JADED-RIP: Joint Angle and Delay Estimator and Detector via Rotational Invariance Properties

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***Abstract*—A novel approach entitled "Joint Angle and Delay Estimator and Detector", or simply JADED, is presented. This approach allows simultaneous estimation of number of coherent/non-coherent sources and joint estimation of the angles and times of arrival of each source. The system is composed of a Uniform Linear Array (ULA) receiving known OFDM symbols from a user in an indoor environment, which is rich in multipath. Therefore, the objective is to apply the JADED approach to this scenario, which allows extraction of the Line-of-Sight component based on the first arriving path. The first method, called JADED-RIP, makes use of the Rotational Invariance Properties (RIP) of ULAs and OFDM symbols, detects the number of multipath components, and estimates the angles and times of arrival of each path by performing a 2D search. The second method is a Computationally Efficient Single Snapshot (CESS) version of the JADED-RIP, i.e. it requires a 1D search followed by a least squares fit, and can only be used when a single OFDM symbol is available. Future insights are given in the Conclusions section.**

***Index Terms*—JADED, Angle-of-Arrival, Time-of-Arrival, Joint Estimation, Detection**

I. INTRODUCTION

Localization has been one challenging topic over the past 60 years. In fact, the location can be determined by estimating signal parameters that are directly related to the users position, such as Angle-of-Arrival (AoA), Time-of-Arrival (ToA), and so forth [1]. To estimate these signal parameters, the Maximum Likelihood (ML) technique was one of the first to be investigated [2]. However, it is highly computational, as it requires a multidimensional search. Moreover, subspace methods, such as MUSIC [3] and ESPRIT [4], were proposed as computationally efficient solutions to estimate signal parameters. However, they perform poorly in case of a single snapshot or coherent sources, i.e. multipath propagation or signal jamming. The spatial smoothing preprocessing technique [5] was discovered to overcome this issue, but it reduces the effective number of antennas. Additionally, ML and subspace techniques require the knowledge of number of sources.

Inspired by the idea of resolving more sources using fewer antennas, Joint Angle and Delay Estimation (JADE) was proposed in [6], which makes joint use of spatial and temporal diversity. As a response, methods were implemented so as to solve the JADE problem, such as [7], [8]. Even these 2D methods couldn't estimate the signal parameters of multiple sources using a single snapshot, unless a 2D preprocessing technique is applied, such as spatio-frequential smoothing [9]. Nevertheless, algorithms that are based on an efficient ML estimator [11] can jointly estimate ToAs and AoAs in the presence of coherent signals and a single snapshot. Other methods that use 2D-Matrix Pencils (2DMP) operate only with a single snapshot, thus coherent sources is not issue for these type of methods. Unfortunately, all these methods require the knowledge of the number of sources or multipath, which is generally unavailable and has to be estimated from data. Source detection is a well known problem in signal processing, where one has to estimate the number of superimposed signals observed by an array. There exists numerous methods on source detection, like AIC and MDL [12], Modified MDL [13], the Benjamin-Hochberg procedure [14], etc. However, these methods assume un-coherent sources, which is not the case in multipath propagation scenarios. Other approaches address source detection in the coherent case, such as [15]–[17]. Unfortunately, they are highly multidimensional, as one has to test all possible number of sources and estimate their corresponding signal parameters. In other words, if one has to test for the presence of three sources, then three AoAs have to be estimated through a 3D search, and so on.

In this paper, we propose a novel approach called Joint Angle and Delay Estimator and Detector, or simply JADED. *Unlike traditional methods, JADED allows efficient source detection and joint angle and delay estimation. Additionally, the algorithm operates with a single or multiple snapshots, and in the presence of coherent sources.* This is of crucial importance in applications, such as Wi-Fi systems [18] in an indoor environment, where a users location should be estimated. The Wi-Fi is equipped with multiple antennas that receive known OFDM symbols[1] from the user. Since indoor environments are rich in multipath (hence coherent sources) and the number of multipath contributions are unknown, then the proposed methods in this paper, which are JADED-based, seem to be suitable for the aforementioned scenario. We, hereby, make use of Rotational Invariance Properties (RIP) of subcarriers in OFDM systems and Uniform Linear Arrays (ULAs), which are known to have a *Vandermonde* structure in their respective dimensions. The first algorithm,

[1]These known OFDM symbols are usually found in the preamble of the OFDM frame, such as the Short-Training-Field (STF).

JADED-RIP, requires a 2D search to estimate the number of multipath and their respective AoAs/ToAs. Furthermore, the second algorithm, named Computationally Efficient Single Snapshot-JADED-RIP (CESS-JADED-RIP), is a faster version of JADED-RIP, as it requires a 1D search followed by a Least Squares fit, and is dedicated for single snapshot scenarios only.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ represent the transpose and transpose-conjugate operators, respectively. The matrix $\boldsymbol{I}_N \in \mathbb{C}^{N\times N}$ is the identity matrix. The operators $\|\boldsymbol{X}\|$, $\Re(\boldsymbol{X})$, and $\Im(\boldsymbol{X})$ denote the *Frobenius* norm, the real part, and the imaginary part of $\boldsymbol{X}$. The symbol $\otimes$ denotes the *Kronecker* product. We index the k^{th} entry of a vector $\boldsymbol{x}$ as $[\boldsymbol{x}]_k$. For a given vector $\boldsymbol{x}$, the operator diag $[\boldsymbol{x}]$, returns a diagonal matrix with diagonal entries equal to those of $\boldsymbol{x}$. Similarly, blkdiag[.] takes a set of matrices as input and outputs a block diagonal matrix. The abbreviation "w.r.t" stands for "with respect to".

II. System Model

Consider an OFDM symbol composed of M subcarriers and centered at a carrier frequency f_c, impinging an array of N antennas via q multipath components. Each path arrives at AoA θ_i and ToA τ_i. After applying an FFT and equalization, we can express the l^{th} OFDM symbol as follows [10]:

$$\boldsymbol{x}(l) = \boldsymbol{A}\boldsymbol{\gamma}(l) + \boldsymbol{w}(l), \qquad l = 1\dots L \tag{1}$$

where $\boldsymbol{x}(l) \in \mathbb{C}^{MN\times 1}$ is given as

$$\boldsymbol{x}(l) = [X_{1,1}^{(l)}\dots X_{1,N}^{(l)}\dots X_{M,1}^{(l)}\dots X_{M,N}^{(l)}]^{\mathrm{T}} \tag{2}$$

with $X_{m,n}^{(l)}$ being the data at the n^{th} antenna and m^{th} subcarrier in the l^{th} OFDM symbol. $\boldsymbol{A} \in \mathbb{C}^{MN\times q}$ contains the ToA/AoA information as

$$\boldsymbol{A} = [\boldsymbol{c}_M(\tau_1)\otimes \boldsymbol{a}_N(\theta_1)\dots \boldsymbol{c}_M(\tau_q)\otimes \boldsymbol{a}_N(\theta_q)] \tag{3}$$

where

$$\boldsymbol{a}_N(\theta) = [1, z_\theta \dots z_\theta^{N-1}]^{\mathrm{T}} \quad \text{with} \quad z_\theta = e^{-j2\pi\frac{d}{\lambda}\sin(\theta)} \tag{4}$$
$$\boldsymbol{c}_M(\tau) = [1, z_\tau \dots z_\tau^{M-1}]^{\mathrm{T}} \quad \text{with} \quad z_\tau = e^{-j2\pi\tau\triangle_f} \tag{5}$$

where $\triangle_f$ is the subcarrier spacing, d is the inter-element spacing, and λ is the signal's wavelength. The $q\times 1$ vector $\boldsymbol{\gamma}(l)$ is composed of the multipath coefficients

$$\boldsymbol{\gamma}(l) = [\gamma_1(l)\dots\gamma_q(l)]^{\mathrm{T}} \tag{6}$$

Note that we have made explicit the dimensions of vectors $\boldsymbol{c}_M(\tau)$ and $\boldsymbol{a}_N(\theta)$, i.e. it should be understood that for any integer $K \geq 1$, the vectors $\big(\boldsymbol{c}_K(\tau), \boldsymbol{a}_K(\theta)\big) \in \mathbb{C}^{K\times 1}$. The vector $\boldsymbol{w}(l)$ is additive Gaussian noise of zero mean and covariance $\sigma^2\boldsymbol{I}$, assumed to be white over space, and frequencies. We are now ready to address the problem:

"Given the data $\{\boldsymbol{x}(l)\}_{l=1}^{L}$, estimate the number of multipath components q and the signal parameters $\{(\tau_j,\theta_j)\}_{j=1}^{q}$."

III. JADED-RIP Algorithm Derivation

A. Data Manipulation

Let $\boldsymbol{X}(l)$ be a matrix formed from the entries of $\boldsymbol{x}(l)$

$$\boldsymbol{X}(l) = \begin{pmatrix} \boldsymbol{X}_1^{(l)} & \boldsymbol{X}_2^{(l)} & \cdots & \boldsymbol{X}_{K_M}^{(l)} \\ \boldsymbol{X}_2^{(l)} & \boldsymbol{X}_3^{(l)} & \cdots & \boldsymbol{X}_{K_M+1}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{X}_{P_M}^{(l)} & \boldsymbol{X}_{P_M+1}^{(l)} & \cdots & \boldsymbol{X}_{M}^{(l)} \end{pmatrix} \tag{7}$$

where $\boldsymbol{X}_i^{(l)}$ is an $P_N \times K_N$ Hankel matrix given by

$$\boldsymbol{X}_i^{(l)} = \begin{pmatrix} X_{i,1}^{(l)} & X_{i,2}^{(l)} & \cdots & X_{i,K_N}^{(l)} \\ X_{i,2}^{(l)} & X_{i,3}^{(l)} & \cdots & X_{i,K_N+1}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{i,P_N}^{(l)} & X_{i,P_N+1}^{(l)} & \cdots & X_{i,N}^{(l)} \end{pmatrix} \tag{8}$$

with

$$K_M = M - P_M + 1 \quad \text{and} \quad K_N = N - P_N + 1 \tag{9}$$

For simplicity of notation, define the following integers

$$K \triangleq K_M K_N \quad \text{and} \quad P \triangleq P_M P_N \tag{10}$$

The matrix $\boldsymbol{X}(l)$ can be written as

$$\boldsymbol{X}(l) = \boldsymbol{L}\boldsymbol{\Gamma}(l)\boldsymbol{R}^{\mathrm{T}} + \boldsymbol{W}(l) \tag{11}$$

where $\boldsymbol{L} \in \mathbb{C}^{P\times q}$ and $\boldsymbol{R} \in \mathbb{C}^{K\times q}$ are given as

$$\boldsymbol{L} = [\boldsymbol{h}_P(\tau_1,\theta_1)\dots\boldsymbol{h}_P(\tau_q,\theta_q)] \tag{12}$$
$$\boldsymbol{R} = [\boldsymbol{h}_K(\tau_1,\theta_1)\dots\boldsymbol{h}_K(\tau_q,\theta_q)] \tag{13}$$

with

$$\boldsymbol{h}_P(\tau,\theta) = \boldsymbol{c}_{P_M}(\tau)\otimes \boldsymbol{a}_{P_N}(\theta) \tag{14}$$
$$\boldsymbol{h}_K(\tau,\theta) = \boldsymbol{c}_{K_M}(\tau)\otimes \boldsymbol{a}_{K_N}(\theta) \tag{15}$$

The matrix $\boldsymbol{\Gamma}(l) \in \mathbb{C}^{q\times q}$ is a diagonal matrix, i.e.

$$\boldsymbol{\Gamma}(l) = \text{diag}\;[\gamma_1(l), \gamma_2(l)\dots\gamma_q(l)] \tag{16}$$

Finally, the matrix $\boldsymbol{W}(l) \in \mathbb{C}^{P\times K}$ is background noise defined in a similar manner as $\boldsymbol{X}(l)$.

B. Introducing Orthogonal Projectors

Let $\boldsymbol{R}_j$ be a matrix defined as $\boldsymbol{R}$ with omitted j^{th} column. Furthermore, define the orthogonal projector matrix that spans the null space of the columns of $\boldsymbol{R}_j$ as

$$\boldsymbol{\mathcal{P}}_j^{\perp} = \boldsymbol{I}_K - \boldsymbol{R}_j\left(\boldsymbol{R}_j^{\mathrm{T}}\boldsymbol{R}_j\right)^{-1}\boldsymbol{R}_j^{\mathrm{T}} \tag{17}$$

In other words, $\boldsymbol{R}_j^{\mathrm{T}}\boldsymbol{\mathcal{P}}_j^{\perp} = \boldsymbol{0}$. Now, let $\boldsymbol{f}_j \in \mathbb{C}^{K\times 1}$ be a vector that lives in the null space of the columns of $\boldsymbol{R}_j$. Therefore, there exists a non-zero vector $\boldsymbol{z} \in \mathbb{C}^{K\times 1}$ such that

$$\boldsymbol{f}_j = \boldsymbol{\mathcal{P}}_j^{\perp}\boldsymbol{z} \tag{18}$$

Post-multiplying the vector $\boldsymbol{f}_j$ with the data matrix $\boldsymbol{X}(l)$ yields

$$\begin{aligned} \boldsymbol{X}(l)\boldsymbol{f}_j &= \left(\boldsymbol{L}\boldsymbol{\Gamma}(l)\boldsymbol{R}^{\mathrm{T}}\right)\boldsymbol{f}_j + \boldsymbol{W}(l)\boldsymbol{f}_j \\ &= \boldsymbol{L}_j\boldsymbol{\Gamma}_j(l)\boldsymbol{R}_j^{\mathrm{T}}\boldsymbol{\mathcal{P}}_j^{\perp}\boldsymbol{z} + \alpha_j(l)\boldsymbol{h}_P(\tau_j,\theta_j) + \widetilde{\boldsymbol{w}}(l) \\ &= \alpha_j(l)\boldsymbol{h}_P(\tau_j,\theta_j) + \widetilde{\boldsymbol{w}}(l), \quad l = 1\dots L \end{aligned} \tag{19}$$

where $\boldsymbol{L}_j$ is defined in a similar manner as $\boldsymbol{R}_j$ and $\boldsymbol{\Gamma}_j(l) \in \mathbb{C}$ is the same as $\boldsymbol{\Gamma}(l)$ in equation (16) but with eliminated j^{th} row and column. Furthermore, $\alpha_j(l) = \gamma_j^{(l)} \boldsymbol{h}_K^{\mathrm{T}}(\tau_j, \theta_j)\boldsymbol{f}_j$. Finally, $\widetilde{\boldsymbol{w}}(l) = \boldsymbol{W}(l)\boldsymbol{f}_j$ is the noise part, which is easily verified to be colored noise.

Equation (19) is key to what follows. In other words, we know that a vector $\boldsymbol{f}_j$ exists, which can select the contribution of the j^{th} source. Next, we derive a Least-Square (LS) estimator of all the unknown parameters.

C. Least-Square Estimator

The parameters concerning the j^{th} source are

$$\boldsymbol{\Theta}_j = [\boldsymbol{f}_j^{\mathrm{T}}, \boldsymbol{\alpha}_j^{\mathrm{T}}, \tau_j, \theta_j] \tag{20}$$

where $\boldsymbol{\alpha}_j = [\alpha_j(1) \dots \alpha_j(L)]^{\mathrm{T}}$. Let's stack all unknown parameters into one vector $\boldsymbol{\Theta}$, i.e.

$$\boldsymbol{\Theta} = [\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2 \dots \boldsymbol{\Theta}_q] = [\boldsymbol{f}^{\mathrm{T}}, \boldsymbol{\alpha}^{\mathrm{T}}, \boldsymbol{\tau}, \boldsymbol{\theta}] \tag{21}$$

where

$$\boldsymbol{f} = [\boldsymbol{f}_1^{\mathrm{T}} \dots \boldsymbol{f}_q^{\mathrm{T}}]^{\mathrm{T}} \text{ and } \quad \boldsymbol{\alpha} = [\boldsymbol{\alpha}_1^{\mathrm{T}} \dots \boldsymbol{\alpha}_q^{\mathrm{T}}]^{\mathrm{T}} \tag{22}$$

$$\boldsymbol{\tau} = [\tau_1 \dots \tau_q] \quad \text{ and } \quad \boldsymbol{\theta} = [\theta_1 \dots \theta_q] \tag{23}$$

All parameters in $\boldsymbol{\Theta}$ have to be jointly estimated. In this section, we propose to estimate these parameters by Least-Squares (LS). In other words, we seek to optimise the following cost function

$$\widehat{\boldsymbol{\Theta}}^{\mathrm{LS}} = \arg\min_{\boldsymbol{\Theta}} g(\boldsymbol{\Theta}) \tag{24}$$

where

$$g(\boldsymbol{\Theta}) = \sum_{j=1}^{q} \sum_{l=1}^{L} \left\| \boldsymbol{X}(l)\boldsymbol{f}_j - \alpha_j(l)\boldsymbol{h}_P(\tau_j, \theta_j) \right\|^2 \tag{25}$$

and $\widehat{\boldsymbol{\Theta}}^{\mathrm{LS}}$ is the LS estimate of $\boldsymbol{\Theta}$. We re-write $g(\boldsymbol{\Theta})$ in a compact way as follows

$$g(\boldsymbol{\Theta}) = \boldsymbol{f}^{\mathrm{H}}\big(\boldsymbol{I}_q \otimes \boldsymbol{Q}\big)\boldsymbol{f} - 2\,\Re\big(\boldsymbol{f}^{\mathrm{H}}\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta})\boldsymbol{\alpha}\big) + P\|\boldsymbol{\alpha}\|^2 \tag{26}$$

where matrices $\boldsymbol{Q}$ and $\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta})$ are given by

$$\boldsymbol{Q} = \boldsymbol{\mathcal{X}}^{\mathrm{H}}\boldsymbol{\mathcal{X}} \tag{27}$$

$$\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta}) = \text{blkdiag } [\boldsymbol{S}(\tau_1, \theta_1) \dots \boldsymbol{S}(\tau_q, \theta_q)] \tag{28}$$

and matrices $\boldsymbol{\mathcal{X}}$ and $\boldsymbol{S}(\tau, \theta)$ are defined as

$$\boldsymbol{\mathcal{X}} = \big[\ \boldsymbol{X}^{\mathrm{H}}(1) \mid \dots \mid \boldsymbol{X}^{\mathrm{H}}(L)\ \big]^{\mathrm{H}} \tag{29}$$

$$\boldsymbol{S}(\tau, \theta) = \boldsymbol{\mathcal{X}}^{\mathrm{H}}\boldsymbol{\mathcal{H}}(\tau, \theta) \tag{30}$$

where

$$\boldsymbol{\mathcal{H}}(\tau, \theta) = \boldsymbol{I}_L \otimes \boldsymbol{h}_P(\tau, \theta) \tag{31}$$

Fixing $(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta})$, we optimise the cost function $g(\boldsymbol{\Theta})$ w.r.t $\boldsymbol{f}$. Hence, setting the derivative of $g(\boldsymbol{\Theta})$ w.r.t $\boldsymbol{f}$ to zero, we get

$$\frac{\partial g(\boldsymbol{\Theta})}{\partial \boldsymbol{f}} = 2\big(\boldsymbol{I}_q \otimes \boldsymbol{Q}\big)\boldsymbol{f} - 2\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta})\boldsymbol{\alpha} = 0 \tag{32}$$

which gives

$$\widehat{\boldsymbol{f}}^{\mathrm{LS}} = \big(\boldsymbol{I}_q \otimes \boldsymbol{Q}\big)^{-1}\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta})\boldsymbol{\alpha} \tag{33}$$

Now, we treat $\widehat{\boldsymbol{f}}^{\mathrm{LS}}$ as a nuissance parameter and plug it in the cost function $g(\boldsymbol{\Theta})$ in equation (26), namely

$$\begin{aligned} g(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta}) &\triangleq g(\widehat{\boldsymbol{f}}^{\mathrm{LS}}, \boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta}) \\ &= \boldsymbol{\alpha}^{\mathrm{H}}\Big(P\boldsymbol{I}_{qL} - \boldsymbol{C}^{\mathrm{H}}(\boldsymbol{\tau}, \boldsymbol{\theta})\big(\boldsymbol{I}_q \otimes \boldsymbol{Q}\big)^{-1}\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta})\Big)\boldsymbol{\alpha} \end{aligned} \tag{34}$$

Due to the block diagonal nature of $\boldsymbol{C}(\boldsymbol{\tau}, \boldsymbol{\theta})$, and using

$$\big(\boldsymbol{I}_q \otimes \boldsymbol{Q}\big)^{-1} = \boldsymbol{I}_q \otimes \boldsymbol{Q}^{-1} \tag{35}$$

The function $g(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta})$ decouples into q positive cost functions

$$g(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta}) = \sum_{j=1}^{q} g_j(\boldsymbol{\alpha}_j, \tau_j, \theta_j) \tag{36}$$

Denoting $g_j \triangleq g_j(\boldsymbol{\alpha}_j, \tau_j, \theta_j)$ for ease of notation, we can say

$$\begin{aligned} g_j &= \boldsymbol{\alpha}_j^{\mathrm{H}}\Big(P\boldsymbol{I}_L - \boldsymbol{S}^{\mathrm{H}}(\tau_j, \theta_j)\boldsymbol{Q}^{-1}\boldsymbol{S}(\tau_j, \theta_j)\Big)\boldsymbol{\alpha}_j \\ &= \boldsymbol{\alpha}_j^{\mathrm{H}}\Big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j, \theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j, \theta_j)\Big)\boldsymbol{\alpha}_j \end{aligned} \tag{37}$$

where the last equality is due to equations (27) and (30). The projector $\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}$ is given as

$$\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp} = \boldsymbol{I}_{LP} - \boldsymbol{\mathcal{X}}(\boldsymbol{\mathcal{X}}^{\mathrm{H}}\boldsymbol{\mathcal{X}})^{-1}\boldsymbol{\mathcal{X}}^{\mathrm{H}} \tag{38}$$

Fixing $(\boldsymbol{\tau}, \boldsymbol{\theta})$ in $g(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta})$, each function g_j is quadratic in $\boldsymbol{\alpha}_j$. Note that minimising $g(\boldsymbol{\alpha}, \boldsymbol{\tau}, \boldsymbol{\theta})$ w.r.t $\boldsymbol{\alpha}$ is equivalent to minimising each g_j w.r.t $\boldsymbol{\alpha}_j$ since $g_j \geq 0$ for all j. In order to prevent a function g_j to be minimized at the trivial solution $\boldsymbol{\alpha}_j = \boldsymbol{0}$, we form the following *Equality Constrained Quadratic Optimisation* [19] problem

$$\begin{cases} \underset{\boldsymbol{\alpha}_j}{\text{minimize}} & g_j(\boldsymbol{\alpha}_j, \tau_j, \theta_j) \\ \text{subject to} & \boldsymbol{\alpha}_j^{\mathrm{H}}\boldsymbol{e}_1 = 1 \end{cases} \tag{39}$$

where $\boldsymbol{e}_1$ is the 1^{st} column of $\boldsymbol{I}_L$. The *Lagrangian* function corresponding to the optimisation problem in (39) is the following:

$$\mathcal{L}(\boldsymbol{\alpha}_j, \lambda) = g_j(\boldsymbol{\alpha}_j, \tau_j, \theta_j) - \lambda\big(\boldsymbol{\alpha}_j^{\mathrm{H}}\boldsymbol{e}_1 - 1\big) \tag{40}$$

Setting the derivative of $\mathcal{L}(\boldsymbol{\alpha}_j, \lambda)$ w.r.t $\boldsymbol{\alpha}_j$ to 0, we get

$$\frac{\partial \mathcal{L}(\boldsymbol{\alpha}_j, \lambda)}{\partial \boldsymbol{\alpha}_j} = 2\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j, \theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j, \theta_j)\boldsymbol{\alpha}_j - \lambda\boldsymbol{e}_{\boldsymbol{1}} = 0 \tag{41}$$

which yields

$$\widehat{\boldsymbol{\alpha}}_j^{\mathrm{LS}} = \frac{\lambda}{2}\big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j, \theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j, \theta_j)\big)^{-1}\boldsymbol{e}_{\boldsymbol{1}} \tag{42}$$

Plugging this expression of $\widehat{\boldsymbol{\alpha}}_j^{\mathrm{LS}}$ in the constraint of the optimisation problem in equation (39), we can solve for the *Lagrangian* multiplier λ as

$$\lambda = \frac{2}{\boldsymbol{e}_1^{\mathrm{H}}\big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j, \theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j, \theta_j)\big)^{-1}\boldsymbol{e}_1} \tag{43}$$

and therefore $\widehat{\boldsymbol{\alpha}}_j^{\mathrm{LS}}$ is obtained by plugging the expression of λ in equation (42), i.e.

$$\widehat{\boldsymbol{\alpha}}_j^{\mathrm{LS}} = \frac{\big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j,\theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j,\theta_j)\big)^{-1}\boldsymbol{e}_1}{\boldsymbol{e}_1^{\mathrm{H}}\big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j,\theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j,\theta_j)\big)^{-1}\boldsymbol{e}_1} \tag{44}$$

hence $\widehat{\boldsymbol{\alpha}}^{\mathrm{LS}}$ is obtained by stacking all $\widehat{\boldsymbol{\alpha}}_j^{\mathrm{LS}}$ into one vector as in equation (22). As done before, we treat $\widehat{\boldsymbol{\alpha}}^{\mathrm{LS}}$ as nuissance parameters and thus we substitute them in $g(\boldsymbol{\alpha},\boldsymbol{\tau},\boldsymbol{\theta})$ to get $g(\boldsymbol{\tau},\boldsymbol{\theta}) \triangleq g(\widehat{\boldsymbol{\alpha}}^{\mathrm{LS}},\boldsymbol{\tau},\boldsymbol{\theta})$, where

$$g(\boldsymbol{\tau},\boldsymbol{\theta}) = \sum_{j=1}^{q} \frac{1}{\boldsymbol{e}_1^{\mathrm{H}}\big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau_j,\theta_j)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau_j,\theta_j)\big)^{-1}\boldsymbol{e}_1} \tag{45}$$

The LS estimates of the ToAs $\boldsymbol{\tau}$ and AoAs $\boldsymbol{\theta}$ are simply

$$(\widehat{\boldsymbol{\tau}}^{\mathrm{LS}},\widehat{\boldsymbol{\theta}}^{\mathrm{LS}}) = \arg\min_{\boldsymbol{\tau},\boldsymbol{\theta}} g(\boldsymbol{\tau},\boldsymbol{\theta}) \tag{46}$$

Since $g(\boldsymbol{\tau},\boldsymbol{\theta})$ is decoupled into q identical functional forms, given in the last equality in equation (45), then one can jointly estimate the ToAs/AoAs by performing a 2D-search as

$$\{(\widehat{\tau}_j^{\mathrm{LS}},\widehat{\theta}_j^{\mathrm{LS}})\}_{j=1}^{\widehat{q}} = \arg\max_{\tau,\theta} f_{\mathrm{JADED}}(\tau,\theta) \tag{47}$$

where

$$f_{\mathrm{JADED}}(\tau,\theta) = \boldsymbol{e}_1^{\mathrm{H}}\big(\boldsymbol{\mathcal{H}}^{\mathrm{H}}(\tau,\theta)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{\mathcal{H}}(\tau,\theta)\big)^{-1}\boldsymbol{e}_1 \tag{48}$$

and $\widehat{q}$ is an estimate of q obtained by the number of peaks in $f_{\mathrm{JADED}}(\tau,\theta)$.

IV. Computationally Efficient Single Snapshot JADED-RIP (CESS-JADED-RIP)

The JADED-RIP algorithm requires a 2D search over the variables (τ,θ). It turns out that for a single snapshot, i.e. $L = 1$, we can propose a computationally more efficient method, which we call here Computationally Efficient Single Snapshot JADED-RIP, or simply CESS-JADED-RIP. For a single snapshot and using equation (31), $f_{\mathrm{JADED}}(\tau,\theta)$ can be expressed as

$$f_{\mathrm{JADED}}(\tau,\theta) = \frac{1}{\boldsymbol{h}_P^{\mathrm{H}}(\tau,\theta)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{h}_P(\tau,\theta)} \tag{49}$$

Using the structure of $\boldsymbol{h}_P(\tau,\theta)$ in equation (14), we can write the denominator in equation (49) as follows

$$\boldsymbol{h}_P^{\mathrm{H}}(\tau,\theta)\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\boldsymbol{h}_P(\tau,\theta) = \boldsymbol{a}_{P_N}^{\mathrm{H}}(\theta)F(\tau)\boldsymbol{a}_{P_N}(\theta) \tag{50}$$

where

$$F(\tau) = \big(\boldsymbol{c}_{P_M}(\tau)\otimes\boldsymbol{I}_{P_N}\big)^{\mathrm{H}}\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}\big(\boldsymbol{c}_{P_M}(\tau)\otimes\boldsymbol{I}_{P_N}\big) \tag{51}$$

Maximising (49) is equivalent to minimizing (50), hence we aim at solving

$$\begin{cases} \underset{\boldsymbol{a}_{P_N}(\theta)}{\text{minimize}} & \boldsymbol{a}_{P_N}^{\mathrm{H}}(\theta)F(\tau)\boldsymbol{a}_{P_N}(\theta) \\ \text{subject to} & \boldsymbol{a}_{P_N}^{\mathrm{H}}(\theta)\boldsymbol{e}_1 = 1 \end{cases} \tag{52}$$

Following similar steps as in equations (40) till (44), the vector $\widehat{\boldsymbol{a}}_{P_N}(\theta)$ that solves the above problem is given as

$$\widehat{\boldsymbol{a}}_{P_N}(\theta) = \frac{F^{-1}(\tau)\boldsymbol{e}_1}{\boldsymbol{e}_1^{\mathrm{H}}F^{-1}(\tau)\boldsymbol{e}_1} \tag{53}$$

Substituting $\widehat{\boldsymbol{a}}_{P_N}(\theta)$ in the objective function of the problem in equation (52) gives us a cost function in τ, and therefore the ToAs are estimated as follows

$$\{\widehat{\tau}_j\}_{j=1}^{\widehat{q}} = \arg\max_{\tau} w(\tau) \tag{54}$$

where

$$w(\tau) = \boldsymbol{e}_1^{\mathrm{H}}F^{-1}(\tau)\boldsymbol{e}_1 \tag{55}$$

Now, we are left with the estimation of the AoAs. Notice that equation (53) maps τ to θ, therefore for each $\widehat{\tau}_j$, we can obtain $\widehat{\boldsymbol{a}}_{P_N}(\widehat{\theta}_j)$ as

$$\widehat{\boldsymbol{a}}_{P_N}(\widehat{\theta}_j) = \frac{F^{-1}(\widehat{\tau}_j)\boldsymbol{e}_1}{\boldsymbol{e}_1^{\mathrm{H}}F^{-1}(\widehat{\tau}_j)\boldsymbol{e}_1}, \qquad j = 1\ldots\widehat{q} \tag{56}$$

Then, we estimate $\widehat{\theta}_j$ from $\widehat{\boldsymbol{a}}_{P_N}(\widehat{\theta}_j)$. This is done by forming the vector of phases of $\widehat{\boldsymbol{a}}_{P_N}(\widehat{\theta}_j)$ as follows

$$\big[\widehat{\boldsymbol{\phi}}_j\big]_k = -\frac{1}{2\pi d}\tan^{-1}\Big(\frac{\Im(\big[\widehat{\boldsymbol{a}}_{P_N}(\widehat{\theta}_j)\big]_k)}{\Re(\big[\widehat{\boldsymbol{a}}_{P_N}(\widehat{\theta}_j)\big]_k)}\Big), \quad j = 1\ldots\widehat{q} \tag{57}$$

After the operation in equation (57), we have $\widehat{\boldsymbol{\phi}}_j$ in the following form: $\widehat{\boldsymbol{\phi}}_j = \boldsymbol{\rho}\sin(\widehat{\theta}_j)$, where $\boldsymbol{\rho} = [0\ldots(P_N-1)]^{\mathrm{T}}$. Finally, we extract $\widehat{\theta}_j$ from $\widehat{\boldsymbol{\phi}}_j$ by the following LS fit

$$\widehat{\theta}_j = \arg\min_{\widehat{\theta}_j} \Big\|\widehat{\boldsymbol{\phi}}_j - \boldsymbol{\rho}\sin(\widehat{\theta}_j)\Big\|^2, \qquad j = 1\ldots\widehat{q} \tag{58}$$

The solution is easily verified to be

$$\widehat{\theta}_j = \sin^{-1}\Big(\boldsymbol{\rho}^{\dagger}\widehat{\boldsymbol{\phi}}_j\Big) = \sin^{-1}\Big(\frac{6\boldsymbol{\rho}^{\mathrm{T}}\widehat{\boldsymbol{\phi}}_j}{P_N(P_N-1)(2P_N-1)}\Big) \tag{59}$$

where $\boldsymbol{\rho}^{\dagger} = (\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\rho})^{-1}\boldsymbol{\rho}^{\mathrm{T}}$.

V. Identifiability conditions

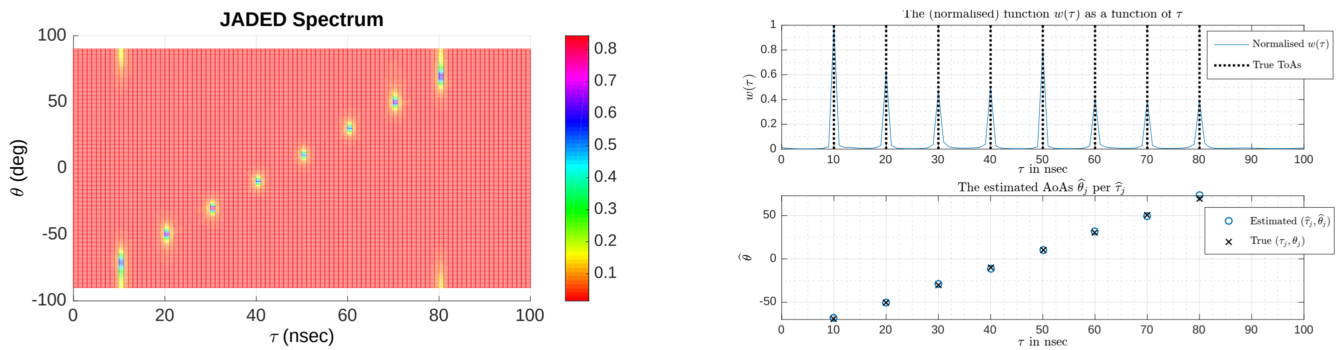
In this section, we derive identifiability conditions for unique estimation and detection of $(\boldsymbol{\tau},\boldsymbol{\theta})$ for JADED-RIP and CESS-JADED-RIP. The first set of conditions are given to guarantee a unique representation of equation (19), which happens when projectors $\{\boldsymbol{\mathcal{P}}_j^{\perp}\}_{j=1}^{q}$, given in equation (17), are uniquely defined. In other terms, these projectors should be full column rank. A sufficient condition for that to occur is when $\boldsymbol{R}$ is full column rank.

Theorem: *Let $\boldsymbol{R} \in \mathbb{C}^{K_M K_N \times q}$ be a matrix defined as in* (13)*, then $\boldsymbol{R}$ is full column rank if*

- $q \le K_M K_N$
- $Q_\tau \le K_N$ and $Q_\theta \le K_M$

where Q_τ is the maximum number of paths arriving with same ToA, but different AoAs; and Q_θ is the maximum number of paths arriving at the same AoA, but different ToAs.
Proof: Ommited due to lack of space.

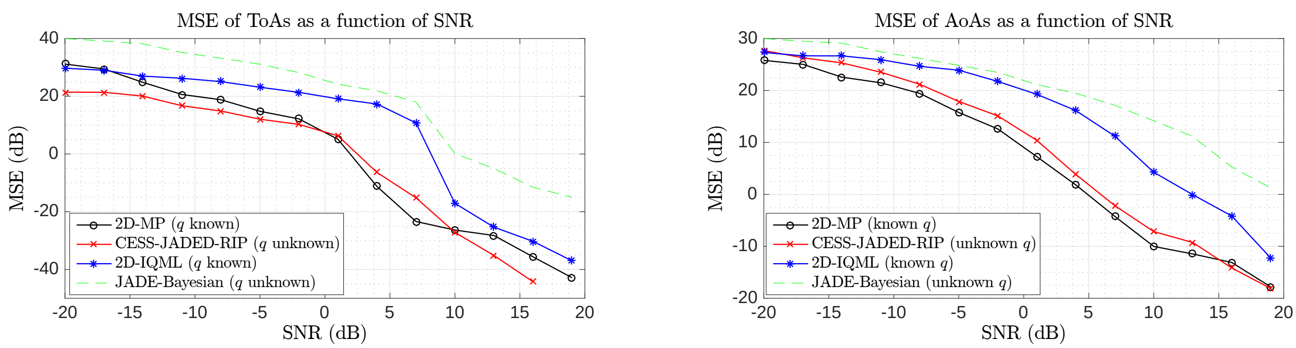
The second projector that should be uniquely defined is the data projector matrix, namely $\boldsymbol{\mathcal{P}}_{\boldsymbol{\mathcal{X}}}^{\perp}$, given in equation (38). A necessary condition is when $\boldsymbol{\mathcal{X}}$ is a tall matrix, namely $LP > K$. Combining ***Theorem 1*** and the condition of the



(a) The JADED-RIP method.

(b) The CESS-JADED-RIP method.

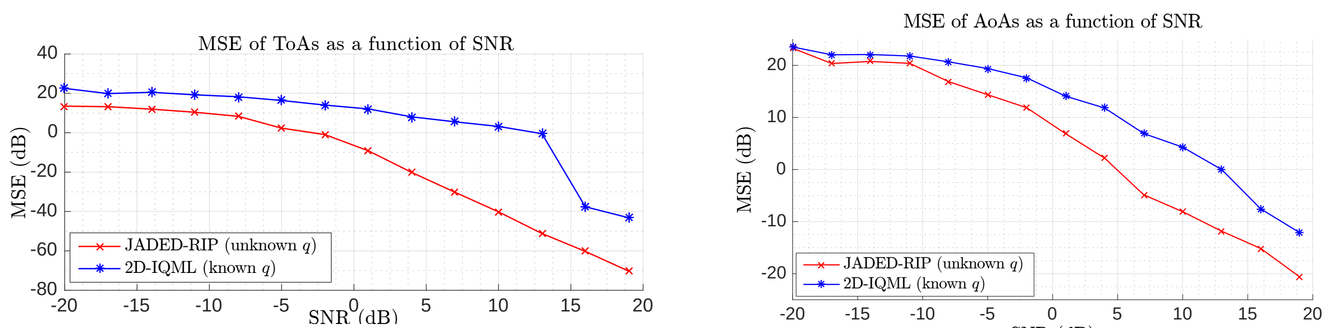
Fig. 1: Experimental plots of the JADED-RIP and CESS-JADED-RIP spectra



(a) MSE of ToAs

(b) MSE of AoAs

Fig. 2: MSE of ToAs/AoAs as a function of SNR of Experiment 2



(a) MSE of ToAs

(b) MSE of AoAs

Fig. 3: MSE of ToAs/AoAs as a function of SNR of Experiment 3

existance of the data projector, the JADED-RIP algorithm should satisfy the following:

- **A1**: $q \leq K_M K_N < L P_M P_N$
- **A2**: $Q_\tau \leq K_N$ and $Q_\theta \leq K_M$

As for CESS-JADED-RIP, the parameter Q_τ should be 1, since the ToAs are estimated through a 1D search over $w(\tau)$ given in equation (55). Therefore, this approach does not allow multiple paths arriving at the same time. Finally, the CESS-JADED-RIP method should satisfy the following:

- **B1**: $q \leq K_M K_N < P_M P_N$
- **B2**: $Q_\tau = 1$ and $Q_\theta \leq K_M$

VI. Simulation Results

This section provides three computer experiments to demonstrate and validate the potential of the proposed methods.

In Experiment 1, i.e. Fig 1, we plot the different spectra of the proposed algorithms. More precisely, Fig 1a plots the 2D-spectrum of the JADED-RIP given in equation (48). Also, Fig 1b plots the 1D-spectrum given in equation (55) (in order to estimate the ToAs) and the scatter plot to estimate the AoAs using the LS fit in equation (59). We have fixed $q = 8$ paths, where $\tau_k = 10k$ nsec and $\theta_k = -70 + 20(k-1)$ degrees, for $k = 1 \ldots 8$. Also the multipath coefficients are chosen to be i.i.d Gaussian of zero mean. The number of antennas used is $N = 3$ with $d = 0.5$ and the OFDM symbol comprises of

$M = 64$ subcarriers occupying a bandwidth of 200 MHz, i.e. $\triangle_f = 3.125$ MHz. We have chosen $P_M = 40$ and $P_N = 2$. The SNR is set to 5 dB. We have collected $L = 10$ OFDM symbols for the JADED-RIP method. It is interesting to see that we do not observe an overestimation of q in both methods, i.e. the peaks correspond to the true and only the true signal parameters.

In Experiment 2, i.e. Fig 2, we plot the MSE of ToA/AoA estimates of CESS-JADED-RIP as a function of SNR. Moreover, the MSE is compared with other existing methods, such as the 2D-MP [10], the 2D-IQML [11], and a straightforward extension of [16] to the 2D case, which we refer to as JADE-Bayesian. We have averaged over 10^3 Monte-Carlo trials. These methods are particularly chosen for this experiment, since they could deal with a single snapshot. We recall that 2D-MP and 2D-IQML require the knowledge of q, whereas JADED and JADE-Bayesian estimate q from data. Note that the value of q is prior known for both 2D-MP and 2D-IQML. To this end, we fix $q = 2$ paths, with $(\tau_1, \theta_1) = (10\text{nsec}, -70°)$ and $(\tau_2, \theta_2) = (20\text{nsec}, 20°)$. The values of N, M, P_N, P_M, d and $\triangle_f$ are the same as those in Experiment 1. The multipath parameters are set to $\boldsymbol{\gamma} = [1; 0.8e^{j\frac{\pi}{4}}]$, i.e. coherent sources. In addition, only $L = 1$ OFDM symbol is available. We see that the performance of CESS-JADED-RIP is very close to that of 2D-MP in terms of MSE of ToA and AoA, according to Fig 2a and Fig 2b, respectively. Also, we can see that CESS-JADED-RIP outperforms 2D-IQML and JADE-Bayesian.

In Experiment 3, i.e. Fig 3, we plot the MSE of ToA/AoA estimates of JADED-RIP and 2D-IQML as a function of SNR, when multiple snapshots are available. This is why we have excluded 2D-MP and JADE-Bayesian, since they only operate with one snapshot. The same parameters are set as in Experiment 2, except for L, which is set to 10. By referring to Fig 3a and Fig 3b, one could observe that the JADED-RIP outperforms 2D-IQML in terms of MSE of ToAs and AoAs, at any given SNR.

VII. Conclusions and Future Work

There are some contributions that should be highlighted: We have proposed a novel approach for joint estimation and detection of Angles and Times of arrival, i.e. JADED. Two methods were derived so as to solve the JADED problem using Rotational Invariance Properties (RIP), which arises when a ULA receives known OFDM symbols. The JADED-RIP method performs a 2D search of a suitable cost function, where each peak indicates a present source with corresponding ToA/AoA. The second algorithm, CESS-JADED-RIP, is a faster version of JADED-RIP, which can be used for single snapshot scenarios only. The algorithms function properly in the presence of coherent sources, since subspace extraction is not needed, as in the case of MUSIC, ESPRIT, and other subspace methods.

Future work should address the following points:

- Improving JADED-RIP, by taking into account the colored noise in equation (19), which leads to an ML estimator.
- Deriving analytic MSE expressions and the optimal values of P_N and P_M.
- Proposing a JADED algorithm that operates for arbitrary arrays, such as uniform circular arrays.
- Taking into account hardware imperfections, such as antenna calibration and mutual coupling, synchronization errors, etc. This could further empower JADED as a competitive candidate among other indoor positioning methods.

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