes the set of NEs of the competitive power management game $\mathcal{G}1$.

B. Cooperating NE in Two-agent Game

To start with, we first consider the case of two-agent networks.

1) Best Response Solution: Consider a two-agent competitive power management game $\mathcal{G}1$. The *conservative coefficient* defined as follows is found to be an important parameter for the solution to the game.

Definition 3 (Conservative Coefficient): The conservative coefficient of agent k on link (k, j) is defined as

$$\Gamma_{kj} \triangleq \frac{V_k}{\mathbf{u}_{kj}^{\text{T}} (\mathbf{J}_k^{\text{o}})^{-2} \mathbf{u}_{kj}}. \tag{11}$$

Denote the two agents as $k, j \in \{1, 2\}$, $k \neq j$. The closed-form solution to the best response problem (8) is given as follows.

Lemma 2 (Best Response Solution): The best response power allocation $x_k^{\text{BR}} = T_{kj}(x_{jk})$ of agents $k, j \in \{1, 2\}$, $k \neq j$, is given by[5]

$$T_{kj}(x_{jk}) = \left\{ \frac{\left(2\sqrt{\xi_{kj}/\Gamma_{kj}} - 1\right) x_{jk}}{1 + 4\xi_{kj}\left(\delta_{kj} + \delta_{jk}\right) x_{jk}} \right\}_0^{P^{(k)}} \tag{12}$$

for asynchronous networks, and

$$T_{kj}(x_{jk}) = \left\{ \frac{\sqrt{\xi_{kj}/\Gamma_{kj}} - 1}{\xi_{kj}\left(\delta_{kj} + \delta_{jk}\right)} - x_{jk} \right\}_0^{P^{(k)}} \tag{13}$$

for synchronous networks, where the parameter δ_{kj} is defined in (5), and $\{x\}_0^P = 0$ if $x < 0$, $\{x\}_0^P = P$ if $x > P$, and $\{x\}_0^P = x$, otherwise.

Proof: Please refer to Appendix A. □

The conservative coefficient Γ_{kj} in (11) measures agent k's incentive for cooperation. Specifically, if Γ_{kj} is larger, agent k is not likely to cooperate on link (k, j), i.e., to allocate zero power on link (k, j) for range measurement with its neighbor j. The conservative coefficient is determined by agent k's power conservative level V_k, the initial localization quality captured by $\mathbf{J}_k^{\text{o}}$, as well as the ranging direction $\mathbf{u}_{kj}$. If agent k has a good initial localization quality (corresponding to small $\text{tr}\{[\mathbf{J}_k^{\text{o}}]^{-1}\}$), the conservative coefficient Γ_{kj} is large.

Remark 1: Lemma 2 implies that in asynchronous networks, if agent j increases the power allocation x_{jk} on link (k, j) for round-trip TOA range measurements, agent k would also increase the power allocation x_{kj} as the best response. In contrast, in synchronous networks, if agent j increases the power x_{jk}, agent k would decrease the power x_{kj} as the best response.

2) Necessary Condition for Cooperation: In order to have $T_{kj}(x_{jk}) > 0$ in (12), the coefficient in the numerator must be positive. Thus, Lemma 2 implies a straightforward necessary condition for cooperation under two-agent competitive game $\mathcal{G}1$.

Proposition 2 (Necessary Condition for Cooperation): For agents $k, j \in \{1, 2\}$, $k \neq j$, a necessary condition on the existence of a cooperating NE $\mathbf{x}^*$ such that $x_{kj}^* > 0$ is

$$\xi_{kj} > \frac{1}{4}\Gamma_{kj} \tag{14}$$

[5] Note that for the two-agent case, the notation $\mathbf{x}_{-k} = x_{jk}$ becomes a scalar.

for asynchronous networks, and

$$\xi_{kj} > \Gamma_{kj} \tag{15}$$

for synchronous network.

Proposition 2 shows how the channel quality ξ_{kj} and the power conservative coefficient Γ_{kj} affect the agent's decision on cooperation. If the channel quality is too poor, i.e., ξ_{kj} is small, agent k prefers not to cooperate, no matter how large x_{jk} is.

Remark 2 (Application of Proposition 2): Proposition 2 yields an important guideline on implementations in that the necessary condition (14) only requires limited local information exchange (only needs to know the direction ϕ_{kj}). As a result, when the necessary condition (14) fails to hold, the two agents can avoid spending communication resources for exchanging the matrices $\mathbf{J}_j^{\mathrm{o}}$ and $\mathbf{J}_k^{\mathrm{o}}$.

3) Uniqueness of the Cooperating NE: The sufficient and necessary condition for cooperation in two-agent networks is now derived as follows.

Theorem 1 (Cooperating NE in Two-agent Game): The two-agent competitive game $\mathcal{G}1$ admits a cooperating NE as $\mathbf{x}^* \neq \mathbf{0}$, if and only if, for $k, j \in \{1, 2\}$, $k \neq j$,

$$\xi_{kj} > \frac{1}{4}\left(\sqrt{\Gamma_{kj}} + \sqrt{\Gamma_{jk}}\right)^2 \tag{16}$$

for asynchronous networks, and

$$\xi_{kj} > \min\{\Gamma_{kj}, \Gamma_{jk}\} \tag{17}$$

for synchronous networks. Moreover, for asynchronous networks, the cooperating NE is *unique*, and for synchronous networks, it is unique when $\Gamma_{kj} \neq \Gamma_{jk}$.

Proof: Please refer to Appendix B. □

From Theorem 1, one can draw the following insights on the *incentive for cooperation* under the competitive power management game:

- *Channel quality* ξ_{kj}: Larger ξ_{kj} leads to higher incentive for cooperation.
- *Conservative coefficient* Γ_{kj}: Smaller Γ_{kj} leads to higher incentive for cooperation. From (11), both V_k and $\mathbf{J}_k^{\mathrm{o}}$ affect Γ_{kj}. In particular, a smaller parameter V_k or a larger quantity $\mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}$ yields a smaller Γ_{kj}.
- *Network synchronism*: Under the same channel quality ξ_{kj}, the agents in synchronous networks have higher incentive for cooperation than the ones in asynchronous networks as $\frac{1}{4}(\sqrt{\Gamma_{kj}} + \sqrt{\Gamma_{jk}})^2 > \min\{\Gamma_{kj}, \Gamma_{jk}\}$.

4) Stability of the NE: In asynchronous networks, when the cooperating NE exists, there are two NEs in the two-agent game. We are thus interested in the *stability* property of the NEs. Consider that the two agents play the iterative best response strategy following the iteration in (9). A natural question is whether the iteration in (9) converges. In addition, when the dynamic equation (9) converges, which NE will it likely converge to?

To address these issues, we make use of the notions of *stable* and *unstable* NE. Specifically, for a stable NE $\mathbf{x}^*$, given any initial point $\mathbf{x}(0)$ that is located in the neighborhood of $\mathbf{x}^*$, the iteration $\mathbf{x}(n)$ generated by the best response iteration (9) eventually converges to $\mathbf{x}^*$. On the other hand, for an unstable NE $\mathbf{x}^*$, there exists an open neighborhood $\mathcal{U}$ of $\mathbf{x}^*$, such that for each open neighbor $\mathcal{V}$ of $\mathbf{x}^*$ there exists an integer N_V such that for any initial point $\mathbf{x}(0) \in \mathcal{U}\backslash\mathbf{x}^*$, the iteration satisfies $\mathbf{x}(n) \notin \mathcal{V}$ for all $n \geq N_V$. In other words, the iteration $\mathbf{x}(n)$ is repelled from $\mathbf{x}^*$ after some small perturbation.

The following theorem shows the stability of the NE and the global convergence of the best response iteration in asynchronous networks.

Theorem 2 (Stability and Convergence in Asynchronous Networks): When the cooperating NE $\mathbf{x}_{\mathrm{c}}^* \neq (0, 0)$ exists, it is stable and the non-cooperating NE $\mathbf{x}^* = (0, 0)$ is unstable. In addition, the best response iteration $\mathbf{x}(n)$ in (9) converges to $\mathbf{x}_{\mathrm{c}}^*$ from any initial point $\mathbf{x}(0) \in \widetilde{\mathcal{P}}\backslash(0, 0)$. On the other hand, when the cooperating NE does not exist, $\mathbf{x}(n)$ converges to the non-cooperating NE.

Proof: Please refer to Appendix B. □

The above result establishes the global convergence property of the best response iteration under two-agent game $\mathcal{G}1$ in asynchronous networks. It also suggests that even though both agents are power conservative and selfish, they are still willing to cooperate when the channel quality is good. On the other hand, when the channel quality deteriorates, the cooperating NE may move to the origin and degenerate to a non-cooperating NE.

Remark 3 (Convergence in Synchronous Networks): From the best response solution (13) for synchronous networks, one can easily show that the best response iteration (9) can converge to the NE, which is stable, from any initial point.

Furthermore, we have the following result on the rate of convergence of the best response iteration (9).

Proposition 3 (Rate of Convergence in Asynchronous Networks): For a sufficiently small $\epsilon > 0$, there exists $\varrho > 0$, such that, for $\|\mathbf{x}(0) - \mathbf{x}^*\| < \varrho$, the sequence $\mathbf{x}(n)$ generated by the best response iteration (9) converges to $\mathbf{x}^*$ linearly, i.e.,

$$\|\mathbf{x}(n) - \mathbf{x}^*\| \leq (r + \epsilon)^n \|\mathbf{x}(0) - \mathbf{x}^*\|$$

where $r = T_{12}^{'}(x_{21}^*)T_{21}^{'}(x_{12}^*) < 1$.

Proof: Please refer to Appendix B. □

Proposition 3 demonstrates that if each agent follows the best response strategy in (12) in asynchronous networks, the solution dynamics converges to the cooperating NE at a linear convergence rate. Note that for synchronous networks, one can easily find that the iteration following the best response in (13) converges in finite steps.

C. Cooperating NE in K-agent Network

In K-agent game $\mathcal{G}1$, the cooperating NE is usually not unique. For example, consider a three-agent asynchronous network. There may be three cooperating NEs, each of which corresponds to allocating zero power to one link and non-zero power to the other two links.[6] Therefore, we focus on the existence condition on the cooperating NE and the algorithm to avoid the non-cooperating NE.

[6]This is due to the gain function (3) in asynchronous networks, where if one agent allocates zero power on the link, the other agent would also allocate zero power as the best response.

Algorithm 1 Power allocation under competitive power management game $\mathcal{G}1$.

1) Each agent k evaluates the necessary condition in (14) for link (k, j) to the neighbor node $j \in \mathcal{N}(k)$. If (14) is not satisfied, $x_{kj} = 0$, and the link (k, j) is eliminated from the network. Otherwise, agent k and j exchange the EFIM $\mathbf{J}_k^{\mathrm{o}}$ and $\mathbf{J}_j^{\mathrm{o}}$.
2) If there is only one cooperating link associated with agent k and j, then the power allocation x_{kj} and x_{jk} is computed by solving the fixed point equation (10).
3) If there is more than one cooperating link, the power allocation x_{kj} and x_{jk} is computed following the best response iteration (9), with a strictly positive initial point $x_{kj}(0), x_{jk}(0) > 0$.

1) Existence of a Cooperating NE: Theorem 1 gives a sufficient and necessary condition for cooperation in a two-agent game. Correspondingly, the notion of a *cooperating link* is introduced for the study of the existence of a cooperating NE in K-agent game $\mathcal{G}1$.

Definition 4 (Cooperating Link): The link (k, j) that connects agent k and j is called a *cooperating link* if the channel quality ξ_{kj} satisfies (16) in asynchronous networks and (17) in synchronous networks.

A cooperating link establishes a sufficient condition on the existence of a cooperating NE in game $\mathcal{G}1$, as is summarized in the following proposition.

Proposition 4 (Existence of a Cooperating NE): There exists a cooperating NE if the network has at least one cooperating link.

Proof: Suppose there is no cooperating NE, but the link (k, j) satisfies condition (16) or (17). Then the network degenerates to a two-agent network for agents k and j. From Theorem 1, there exists a cooperating NE on the link (k, j), and hence this yields a contradiction. $\square$

2) Stability of the NE: Using similar techniques to those in Section III-B, the stability properties of the NE in K-agent game $\mathcal{G}1$ can be characterized as follows.

Proposition 5 (Stability Properties of the NE): If a cooperating NE exists, the non-cooperating NE $\mathbf{x}^* = \mathbf{0}$ is unstable.

Proposition 5 suggests a way to avoid the non-cooperating NE by simply choosing the initial point $\mathbf{x}(0)$ to be strictly positive on each of its elements.

Using the theoretical results in this section, an efficient power allocation algorithm under the competitive power management game $\mathcal{G}1$ is given in Algorithm 1.

Remark 4: Note that the necessary condition in (14) is only valid in two-agent networks. Therefore, Step 1 in Algorithm 1 yields a sub-optimal solution. Nevertheless, Algorithm 1 still gives good performance, as demonstrated by the numerical results in Section V.

Remark 5: Using the results in Theorem 2, Algorithm 1 can be shown to converge to the cooperative NE in two-agent networks globally from any initial points except the origin. However, in K-agent networks, the conditions for the convergence to a cooperative NE is not known.

D. Performance Evaluation

To evaluate the performance of the power allocation algorithm from the competitive power management game $\mathcal{G}1$, consider a *full* power allocation strategy $\mathbf{x}_k^{\mathrm{f}}$ for agent k as the solution to the following SPEB minimization problem

$$\begin{array}{ll} \underset{\mathbf{x}_k \succeq \mathbf{0}}{\text{minimize}} & \mathrm{tr}\{\mathbf{J}_k^{-1}(\mathbf{x}_k, \mathbf{x}_{-k})\} \\ \text{subject to} & \sum_{j \neq k} x_{kj} = P^{(k)} \end{array}$$

Correspondingly, let $\mathbf{x}_k^*$ be the solution of agent k from game $\mathcal{G}1$ given the power allocation $\mathbf{x}_{-k}$ from the other agents.

Moreover, consider the scenario where the power conservation level $V_k = V_k(P^{(k)})$ is a decreasing function of the power budget $P^{(k)}$, since a higher power budget $P^{(k)}$ usually implies relatively lower cost in power consumption, and hence, there should be a smaller V_k. In addition, let $\mathcal{I}_k(\mathbf{x}_k, \mathbf{x}_{-k}) = \mathrm{tr}\{\mathbf{J}_k^{-1}(\mathbf{x}_k, \mathbf{x}_{-k})\}$ be the SPEB for agent k achieved by the power allocation $(\mathbf{x}_k, \mathbf{x}_{-k})$. The following theorem evaluates the asymptotic performance of the proposed power allocation strategy.

Theorem 3 (Asymptotic Performance): Suppose $V_k(P^{(k)}) \to 0$ and $V_k(P^{(k)})^{\frac{1}{2}} P^{(k)} \to \infty$ as $P^{(k)} \to \infty$ for all k. Then, for both asynchronous and synchronous networks

$$\mathcal{I}_k(\mathbf{x}_k^*, \mathbf{x}_{-k}) \to \mathcal{I}_k(\mathbf{x}_k^{\mathrm{f}}, \mathbf{x}_{-k}), \text{ and } \sum_{j \in \mathcal{N}(k)} \frac{x_{kj}^*}{P^{(k)}} \to 0$$

as $P^{(k)} \to \infty$, for all channel qualities ξ_{kj}.

Proof: Please refer to Appendix C. $\square$

From Theorem 3, the SPEB performance achieved by the proposed power allocation strategy $\mathcal{X}$ can be arbitrarily close to the performance lower bound of the full power allocation $\mathcal{X}_{\mathrm{f}}$ when the power budget $P^{(k)}$ is sufficiently large. At the same time, the strategy $\mathcal{X}$ requires much less power compared to $\mathcal{X}_{\mathrm{f}}$.

Condition $V_k(P^{(k)}) \to 0$ leads to sufficiently high power allocation x_{kj}^* as $P^{(k)} \to \infty$, and condition $V_k(P^{(k)})^{\frac{1}{2}} P^{(k)} \to \infty$ guarantees that x_{kj}^* scales (order-wisely) slower than $P^{(k)}$ does. The intuition from Theorem 3 is that, in high power budget regions, the range measurement has sufficiently high accuracy and the performance improvement from cooperation is limited by the EFIMs $\{\mathbf{J}_k^{\mathrm{o}}\}$ before cooperation. As a result, as the power budget increases, the additional gain in terms of the SPEB achieved by allocating more power to the cooperative range measurement diminishes. This result demonstrates the energy efficiency of the power management game.

IV. Coordinated Power Allocation with Global Information

In this section, we focus on the scenario when the global information (such as network topology, channel quality, and position estimates) is available to all the agents, which thus can fully coordinate with each other. This is of particular interest in a small network. We are interested in a *Pareto optimal* strategy, under which only this strategy can reduce the cost (objective value) for at least one agent but not increase the cost of any other agents. In fact, there may exist an infinite number of Pareto optimal points, and each corresponds to a different fairness among the agents. In particular, an axiom-based fairness for cost distribution among agents is considered, which leads to the Nash bargaining solution [34], [45] for the power allocation in cooperating localization.

A. Coordinated Game via Nash Bargaining Solutions

The bargaining process is introduced as follows. Define $\mathcal{F}$ as the set of all possible costs that the agents can achieve, i.e., $\mathcal{F} = \{\mathbf{f}(\mathbf{x}) \big| \mathbf{x} \in \prod_{k=1}^{K} \mathcal{P}_k\}$, where $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_K(\mathbf{x}))$ is the vector of the costs (objective value) of the agents. Consider that the agents conduct a bargaining process to negotiate the power allocation on all the links. Each agent k has a requirement on the outcome f_k, such that the resulting cost from the bargaining is smaller than some threshold d_k (*disagreement point*). Otherwise, agent k would not participate in the global coordination for power allocation. Let $\mathbf{d} = (d_1, d_2, \dots, d_K) \in \mathcal{F}$ be the disagreement point of the bargaining, which means if after the bargaining, any agent obtains a higher cost $f_k > d_k$, then the global coordination fails and the final global cost becomes $\mathbf{f} = \mathbf{d}$.

The Nash bargaining problem is to assign the pair $(\mathcal{F}, \mathbf{d})$ to a unique cost vector $\mathbf{f}^*$ that is Pareto optimal, i.e., there does not exist a point $\mathbf{f} \in \mathcal{F}$ such that $f_k \leq f_k^*$ for all the agents and at least one of them holds the strict inequality. Nash showed that there exists a unique cost vector $\mathbf{f}^*$ that is Pareto optimal and satisfies three other axioms.[7] Such cost vector $\mathbf{f}^*$ is obtained by solving the following problem [34], [45]:

$$\underset{\mathbf{f} \in \mathcal{F}}{\text{minimize}} \quad \prod_{k=1}^{K} (d_k - f_k) \tag{18}$$

$$\text{subject to} \quad f_k \leq d_k, \quad k \in \mathcal{K} = \{1, 2, \dots, K\}. \tag{19}$$

However, the original form of the Nash bargaining problem (18)-(19) may not be a good formulation to find the power allocation in cooperative localization. First of all, the uniqueness of the cost vector $\mathbf{f}^*$ in (18) does not directly imply the uniqueness of the power allocation $\mathbf{x}^*$ to achieve $\mathbf{f}(\mathbf{x}^*) = \mathbf{f}^*$. Second, for some choices of d_k, it may happen that $f_k \geq d_k$ for all $\mathbf{f} \in \mathcal{F}$, i.e., agent k cannot further reduce its cost under all possible power allocations, in which case the objective in (18) trivially becomes 0, and there exist multiple solutions for $\mathbf{x}^*$.

To address these issues, we first propose a reasonable choice of the disagreement point $\mathbf{d}$, that is, the outcome $\mathbf{f}(\mathbf{0})$ of a non-cooperating NE $\mathbf{x} = \mathbf{0}$, which corresponds to the non-cooperating scenario. This is because the performance improvement is expected for at least one agent through global coordination compared to the non-cooperating strategies. A notion of strict feasibility is then introduced as follows.

Definition 5 (Strict Feasibility): Given $f_k^{\mathrm{d}} = f_k(\mathbf{0})$ for $k = 1, 2, \dots, K$, the Nash bargaining problem (18)-(19) is strictly feasible if there exists $\mathbf{x} \in \widetilde{\mathcal{P}}$ such that $f_k(\mathbf{x}) < f_k^{\mathrm{d}}$ for at least one k.

[7] The three other axioms specify three properties of the mapping $(\mathcal{F}, \mathbf{d}) \mapsto \mathbf{f}^*$: linearity, symmetry, and independence of irrelevant alternatives properties. Please refer to [34], [45] for details.

The power management game is formulated as follows.

<u>*Coordinated power management game:*</u>

$$(\mathcal{G}2): \quad \begin{array}{ll} \underset{\mathbf{x} \succeq \mathbf{0}}{\text{maximize}} & \sum_{k \in \mathcal{K}_1} \log(f_k^{\mathrm{d}} - f_k(\mathbf{x})) \\ \text{subject to} & f_k(\mathbf{x}) < f_k^{\mathrm{d}}, \qquad k \in \mathcal{K}_1 \\ & f_k(\mathbf{x}) \leq f_k^{\mathrm{d}}, \qquad k \in \mathcal{K}_0 \\ & \sum_{j \neq k} x_{kj} \leq P^{(k)}, \quad k \in \mathcal{K} \end{array} \tag{20}$$

where $f_k^{\mathrm{d}} = f_k(\mathbf{0})$ for $k \in \mathcal{K}$, and agent sets $\mathcal{K}_1$ and $\mathcal{K}_0$ are disjoint partition of $\mathcal{K}$ (i.e., $\mathcal{K}_1 \cap \mathcal{K}_0 = \varnothing$ and $\mathcal{K}_1 \cup \mathcal{K}_0 = \mathcal{K}$), such that (i) there exists $\mathbf{x} \in \widetilde{\mathcal{P}}$, such that $f_k < f_k^{\mathrm{d}}$ for all $k \in \mathcal{K}_1$; and (ii) for all $\mathbf{x} \in \widetilde{\mathcal{P}}$, we have $f_k(\mathbf{x}) \geq f_k^{\mathrm{d}}$ for at least one $k \in \mathcal{K}_0$.

If $\mathcal{K}_1 = \mathcal{K}$, the optimal power allocation $\mathbf{x}^*$ for problem (20) yields the optimal cost vector $\mathbf{f}(\mathbf{x}^*) = \mathbf{f}^*$ in the Nash bargaining problem (18)-(19), and $\mathbf{x}^*$ is Pareto optimal. If $\mathcal{K}_1 = \varnothing$, game $\mathcal{G}2$ is not strictly feasible, and the solution degenerates to $\mathbf{x}^* = \mathbf{0}$, implying that a Pareto optimal solution is to not cooperate. In general when $\mathcal{K}_1 \neq \varnothing$, there could be multiple choices of the agent partition $(\mathcal{K}_0, \mathcal{K}_1)$, and a low complexity algorithm for a sub-optimal agent partition is given in Algorithm 2.

B. Unique Solution and the Strict Feasibility

We next show that given an agent partition $(\mathcal{K}_0, \mathcal{K}_1)$, $\mathcal{G}2$ admits a unique solution, by proving its convexity.

Lemma 3 (Convexity): For both asynchronous and synchronous networks, the function $f_k(\mathbf{x})$ is convex in $\mathbf{x} \in \widetilde{\mathcal{P}}$, and the function $\sum_{k \in \mathcal{K}_1} \log(f_k^{\mathrm{d}} - f_k(\mathbf{x}))$ is concave in $\mathbf{x} \in \{\mathbf{x} \in \widetilde{\mathcal{P}} : f_k(\mathbf{x}) < f_k^{\mathrm{d}},\ k \in \mathcal{K}_1\}$.

Proof: Please refer to Appendix D. □

Lemma 3 implies that $\mathcal{G}2$ is convex, and hence there is a unique local optimal solution and it can be computed using efficient convex optimization techniques.

Remark 6 (Interpretation of Algorithm 2): Denote $(\mathcal{K}_0^*, \mathcal{K}_1^*)$ as the optimal agent partition to achieve the maximum objective value in (20). Under cases $\mathcal{K}_1^* = \mathcal{K}$ or $\mathcal{K}_1^* = \varnothing$, Step 2 with $\mu = 0$ solves a standard feasibility problem, which finds the optimal active agent set $\mathcal{K}_1^*$. Under the other cases, Steps 2-3 with $\mu = 1$ add agents one by one into set $\mathcal{K}_1$ in a greedy way.

As a special case for two-agent scenario, we now study the condition on the strict feasibility of $\mathcal{G}2$. It suffices to check the condition under which both agents allocate non-zero power for the cooperation. The result is summarized in the following theorem.

Theorem 4 (Cooperation Condition via Game $\mathcal{G}2$): Game $\mathcal{G}2$ is strictly feasible in a two-agent network, if and only if the channel quality ξ_{kj} between the two agents k and j satisfies

$$\xi_{kj} > \frac{1}{4} (\Gamma_{kj} + \Gamma_{jk}) \tag{22}$$

in asynchronous networks, and

$$\xi_{kj} > \frac{\Gamma_{kj} \Gamma_{jk}}{\Gamma_{kj} + \Gamma_{jk}} \tag{23}$$

in synchronous networks.

Proof: Please refer to Appendix E. □

Algorithm 2 Power allocation under coordinated power management game $\mathcal{G}2$.

1) Choose a sufficiently small parameter $\epsilon > 0$ and set $\mu = 0$. Initialize $\mathcal{K}_1 = \varnothing$.
2) Obtain the solution $(\mathbf{x}^t, t^*)$ to the following problem

$$\begin{aligned} \underset{\mathbf{x},t}{\text{minimize}} \quad & t + \mu \sum_{k\in\mathcal{K}\backslash\mathcal{K}_1} \big(f_k(\mathbf{x}) - f_k^{\mathrm{d}}\big) \qquad (21) \\ \text{subject to} \quad & f_k(\mathbf{x}) - f_k^{\mathrm{d}} \le t, \quad \forall k \in \mathcal{K}\backslash\mathcal{K}_1 \\ & f_k(\mathbf{x}) - f_k^{\mathrm{d}} \le -\epsilon, \quad \forall k \in \mathcal{K}_1 \\ & \mathbf{x} \in \widetilde{\mathcal{P}} \end{aligned}$$

and add agents $k \in \mathcal{K}\backslash\mathcal{K}_1$ into $\mathcal{K}_1$ if $f_k(\mathbf{x}^t) - f_k^{\mathrm{d}} \le -\epsilon$.
3) If $\mathcal{K}_1 \neq \varnothing$ and $\mathcal{K}_1 \neq \mathcal{K}$, then set $\mu = 1$ and repeat Step 2) until $\mathcal{K}_1$ stays invariant.
4) Let $\mathcal{K}_0 = \mathcal{K}\backslash\mathcal{K}_1$. Obtain the power allocation $\mathbf{x}^*$ by solving problem (20).

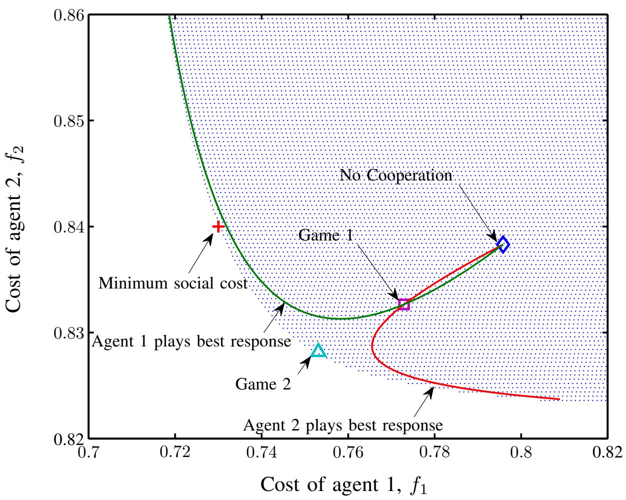


Fig. 2. Comparison of the performance achieved by different schemes in a two-agent asynchronous network. The blue curve illustrates the cost (f_1, f_2) under all possible power allocation x_{21} of agent 2, where agent 1 plays the best response power allocation $T_{12}(x_{21})$ to x_{21}, and the red curve is vice versa.

Note that condition (16) on the existence of a cooperating NE implies (22). Similarly, condition (17) implies (23).[8] These results are intuitive because if there exists a non-zero equilibrium under other games, the non-cooperating NE $\mathbf{x}^* = \mathbf{0}$ is not Pareto optimal and game $\mathcal{G}2$ is then strictly feasible. This result suggests that knowing the global information, the agents are more proactive in cooperating.

C. Comparison between $\mathcal{G}1$, $\mathcal{G}2$ and Social Cost Minimization

It is interesting to compare the proposed game theoretical approaches with the traditional approaches, which optimize a single objective over the whole network as in [9], [27]. In contrast to the proposed game theoretical approaches which try to balance the costs f_k of all the agents, the non-game power allocation approach in [9] corresponds to minimizing the cost of the whole network. Specifically, consider a centralized power allocation obtained by the following minimization problem

$$\underset{\mathbf{x}\in\widetilde{\mathcal{P}}}{\text{minimize}} \quad \sum_{k\in\mathcal{K}} f_k(\mathbf{x}) \qquad (24)$$

where $\sum_{k\in\mathcal{K}} f_k(\mathbf{x})$ is defined as the *social cost* of the network, i.e., the total SPEB penalized by the total power spent in the network. The following results give the condition on cooperation under social cost minimization (24) over a two-agent network.

Proposition 6 (Cooperation Condition via Social Cost Minimization): The power allocation solution for minimizing the social cost in (24) is non-zero in a two-agent network, if and only if the channel quality ξ_{kj} between the two agents k and j satisfies

$$\xi_{kj} > \frac{1}{4}\frac{(\sqrt{V_k}+\sqrt{V_j})^2}{V_k\Gamma_{kj}^{-1}+V_j\Gamma_{jk}^{-1}} \qquad (25)$$

for asynchronous networks, and

$$\xi_{kj} > \frac{\min\{V_k, V_j\}}{V_k\Gamma_{kj}^{-1}+V_j\Gamma_{jk}^{-1}} \qquad (26)$$

for synchronous networks.

Proof: The derivation follows that of Theorem 4 in Appendix E, and hence is omitted here for brevity. □

Proposition 6 suggests that social cost minimization (24) requires the lowest channel quality ξ_{kj} for cooperation in two-agent networks as compared to games $\mathcal{G}1$ and $\mathcal{G}2$.

However, the social cost minimization may require some agents to sacrifice their performance for the whole network. In general, the major difference between the two proposed game theoretical power allocations and the non-game power allocation via social cost minimization is that, the game approaches balance the performance of all the agents, whereas the social cost minimization focuses on the overall performance in the network. As a result, the social cost minimization may cause performance deterioration for some agents.

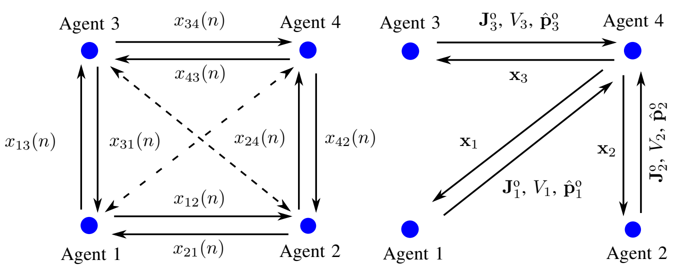
An numerical example is given in Fig. 2, which illustrates the comparison of the costs f_k achieved by different schemes in a two-agent asynchronous network. The shaded region represents the domain $\mathcal{F}$ as the region of the cost (f_1, f_2) that can be achieved by all feasible power allocations. The solutions from game $\mathcal{G}1$ and game $\mathcal{G}2$ improve the performance for both of the agents, whereas the power allocation from social cost minimization (24) degrades the performance of agent 2.[9] In this particular example, social cost minimization is unfair to agent 2, which participates in cooperation but achieves worse performance.

The key properties of game $\mathcal{G}1$, $\mathcal{G}2$, and the reference scheme based on social cost minimization are summarized in Table I.

[8]Note that $\frac{ab}{a+b} < \min\{a, b\}$ for $a, b > 0$.

	Competitive Game $\mathcal{G}1$	**Coordinated Game $\mathcal{G}2$**	**Social Cost Minimization**
Information Structure	Each agent knows the tentative power allocation of its neighbors	All the information is available to the network	All the information is available to the network
Solution Concept	Cooperating Nash Equilibrium	Nash Bargaining Solution	Global Objective Minimizer
Computation	Distributive	Centralized	Centralized
Cond. on Cooperation (2-agent ASYN case)	$\xi_{12} > \frac{1}{4}(\sqrt{\Gamma_{12}} + \sqrt{\Gamma_{21}})^2$	$\xi_{12} > \frac{1}{4}\left(\Gamma_{12} + \Gamma_{21}\right)$	$\xi_{12} > \frac{1}{4}\frac{(\sqrt{V_1}+\sqrt{V_2})^2}{V_1\Gamma_{12}^{-1}+V_2\Gamma_{21}^{-1}}$
Cond. on Cooperation (2-agent SYN case)	$\xi_{12} > \min\{\Gamma_{12}, \Gamma_{21}\}$	$\xi_{12} > \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{12}+\Gamma_{21}}$	$\xi_{kj} > \frac{\min\{V_1,V_2\}}{V_1\Gamma_{12}^{-1}+V_2\Gamma_{21}^{-1}}$
Performance (two-agent case)	Both agents improve (lower cost), i.e., $\mathbf{f}(\mathcal{G}1) \preceq \mathbf{f}(\mathbf{0})$	Both agents improve (lower cost), i.e., $\mathbf{f}(\mathcal{G}2) \preceq \mathbf{f}(\mathbf{0})$	Some agents may deteriorate, i.e., $f_k(\mathbf{x}^*) > f_k(\mathbf{0})$

Table I
COMPARISON BETWEEN $\mathcal{G}1$, $\mathcal{G}2$ AND SOCIAL COST MINIMIZATION



(a) Game $\mathcal{G}_1$ (b) Game $\mathcal{G}_2$

Fig. 3. An implementation example of message exchange under the two different games. (a) Under game $\mathcal{G}1$, power allocation is iterated among agents over the links (solid lines) that satisfy the necessary condition ((14) or (15)) for cooperation. The links that do not satisfy the condition (dash lines) are ruled out from power iteration. (b) Under game $\mathcal{G}2$, all the agents pass the information (EFIMs and parameters V's etc.) to a leader (Agent 4), which computes the power allocation and feeds back to the other agents.

D. Implementation Example

As an implementation example, consider a four-agent network illustrated in Fig. 3, where the links $(1, 4)$ and $(2, 3)$ do not satisfy the necessary condition ((14) or (15)) for cooperation, because the agents on both ends of the link are far away from each other. The example message passing among agents under the two power management games is illustrated. Under game $\mathcal{G}1$, power allocation is iterated among agents, and agents only need to communicate with their neighbors that have potential to cooperate (based on the necessary condition). Under game $\mathcal{G}2$, one agent is selected as the leader (Agent 4), and all the other agents pass the information (EFIMs, the channel qualities, and parameters V's etc.) to the leader, which computes the power allocation for all the agents.

Note that under the proposed power management games, some communication overhead is expected for better localization performance and more efficient power utilization. Nevertheless, since the ranging signals usually consume a large amount of time-frequency resources, the communication overhead to exchange the power allocation variables may be negligible.

[9]Geometrically, the minimizer of the social cost corresponds to the intersection point at which the tangent line $f_1 + f_2 = c$ just touches the Pareto boundary.

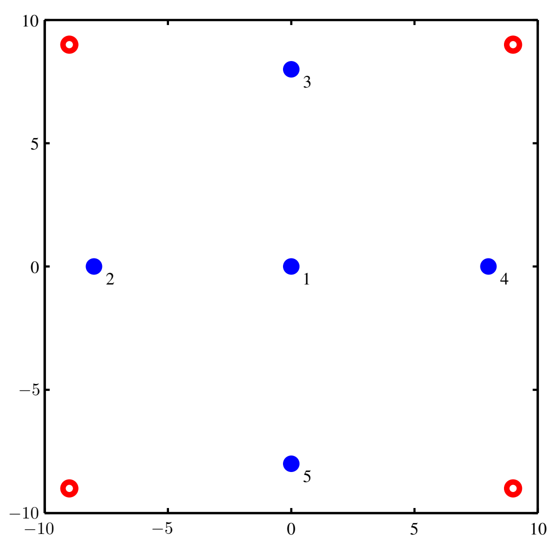


Fig. 4. The deployment of anchors (red circles) and agents (blue dots) in an asynchronous network.

V. NUMERICAL RESULTS

In this section, the performance of the game theoretical power management strategies is evaluated through numerical studies. Specifically, we evaluate MSE, average power consumption, and the objective value under different power allocation strategies for the cooperative localization based on round-trip TOA ranging in asynchronous networks.

The network topology is depicted in Fig. 4, where there are four anchors (red circles) and five agents (blue dots). The extended WINNER channel model in the line-of-sight (LOS) case under the *indoor small office* scenario [49] is adopted to model the path loss, which is specified as $\mathrm{PL[dB]} = 46.4 + 18.7\log_{10} d[\mathrm{m}]$, with shadow fading standard deviation being $\sigma = 3.1$ dB. The multipath effect is modeled using the Rician distribution with K-factor 4.7 dB [49]. In addition, the noise is normalized such that the average channel ranging quality ξ_{kj} over all the links is -6 dB. As a result, the parameters ξ_{kj} are Rician random variables scaled by the path loss and

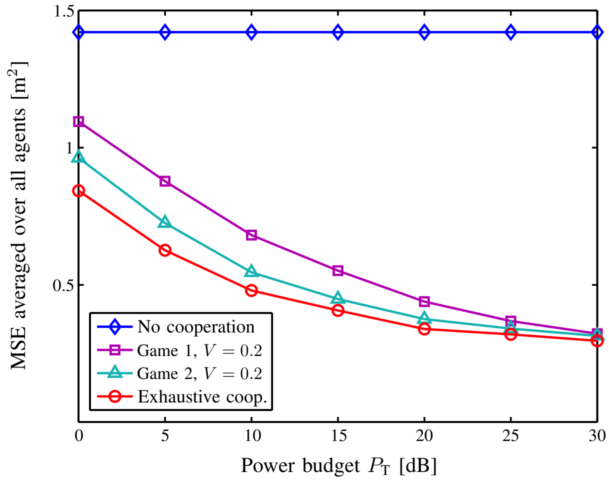


Fig. 5. MSE of the estimated positions averaged over all the agents versus the power budget P_T under $P^{(k)} = P_\mathrm{T}, \forall k$.

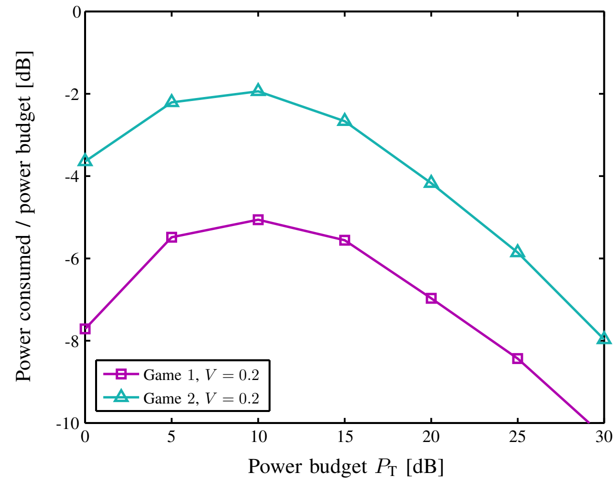


Fig. 6. Average power consumed over all the agents under the power budget P_T for $P^{(k)} = P_\mathrm{T}, \forall k$.

the noise.[10]

The cooperative localization is carried out in two phases. In the first phase, the anchors broadcast one-way ranging signals to the network. Each agent obtains an initial position estimation $\hat{\mathbf{p}}_k^\mathrm{o}$. The estimation error is roughly considered to be Gaussian distributed with zero mean and covariance matrix $(\mathbf{J}_k^\mathrm{o})^{-1}$, where $\mathbf{J}_k^\mathrm{o}$ is the EFIM obtained according to [8, Theorem 1].

In the second phase, the agents first exchange $\hat{\mathbf{p}}_k^\mathrm{o}$ and $\mathbf{J}_k^\mathrm{o}$ with the neighbors, and then follow Algorithm 1 (or Algorithm 2) to determine the power allocation on each link for cooperative localization. For each link with non-zero power allocation, the two agents perform round-trip TOA range measurements. The range measurement between agent k and j is modeled as $z_{kj} = d_{kj} + w_{kj}$, where d_{kj} is the true distance between the two agents and $w_{kj} \sim \mathcal{N}(0, \lambda_{kj}^{-1})$ is a Gaussian random variable, in which $\lambda_{kj} = 4x_{kj}x_{jk}\xi_{kj}/(x_{kj}+x_{jk})$ [20, Remark 3]. Based on the range measurements, the agents update the position estimate $\hat{\mathbf{p}}_k$ using the maximum a posteriori (MAP) algorithm given as follows

$$\hat{\mathbf{p}}_k = \arg\min_{\mathbf{p}_k \in \mathbb{R}^2} \sum_{j: x_{kj}>0} \frac{\lambda_{kj}}{1+\lambda_{kj}\delta_{kj}} (z_{kj} - \|\mathbf{p}_k - \hat{\mathbf{p}}_j^\mathrm{o}\|)^2 + (\mathbf{p}_k - \hat{\mathbf{p}}_k^\mathrm{o})^\mathrm{T} \mathbf{J}_k^\mathrm{o} (\mathbf{p}_k - \hat{\mathbf{p}}_k^\mathrm{o}).$$

For the proposed algorithms, the parameter V_k is chosen as $V_k = V/P^{(k)}$ for each agent k and some $V > 0$. The performance of the proposed game theoretical power management strategies is compared with the following two baselines as a performance benchmark. **Baseline 1**: exhaustive uniform power allocation, where $x_{kj} \equiv P^{(k)}/(K-1)$, $\forall j \neq k$; and **Baseline 2**: non-cooperative power allocation, where $x_{kj} \equiv 0$. The simulation results are averaged over $N = 10^5$ independent channel realizations.

[10]In practice, specific channel scenarios may be applied to model ξ_{kj}. Here, the simple Rician distribution for ξ_{kj} is for the ease of demonstration on the power allocation algorithm in general scenarios.

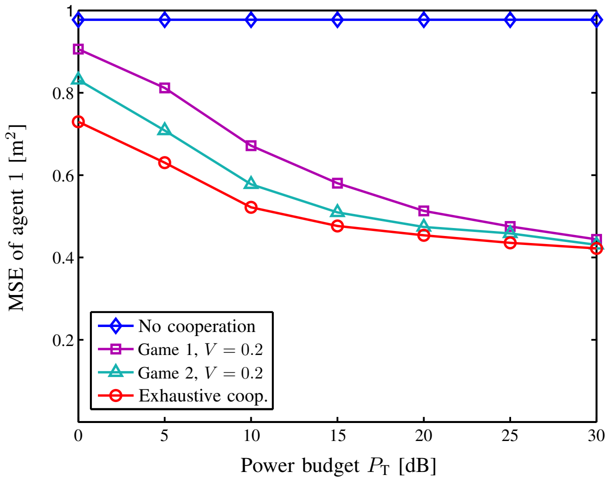
A. Performance of the Network

Fig. 5 shows the MSE of the estimated positions averaged over all the agents versus the power budget P_T under $P^{(k)} = P_\mathrm{T}, \forall k$. First, all the cooperating schemes decrease the MSE of the position estimation compared to the non-cooperating scheme, even at 0 dB power budget. Second, as the power budget increases, the MSE from the cooperating schemes decreases, but the reduction becomes marginal in high power budget regimes. This is because the MSE is dominated by the error of initial estimates before cooperation. Third, both of the game theoretical power management strategies significantly reduce the MSE of the position estimation, and in particular, they perform almost as well as the exhaustive power allocation strategy but with much lower power consumption as will be shown in Fig. 6. In particular, the Pareto optimal strategy achieved by game $\mathcal{G}2$ outperforms $\mathcal{G}1$ as expected.

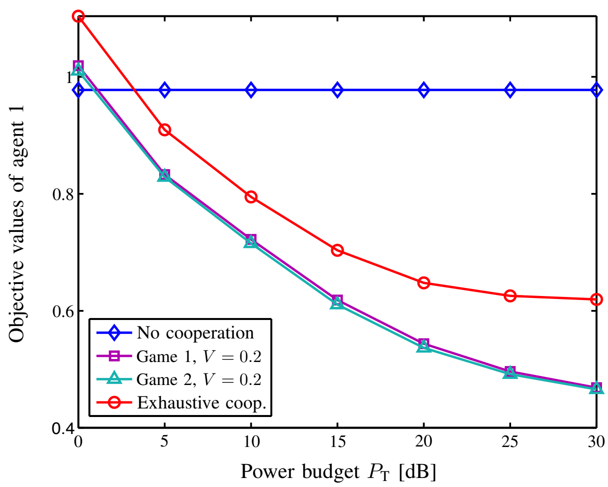
Fig. 6 illustrates the average power consumed over all the agents under the power budget P_T for $P^{(k)} = P_\mathrm{T}, \forall k$. First, in the low power budget regime, the power consumption ratio is low because the agents cooperate only when the channel quality is very good. On the other hand, the power consumption ratio decreases quickly when the power budget increases in the high power budget regime because the position accuracy is limited by the accuracy of the initial estimates before cooperation. Second, both of the game theoretical power management strategies require much less power over the exhaustive power allocation scheme (0 dB power consumption ratio). Finally, although game $\mathcal{G}2$ consumes more power than game $\mathcal{G}1$, it achieves a lower cost than game $\mathcal{G}1$, which means that game $\mathcal{G}2$ utilizes the power more efficiently.

B. Performance of an Individual Agent

Fig. 7 demonstrates the MSE of the estimated position of agent 1 versus the power budget P_T under $P^{(k)} = P_\mathrm{T}, \forall k$. The game theoretical power management strategies achieve performance almost as good as that of the exhaustive power allocation strategy. Although the exhaustive power allocation



(a) MSE



(b) Objective value

Fig. 7. MSE of the estimated position and the objective value of agent 1 versus the power budget P_T under $P^{(k)} = P_\mathrm{T}, \forall k$.

strategy achieves slightly lower MSE [see Fig. 7(a)], it results in a much higher cost [see Fig. 7(b)], which suggests that it does not use the power efficiently. In contrast, the games $\mathcal{G}1$ and $\mathcal{G}2$ achieve low MSE while they also decrease the objective value simultaneously.

Fig. 8 shows the MSE of the estimated position of agent 1 versus the network size of cooperation under power budget $P_\mathrm{T} = 5$ dB. The agents labeled by two to five in Fig. 4 are added to the localization network one by one. First, when more agents participate in the cooperation, the MSE of agent 1 can be reduced more. Second, both of the game theoretical power management strategies benefit from cooperation at the same scale as the exhaustive strategy does.

VI. Conclusion

In this paper, we proposed and analyzed two power management games for network localization with agent cooperation in both asynchronous and synchronous networks. The goal

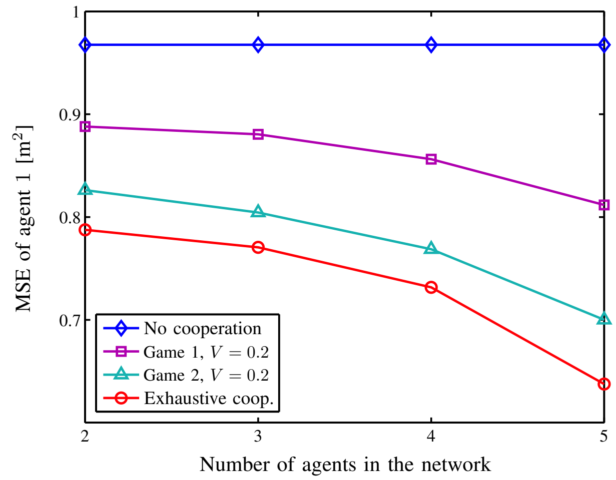


Fig. 8. MSE of the estimated position of agent 1 versus the network size of cooperation under power budget $P_\mathrm{T} = 5$ dB.

is to minimize the individual power-penalized cost function to achieve a better tradeoff between SPEB performance and power consumption for each agent. The notion of cooperating Nash equilibrium has been defined to analyze agents' preference for cooperation. It has been found that the agents prefer to cooperate when the channel quality is good. Meanwhile, the agents are more likely to cooperate when global information is available compared to when only local information is known. In addition, agents require higher channel quality for cooperation in asynchronous networks than in synchronous networks. Furthermore, we developed power management strategies based on game theoretical approaches. It is shown that if the agents have a sufficiently large power budget, the proposed power management strategies achieve SPEB arbitrarily close to that achieved by the exhaustive power allocation strategy, for which each agent always uses the entire power budget for cooperation.

Appendix A
The Best Response Solution for Two-agent Network

Due to page limit, we only prove the asynchronous network case, and the proof for synchronous case follows similarly.

For notational convenience, without loss of generality, we focus on agent 1, and let $k = 1$ and $j = 2$. Denote the power allocation variables as $x_1 = x_{12}$ and $x_2 = x_{21}$. Moreover, denote the perpendicular direction vector as $\mathbf{v}_{12} = [-\sin\phi_{12} \;\; \cos\phi_{12}]^\mathrm{T}$.

The proof is derived in three steps.

Step 1: $\phi_{12} = 0$.

Correspondingly, $\mathbf{u}_{12} = [1 \;\; 0]^\mathrm{T}$, $\mathbf{v}_{12} = [0 \;\; 1]^\mathrm{T}$, and the individual EFIM becomes

$$\begin{aligned}\mathbf{J}_1 &= \mathbf{J}_1^\mathrm{o} + g_{12}(x_1, x_2)\mathbf{u}_{12}\mathbf{u}_{12}^\mathrm{T} \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + g_{12}(x_1, x_2)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\end{aligned}$$

and the objective function can be written as

$$\begin{aligned} f_1(x_1;x_2) &= \text{tr}\left\{\mathbf{J}_1^{-1}\right\} + V_1 x_1 \\ &= \text{tr}\left\{\left[\begin{array}{cc} a+g_{12}(x_1;x_2) & b \\ c & d \end{array}\right]^{-1}\right\} + V_1 x_1 \\ &= \frac{a+d+g_{12}(x_1;x_2)}{ad+d\cdot g_{12}(x_1;x_2)-bc} + V_1 x_1 \end{aligned}$$

where $g_{12}(x_1;x_2) = \frac{4x_1x_2\xi_{12}}{x_1+x_2+4x_1x_2\xi_{12}\delta_{1,2}}$.

Solving the optimality condition $\frac{\partial}{\partial x_1} f(x_1;x_2) = 0$ for x_1, we obtain

$$\hat{x}_1 = \frac{\left[2V_1^{-\frac{1}{2}}(bc+d^2)^{\frac{1}{2}}\xi_{12}^{\frac{1}{2}} - (ad-bc)\right]x_2}{(ad-bc)+4\left[(ad-bc)\delta_{1,2}+d\right]\xi_{12}x_2}.$$

Considering the constraint $0 \le x_1^* \le P^{(1)}$, we can obtain the optimal solution via projections $x_1^* = \{\hat{x}_1\}_0^{P^{(1)}}$.

Moreover, it can be verified that

$$\mathbf{v}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^2\mathbf{v}_{12} = \left[\begin{array}{c} 0 \\ 1 \end{array}\right]^{\mathrm{T}} \left[\begin{array}{cc} a & b \\ c & d \end{array}\right]^2 \left[\begin{array}{c} 0 \\ 1 \end{array}\right] = bc + d^2$$

and $\mathbf{v}_{12}^{\mathrm{T}}\mathbf{J}_1^{\mathrm{o}}\mathbf{v}_{12} = d$, $|\mathbf{J}_1^{\mathrm{o}}| = ad - bc$. Therefore, the solution can be written as

$$x_1^*(x_2) = \left\{\frac{\left(2\sqrt{\xi_{12}/V_1}\sqrt{\mathbf{v}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^2\mathbf{v}_{12}} - |\mathbf{J}_1^{\mathrm{o}}|\right)x_2}{|\mathbf{J}_1^{\mathrm{o}}| + \left(|\mathbf{J}_1^{\mathrm{o}}|\,\delta_{1,2} + \mathbf{v}_{12}^{\mathrm{T}}\mathbf{J}_1^{\mathrm{o}}\mathbf{v}_{12}\right)4\xi_{12}x_2}\right\}_0^{P^{(k)}}. \tag{27}$$

Step 2: $\phi_{12} \neq 0$.

Consider rotating the coordinate system by ϕ_{12}, such that in the new coordinate system, $\hat{\mathbf{u}}_{12} = \mathbf{R}_\phi \mathbf{u}_{12} = [1\ 0]^{\mathrm{T}}$, where $\mathbf{R}_\phi = \left[\begin{array}{cc} \cos(-\phi_{12}) & -\sin(-\phi_{12}) \\ \sin(-\phi_{12}) & \cos(-\phi_{12}) \end{array}\right]$ is a rotation matrix. Denote $\hat{\mathbf{J}}_1^{\mathrm{o}}$ as the initial EFIM in the new coordinate system. Then, we have $\hat{\mathbf{J}}_1^{\mathrm{o}} = \mathbf{R}_\phi \mathbf{J}_1^{\mathrm{o}} \mathbf{R}_\phi^{\mathrm{T}}$ and $\hat{\mathbf{v}}_{12} = \mathbf{R}_\phi \mathbf{v}_{12}$. The optimal solution x_1^* in terms of $\hat{\mathbf{J}}_1^{\mathrm{o}}$ and $\hat{\mathbf{v}}_{12}$ can be obtained using the method in step 1.

Note that

$$\begin{aligned} \hat{\mathbf{v}}_{12}^{\mathrm{T}}(\hat{\mathbf{J}}_1^{\mathrm{o}})^2\hat{\mathbf{v}}_{12} &= \text{tr}\left\{(\hat{\mathbf{J}}_1^{\mathrm{o}})^2\hat{\mathbf{v}}_{12}\hat{\mathbf{v}}_{12}^{\mathrm{T}}\right\} \\ &= \text{tr}\left\{(\mathbf{R}_\phi\mathbf{J}_1^{\mathrm{o}}\mathbf{R}_\phi^{\mathrm{T}})(\mathbf{R}_\phi\mathbf{J}_1^{\mathrm{o}}\mathbf{R}_\phi^{\mathrm{T}})(\mathbf{R}_\phi\mathbf{v}_{12}\mathbf{v}_{12}^{\mathrm{T}}\mathbf{R}_\phi^{\mathrm{T}})\right\} \\ &= \text{tr}\left\{\mathbf{J}_1^{\mathrm{o}}(\mathbf{R}_\phi^{\mathrm{T}}\mathbf{R}_\phi)\mathbf{J}_1^{\mathrm{o}}(\mathbf{R}_\phi^{\mathrm{T}}\mathbf{R}_\phi)\mathbf{v}_{12}\mathbf{v}_{12}^{\mathrm{T}}(\mathbf{R}_\phi^{\mathrm{T}}\mathbf{R}_\phi)\right\} \\ &= \text{tr}\left\{(\mathbf{J}_1^{\mathrm{o}})^2\mathbf{v}_{12}\mathbf{v}_{12}^{\mathrm{T}}\right\} \\ &= \mathbf{v}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^2\mathbf{v}_{12}. \end{aligned}$$

Similarly, one can show that $\hat{\mathbf{v}}_{12}^{\mathrm{T}}\hat{\mathbf{J}}_1^{\mathrm{o}}\hat{\mathbf{v}}_{12} = \mathbf{v}_{12}^{\mathrm{T}}\mathbf{J}_1^{\mathrm{o}}\mathbf{v}_{12}$ and $\left|\hat{\mathbf{J}}_1^{\mathrm{o}}\right| = |\mathbf{J}_1^{\mathrm{o}}|$. Then the same expression as in (27) can be obtained in the $\phi_{12} \neq 0$ case.

Step 3: We need to make use of the following lemma.

Lemma 4: For any 2×2 non-singular matrix $\mathbf{A}$ and a unit vector $\mathbf{u}$, we have

$$|\mathbf{A}|\mathbf{u}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{u} = \mathbf{v}^{\mathrm{T}}\mathbf{A}\mathbf{v} \tag{28}$$

where $\mathbf{v}$ is a unit vector perpendicular to $\mathbf{u}$, i.e., $\|\mathbf{v}\| = 1$ and $\mathbf{u}^{\mathrm{T}}\mathbf{v} = 0$.

Proof: Denote $\mathbf{A} = \left[\begin{array}{cc} a & b \\ c & d \end{array}\right]$ and $\mathbf{u} = [\cos\phi\ \ \sin\phi]$, $\phi \in [0, 2\pi)$. Without loss of generality, denote $\mathbf{v} = [-\sin\phi\ \ \cos\phi]$. We have

$$\mathbf{u}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{u} = |\mathbf{A}|^{-1}(d\cos^2\phi - (b+c)\cos\phi\sin\phi + a\sin^2\phi).$$

On the other hand,

$$\begin{aligned} \mathbf{v}^{\mathrm{T}}\mathbf{A}\mathbf{v} &= a\sin^2\phi - (b+c)\sin\phi\cos\phi + d\cos^2\phi \\ &= |\mathbf{A}|\mathbf{u}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{u}. \end{aligned}$$

□

Using the above lemma, we have

$$\mathbf{v}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^2\mathbf{v}_{12} = |\mathbf{J}_1^{\mathrm{o}}|^2\mathbf{u}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^{-2}\mathbf{u}_{12}$$

and

$$\mathbf{v}_{12}^{\mathrm{T}}\mathbf{J}_1^{\mathrm{o}}\mathbf{v}_{12} = |\mathbf{J}_1^{\mathrm{o}}|\mathbf{u}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^{-1}\mathbf{u}_{12}.$$

In addition, we note that $\mathbf{u}_{12}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^{-1}\mathbf{u}_{12} = \mathbf{u}_{21}^{\mathrm{T}}(\mathbf{J}_1^{\mathrm{o}})^{-1}\mathbf{u}_{21} = \delta_{2,1}$, by the definition of $\delta_{k,l}$ in (5). Using the general notation for agent pair (k, j), (27) can be simplified into (12).

Appendix B
Stability and Convergence Results under Two-agent Networks

Due to page limit, we only prove the asynchronous network case, and the proof for the synchronous case follows similarly.

For notation convenience, without loss of generality, let $k = 1$ and $j = 2$. In addition, denote the power allocation variables as $x_1 = x_{12}$ and $x_2 = x_{21}$.

Proof of Theorem 1:

From (12), the best response functions have the following form:

$$T_{12}(x_2) = \left\{\frac{a_1x_2}{1+b_1x_2}\right\}_0^{P^{(1)}} \quad \text{and} \quad T_{21}(x_1) = \left\{\frac{a_2x_1}{1+b_2x_1}\right\}_0^{P^{(2)}} \tag{29}$$

where $a_1, a_2, b_1, b_2 > 0$ under the necessary condition for cooperation in Proposition 2. Consider that the upper level projections are not active. Solving the system of fixed point equations $T_{12}(x_2^*) = x_1^*$ and $T_{21}(x_1^*) = x_2^*$, two solutions are obtained, a trivial solution $(x_1^*, x_2^*) = (0, 0)$ and

$$x_1^* = \frac{a_1a_2 - 1}{b_2 + b_1a_2} \quad \text{and} \quad x_2^* = \frac{a_1a_2 - 1}{b_1 + b_2a_1}.$$

The cooperating NE requires that $x_1^*, x_2^* > 0$. Equivalently, we need $a_1a_2 - 1 > 0$, which yields the condition (16).

Correspondingly, a necessary condition for the upper level projections being active is $a_1a_2 - 1 > 0$. Moreover, if the projection for x_1 is active, i.e., $x_1^* = P^{(1)}$, then x_2^* is uniquely determined by $x_2^* = \left\{\frac{a_2P^{(1)}}{1+b_2P^{(1)}}\right\}_0^{P^{(2)}}$. Therefore, the projections $\{\cdot\}_0^P$ do not change the condition on the existence and uniqueness of the cooperating NE.

Proof of Theorem 2:

Using the simplified notations $x_1 = x_{12}$ and $x_2 = x_{21}$, the best response iteration for agent 1 can be written as $x_1(n+2) = T_{12}(x_2(n+1)) = T_{12}(T_{21}(x_1(n)))$. Define $Q_1(x) \triangleq T_{12}(T_{21}(x))$.

Case (i): Suppose the cooperating NE $\mathbf{x}_c^*$ is in the interior of the feasible domain $\widetilde{\mathcal{P}}$ (i.e., the projections in $T_{12}(\cdot)$ and $T_{21}(\cdot)$ are not active). From the best response mappings in (29), for $x \geq x_1^*$

$$\begin{aligned}\frac{d}{dx}Q_1(x) &= \frac{\partial T_{12}(u)}{\partial u}\bigg|_{u=T_{21}(x)} \cdot \frac{\partial T_{21}(x)}{\partial x} \\ &= \frac{a_1 a_2}{[(b_2 + b_1 a_2)x + 1]^2} \\ &< \frac{a_1 a_2}{[(b_2 + b_1 a_2)x_1^* + 1]^2} \\ &= \frac{1}{a_1 a_2}\end{aligned}$$

which is less than 1 due to the condition $a_1 a_2 - 1 > 0$ for the existence of a cooperating NE as studied in the proof of Theorem 1. Then by the fixed point theory [50], the sequence $x_1(0),\ x_1(2),\ \dots,\ x_1(2n),\ \dots$ generated by $x_1(n+2) = Q_1(x_1(n))$ from $x_1(0) \geq x_1^*$ converges to x_1^* as $n \to \infty$.

On the other hand, for $0 < x < x_1^*$,

$$\begin{aligned}Q_1(x) - x &= \frac{a_1 a_2 x}{(b_2 + b_1 a_2)x + 1} - x \\ &> \frac{a_1 a_2 x}{(b_2 + b_1 a_2)x_1^* + 1} - x \\ &= 0\end{aligned}$$

which yields $Q_1(x) > x$. It follows that $x_1(n+2) = Q_1(x_1(n)) > x_1(n)$, which means the sequence $x_1(0), x_1(2), \dots, x_1(2n), \dots$ is strictly increasing unless $x_1(m) \geq x_1^*$ for some m. Since there is only one fixed point from Theorem 1, the sequence must converge to x_1^*.

The same properties apply to the sequence $x_1(1)$, $x_1(3)$, $\dots$, $x_1(2n+1)$, $\dots$. This concludes that $x_1(n)$ converges to x_1^* from any initial point $x_1(0) > 0$. It can be easily verified that the same results apply to the sequence $x_2(n)$.

Now suppose the cooperating NE $\mathbf{x}_c^*$ is on the boundary of $\widetilde{\mathcal{P}}$, and without loss of generality, assume $x_1^* = P^{(1)}$. Following similar steps, one can show that both of the sequences $x_1(0), x_1(2), \dots, x_1(2n), \dots$ and $x_1(1), x_1(3), \dots, x_1(2n+1), \dots$ are increasing until reaching $x_1^* = P^{(1)}$. Since $T_1(x_1^*) = x_2^*$, $x_2(n)$ converges correspondingly.

These show that the best response sequence $\mathbf{x}(n)$ converges to $\mathbf{x}_c^*$ globally, and hence $\mathbf{x}_c^*$ is stable and the non-cooperating NE at the origin is unstable.

Case (ii): When the cooperating NE does not exist, $\mathbf{x}(n)$ converges to the origin (the unique NE) by the fixed point theory, because $\frac{d}{dx_1}Q_1(x_1) < 1$ for all $x_1 \in \mathcal{P}_1$.

Proof of Proposition 3:

The derivative of the best response mapping $\widetilde{\mathbf{T}}$ in (9) is

$$\nabla \widetilde{\mathbf{T}} = \begin{bmatrix} 0 & \frac{\partial T_{12}(x_2^*)}{\partial x_2} \\ \frac{\partial T_{21}(x_1^*)}{\partial x_1} & 0 \end{bmatrix}$$

in which the *radius* of the matrix $\nabla \widetilde{\mathbf{T}}$ at the cooperating NE $\mathbf{x}_c^*$ is given by

$$r(\nabla \widetilde{\mathbf{T}}\big|_{\mathbf{x}_c^*}) = \left(\frac{\partial T_{12}(x_2^*)}{\partial x_2}\frac{\partial T_{21}(x_1^*)}{\partial x_1}\right)^{\frac{1}{2}} = \frac{d}{dx_1}Q_1(x_1^*) < 1.$$

The result on the rate of convergence follows directly from [50, Theorem 4.C].

Appendix C
Proof of Theorem 3

When the power budget $P^{(k)}$ is large enough, the power conservation levels $V_k(P^{(k)})$ are small enough and all the links are cooperating. Moreover, under the proposed strategy $\mathbf{x}_k^*$ from game $\mathcal{G}1$, all the links will have non-zero power allocation $x_{kj}, x_{jk} > 0$, because the solutions with zero power on the cooperating link will be unstable under the iteration in game $\mathcal{G}1$. In addition, x_{kj}^* increases as $P^{(k)}$ increases because the power penalty V_k is asymptotically small.

Consider asynchronous networks. Specifically, for agent k on link (k, j), the optimality condition on the power allocation x_{kj} from solving the best response problem (8) is

$$V_k(P^{(k)}) = \mathrm{tr}\bigg\{\bigg[\mathbf{J}_k^{\mathrm{o}} + \sum_{l=\mathcal{N}(k)} g_{kl}(x_{kl}, x_{lk})\mathbf{D}_{kl}\bigg]^{-2}\mathbf{D}_{kj}\bigg\} \times \frac{\partial}{\partial x_{kj}} g_{kj}(x_{kj}, x_{jk}) \qquad (30)$$

where $\mathbf{D}_{kj} = \mathbf{u}_{kj}\mathbf{u}_{kj}^{\mathrm{T}}$.

Note that, from the derivative of the gain function

$$\frac{\partial}{\partial x_{kj}} g_{kj}(x_{kj}, x_{jk}) = \frac{4x_{jk}^2 \xi_{kj}}{(x_{kj} + x_{jk} + 4x_{kj}x_{jk}\delta_{kj}\xi_{kj})^2}$$

the right hand side (R.H.S.) of (30) scales as $\mathcal{O}(x_{kj}^{-2})$ as $x_{kj}(P^{(k)}) \to \infty$. Therefore, the optimality condition (30) can be written as

$$V_k(P^{(k)}) = C x_{kj}^*(P^{(k)})^{-2} + o(x_{kj}^{*-2})$$

for large $P^{(k)}$ (and hence large $x_{kj}^*(P^{(k)})$), and can be solved as

$$x_{kj}^*(P^{(k)}) = \sqrt{C} V_k(P^{(k)})^{-\frac{1}{2}} + o(1).$$

This applies to all the variables x_{kj}, $(k, j) \in \mathcal{E}$, where $\mathcal{E}$ is the set of all the (directed) links in the network. As a result,

$$x_{kj}^*(P^{(k)})/P^{(k)} = \sqrt{C} V_k(P^{(k)})^{-\frac{1}{2}}/P^{(k)} + o(1) \to 0$$

as $P^{(k)} \to \infty$ for all k.

In addition, since $x_{kj}^*(P^{(k)}), x_{jk}^*(P^{(j)}) \to \infty$, the gain function $g_{kj}(x_{kj}, x_{jk}) \to \delta_{kj}^{-1}$ as $P^{(k)}, P^{(j)} \to \infty$. Therefore,

$$\begin{aligned}\mathcal{I}_k(\mathbf{x}_k^*, \mathbf{x}_{-k}) &= \mathrm{tr}\{[\mathbf{J}_k^{\mathrm{o}} + \textstyle\sum_{j\neq k} g_{kj}(\cdot)\mathbf{D}_{kj}]^{-1}\} \\ &\to \mathrm{tr}\{[\mathbf{J}_k^{\mathrm{o}} + \textstyle\sum_{j\neq k}\delta_{k,j}^{-1}\mathbf{D}_{kj}]^{-1}\} \\ &= \mathcal{I}_k(\mathbf{x}_k = \infty, \mathbf{x}_{-k})\end{aligned}$$

and $\mathcal{I}_k(\mathbf{x}_k^{\mathrm{f}}, \mathbf{x}_{-k}) \to \mathcal{I}_k(\mathbf{x}_k = \infty, \mathbf{x}_{-k})$, as $P^{(k)} \to \infty$. Hence, $\mathcal{I}_k(\mathbf{x}_k^*, \mathbf{x}_{-k}) \to \mathcal{I}_k(\mathbf{x}_k^{\mathrm{f}}, \mathbf{x}_{-k})$.

Similar arguments apply to synchronous networks.

Appendix D
Proof of Lemma 3

To show the convexity of $f_k(\mathbf{x})$, we first note that $\mathrm{tr}\{\mathbf{X}^{-1}\}$ is convex in $\mathbf{X}$. Since $\mathbf{X} = \mathbf{X}_0 + \sum_j g_{kj}\mathbf{u}_{kj}\mathbf{u}_{kj}^{\mathrm{T}}$ is linear in $\{g_{kj}\}$, the function f_k is convex in $\{g_{kj}\}$. As a result, $\frac{\partial f_k}{\partial g_{kj}} < 0$ and $\frac{\partial^2 f_k}{\partial g_{kj}^2} \geq 0$. In addition, the gain functions $g_{kl}(x_{kl}, x_{lk})$

in (3) and (4) are both concave in (x_{kl}, x_{lk}); i.e., $\nabla^2 g_{kl}$ is negative semidefinite. Therefore, from the chain rule, we have

$$\begin{aligned}\nabla^2 f_k(\mathbf{x}) &= \nabla\left(\sum_j \frac{\partial f_k}{\partial g_{kj}} \nabla g_{kj}(\mathbf{x})\right)\\ &= \sum_j \sum_l \frac{\partial^2 f_k}{\partial g_{kl}\partial g_{kj}} \nabla g_{kl}(\mathbf{x}) \nabla g_{kj}(\mathbf{x})^{\mathrm{T}}\\ &\qquad + \sum_j \frac{\partial f_k}{\partial g_{kj}} \nabla^2 g_{kj}(\mathbf{x})\\ &\overset{(a)}{=} \sum_j \frac{\partial^2 f_k}{\partial g_{kj}^2} \nabla g_{kj}(\mathbf{x}) \nabla g_{kj}(\mathbf{x})^{\mathrm{T}} + \sum_j \frac{\partial f_k}{\partial g_{kj}} \nabla^2 g_{kj}(\mathbf{x})\\ &\succeq \mathbf{0}\end{aligned}$$

where $\overset{(a)}{=}$ is due to the fact that $\nabla g_{kl}(\mathbf{x}) \nabla g_{kj}(\mathbf{x})^{\mathrm{T}} = \mathbf{0}$ for $l \neq j$. Hence $f_k(\mathbf{x})$ is convex in $\mathbf{x}$.

To show the concavity of $L_k(\mathbf{x}) = \log(f_k^{\mathrm{d}} - f_k(\mathbf{x}))$, we have $\nabla L_k(\mathbf{x}) = -\frac{\nabla f_k(\mathbf{x})}{f_k^{\mathrm{d}} - f_k(\mathbf{x})}$, and

$$\nabla^2 L_k(\mathbf{x}) = -\frac{\nabla f_k(\mathbf{x}) \nabla f_k(\mathbf{x})^{\mathrm{T}}}{(f_k^{\mathrm{d}} - f_k(\mathbf{x}))^2} - \frac{\nabla^2 f_k(\mathbf{x})}{f_k^{\mathrm{d}} - f_k(\mathbf{x})} \preceq \mathbf{0}.$$

Hence $L_k(\mathbf{x})$ is concave in $\mathbf{x}$.

Appendix E
Proof of Theorem 4

We only prove for the case of asynchronous networks, and the derivation for synchronous networks is similar.

Note that if there exists one strictly feasible point, then the optimal solution to game $\mathcal{G}2$ satisfies $\mathbf{x}^* \neq \mathbf{0}$. Therefore, we only need to find the condition under which game $\mathcal{G}2$ is strictly feasible.

Without loss of generality, suppose $\mathbf{x}^1 = (x_{kj}^1 = v, x_{jk}^1 = \rho v)$ is a strictly feasible point, for some ρ. Since the functions $f_k(\mathbf{x})$ are strictly convex, the feasibility region of the problem in game $\mathcal{G}2$ is convex and $\mathbf{0}$ is an accumulation point of the strictly feasible region. As a result, every point on the line segment $t\mathbf{x}^1$, $0 < t \leq 1$, is strictly feasible. Moreover, the function $h(t) \triangleq f_k(\mathbf{0} + t\mathbf{x}^1)$ is strictly convex in $0 \leq t \leq 1$, and thus $h(1) > h(0) + h'(0)(1-0)$ by the property of a convex function. Since $\mathbf{x}^1$ is strictly feasible, $f_k^{\mathrm{d}} = f_k(\mathbf{0}) = h(0) > h(1) = f_k(\mathbf{x}^1)$, which implies that $\lim_{t\to 0+} h'(t) < 0$. Thus, we have

$$\begin{aligned}\frac{dh(t)}{dt} &= \left(\frac{df_k}{dx_{kj}}\frac{dx_{kj}}{dt} + \frac{df_k}{dx_{jk}}\frac{dx_{jk}}{dt}\right)\Bigg|_{(x_{kj}=tv, x_{jk}=t\rho v)}\\ &= \frac{\partial f_k}{\partial g_{kj}}\left(\frac{\partial g_{kj}}{\partial x_{kj}}\frac{dx_{kj}}{dt} + \frac{\partial g_{kj}}{\partial x_{jk}}\frac{dx_{jk}}{dt}\right) + \frac{\partial f_k}{\partial x_{kj}}\frac{dx_{kj}}{dt}\\ &= -\mathrm{tr}\{(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}\mathbf{u}_{kj}^{\mathrm{T}}\}\\ &\qquad \times \frac{4x_{jk}^2\xi_{kj}\cdot v + 4x_{kj}^2\xi_{kj}\cdot\rho v}{(x_{kj} + x_{jk} + 4x_{kj}x_{jk}\xi_{kj}\delta_{k,j})^2} + V_k v\\ &= -\mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}\frac{4(t\rho v)^2\xi_{kj}v + 4(tv)^2\xi_{kj}\rho v}{(tv + t\rho v + 4t^2\rho v^2\xi_{kj}\delta_{k,j})^2} + V_k v.\end{aligned}$$

By setting $\lim_{t\to 0+}\frac{dh(t)}{dt} < 0$, we have $-\mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}\frac{4\rho\xi_{kj}}{\rho+1} + V_k < 0$, and hence

$$4\xi_{kj} > \frac{\rho+1}{\rho}\frac{V_k}{\mathbf{u}_{kj}^{\mathrm{T}}(\mathbf{J}_k^{\mathrm{o}})^{-2}\mathbf{u}_{kj}} = \frac{\rho+1}{\rho}\Gamma_{kj}.$$

Similarly, we obtain $4\xi_{kj} = 4\xi_{jk} > (\rho+1)\Gamma_{jk}$ by evaluating the inequality $\lim_{t\to 0+}\frac{df_j(\mathbf{0}+t\mathbf{x}^1)}{dt} < 0$.

The existence of a strictly feasible point $\mathbf{x}^1$ corresponds to the condition that the above inequalities are satisfied for at least one ρ; i.e.,

$$\xi_{kj} > \frac{1}{4}\inf_{\rho}\ \max\{\frac{\rho+1}{\rho}\Gamma_{kj}, (\rho+1)\Gamma_{jk}\}. \tag{31}$$

Note that as $\frac{\rho+1}{\rho}\Gamma_{kj}$ decreases with respect to (w.r.t.) ρ and $(\rho+1)\Gamma_{jk}$ is increasing, the optimal value on the right hand side of (31) is achieved at $\frac{\rho^*+1}{\rho^*}\Gamma_{kj} = (\rho^*+1)\Gamma_{jk}$, which gives $\rho^* = \Gamma_{kj}/\Gamma_{jk}$. Therefore, we must have $\xi_{kj} > \frac{1}{4}(\rho^*+1)\Gamma_{jk} = \frac{1}{4}(\Gamma_{kj} + \Gamma_{jk})$.

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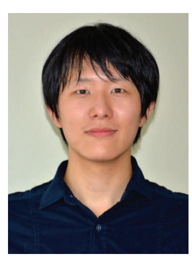

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