Mixed Time Scale Weighted Sum Rate Maximization for Hybrid Beamforming in Multi-Cell MU-MIMO Systems

Christo Kurisummoottil Thomas   Dirk Slock
EURECOM, Sophia-Antipolis, France, Email:{kurisumm,slock}@eurecom.fr

***Abstract*—This work deals with hybrid beamforming for the MIMO Interfering Broadcast Channel (IBC), i.e. the Multi-Input Multi-Output (MIMO) Multi-User Multi-Cell downlink. In the case of massive MIMO with a large number of antennas at the base station, it may not be feasible to have as many RF chains M as the number of antennas N_t. Hybrid beamforming (BF) involves two stage precoders with an analog stage in RF and a digital stage in baseband. Key assumptions here are that the fully connected analog stage consists of phase shifters and cannot be adapted at the fast fading rate. We consider BF design by maximizing the Weighted Sum Rate (WSR) for the case of Perfect Channel State Information at the Transmitter (CSIT). However, whereas analog and digital beamformers are optimized jointly only from time to time, only the digital beamformers are updated at the fast fading rate with the analog beamformer stage being frozen over the slow fading coherence times. Simulation results show that even if the analog beamformer gets outdated, close to optimal performance gets obtained thanks to the self averaging of large antenna systems. The simulations also show significant improvements compared to suboptimal covariance CSIT based designs for the analog BF.**

I. INTRODUCTION

In this paper, Tx may denote transmit/transmitter/ transmission and Rx may denote receive/receiver/reception. As Multi-Input Multi-Output (MIMO) systems allow spatial multiplexing, Massive MIMO systems employ large numbers of antennas at the base station to increase the spectral efficiency of the system and possibly simplify beamforming techniques. With a large number of antennas though, it may not be feasible to have as many RF chains as the antennas due to the increased cost of the number of RF chains required (which includes Analog- to-Digital and Digital-to-Analog converters (ADCs/DACs), power amplifiers and low noise amplifiers). So signal processing techniques called hybrid beamforming have been developed to take care of the case where the number of RF chains is less than the number of antennas.

In hybrid beamforming the baseband precoder or the digital precoder is a low dimensional matrix which multiplex the data streams to the number of RF chain which is much less than the number of antennas. The analog precoder further converts the output from the RF chains to the number of antennas. This technique was first introduced in [1] . In [1], a phase shifter constraint(unit modulus) is applied on the analog precoder elements. And the optimal analog precoder matrix is being derived for the case where there is only one data stream to be transmitted. In this case the maximum performance is obtained only when there are atleast 2 RF chains.

Most of the prior work on hybrid beamforming assumes perfect channel knowledge (CSIT) at the transmitter. In practice, this is very difficult to obtaint at fast fading rate, since for a massive mimo system it increases the feedback in the uplink substantially reducing the spectral efficiency. In [2] a scheme called joint spatial division and multiplexing (which is a two stage precoding scheme) is proposed such that a prebeamforming matrix is used which groups the users based on the spatial channel covariance. Users are separated in the spatial domain through a prebeamforming matrix and users in the same group are further multiplexed through a linear MU-MIMO precoding which considers the effective channel as that including the prebeamforming matrix. Some of the prior works on designing the hybrid beamformers are described in [3]-[6]. In [3] , some compressed sensing based schemes are proposed to estimate the channel in a hybrid struture utilizing the sparsity of the channel. In [4], the phase shifter analog precoding matrix is seen as a two step problem. In the first step, conditions for an unconstrained analog precoder matrix is derived assuming regularized zero forcing precoder for the baseband. Also it is assumed that in the large system analysis limit, the SLNR for all users are equal. Also the zero forcing precoding for digital beamforming makes it a suboptimal scheme. Once the unconstrained precoder is obtained , the phase shifter constrained analog precoder is obtained through an iterative algorithm by minimising the euclidean distance between unconstrained and the phase shifter constrained matrices. In [7] , various architectures for the phase shifter matrix of analog precoder are given.

In this paper, we are directly considering the design of analog precoder matrix where all the elements are phase elements by maximisation of the weighted sum rate. Each element of the phase shifter matrix (analog precoder) is obtained through an iterative process similar to [5]. But in [5], for the digital beamformers the authors assume zero forcing precoders with effective channel including the analog precoder. With this approach, WSR problem simplifies to a waterfilling algorithm for the power. For the analog precoder, taking the minimization of power as objective, they optimize each element of the analog precoder matrix. Again this is a suboptimal approach.

The contributions of this paper are:

- In this paper, for a multi-cell multi-user MIMO, we derive the hybrid digital and analog beamformers based on the maximization of weighted sum rate which is formulated as a weighted sum MSE (WSMSE) problem. The analog precoder is assumed to have all elements with unit modulus (or only phase elements) and each of the elements in this matrix is optimized iteratively in an alternating optimization fashion, as also the digital beamformers and auxiliary quantities (receivers and weights).
- Simulations are performed by alternating between joint updating of analog and digital beamformer using perfect CSIT, and then updating the digital beamformers on uncorrelated channel realizations (but with same covariance) while the analog beamformers are frozen. Results shows that even with the analog precoder based on outdated CSIT, close to optimal performance (WSR) is attained. Thus reducing the CSIT feedback overhead for a large antenna array system. Practically, this is even more of interest for an OFDM system, where we cannot afford to lose throughput due to large feedback for CSIT. The results suggest indeed that we could afford updating the analog beamformers only once every so many channel uses across time or frequency in an OFDM system.

For the work in this paper, the extension from perfect (instantaneous) CSIT to imperfect CSIT could be immediate, at least if one uses Expected WSMSE along the lines of [11].

Notation: In the following, boldface lower-case and upper-case characters denote vectors and matrices respectively. The operators $E[\cdot]$, $tr(\cdot)$, $(\cdot)^H$, $(\cdot)^T$ represents expectation, trace , conjugate transpose and transpose respectively. $(\cdot)^*$ represents conjugate of a complex scalar. A circularly complex Gaussian random vector with mean μ and covariance matrix Θ is distributed as $x \sim \mathcal{CN}(\mu, \Theta)$.

II. Multi-User MIMO System Model

Consider a Multi-User MIMO system with N_t^c transmit antennas in cell c and K multi-antenna users. In the rest of this paper we shall consider a per stream approach (which in the perfect CSI case would be equivalent to per user). In an IBC formulation, one stream per user can be expected to be the usual scenario. In the development below, in the case of more than one stream per user, we shall treat each stream as an individual user. So, consider an IBC with C cells with a total of K users. We shall consider a system-wide numbering of the users. User k is served by BS b_k. User k is equipped with N_k antennas. The $N_k \times 1$ received signal at user k in cell b_k can be written as

$$y_k = \underbrace{H_{k,b_k}\, V^{b_k}\, g_k\, s_k}_{\text{signal}} + \underbrace{\sum_{i \neq k, b_i = b_k} H_{k,b_k}\, V^{b_k}\, g_i\, s_i}_{\text{intracell interf.}} + \underbrace{\sum_{c \neq b_k} \sum_{i: b_i = c} H_{k,c}\, V^{b_i}\, g_i\, s_i}_{\text{intercell interf.}} + v_k \tag{1}$$

where s_k is the intended (white, unit variance) scalar signal stream, H_{k,b_k} is the $N_k \times N_t^{b_k}$ channel from BS b_k to user k. BS c serves $U_c = \sum_{i:b_i=c} 1$ users. We considering a noise whitened signal representation so that we get for the noise $v_k \sim \mathcal{CN}(0, I_{N_k})$. The $N_t^{b_k} \times 1$ spatial Tx filter or beamformer (BF) is g_k. The analog beam former for base station c , V^c is of dimension $N_t^c \times M^c$. M^c is the number of RF chains at BS c. Treating interference as noise, user k will apply a linear Rx filter f_k (of dimension $N_k \times 1$) to maximize the signal power (diversity) while reducing any residual interference that would not have been (sufficiently) suppressed by the BS Tx. The Rx filter output is $\widehat{s}_k = f_k^H y_k$, hence

$$\widehat{s}_k = f_k^H H_{k,b_k} V^{b_k} g_k\, s_k + \sum_{i=1, \neq k}^{K} f_k^H H_{k,b_i} V^{b_i} g_i\, s_i + f_k^H v_k. \tag{2}$$

The transmit power constraints could be written as

$$\mathrm{tr}(V^c \left(\sum_{i: b_i = c}^{K} g_i\, g_i^H \right) V^{b_k\, H}) \leq P_c \tag{3}$$

where P_c is the transmit power constraint at BS c.

III. WSR Optimization in terms of WSMSE

We consider the problem of maximizing the weighted sum rate of the MIMO IBC system. This could be written as

$$[g_1^{WSR}, ..., g_K^{WSR}, V^{1,WSR}, ..., V^{C,WSR}] = \arg\max_{g,V} \sum_{k=1}^{K} u_k\, R_k. \tag{4}$$

where the u_k are rate weights. Here the optimization is over analog beamformers for C cells and digital beamformers for the K users. Under Gaussian signaling and optimal single user decoding, rate R_k for user k is defined as

$$R_k = \max_{f_k} \ln(1 + \gamma_k). \tag{5}$$

where γ_k is the SINR (Signal to Interference plus Noise Ratio) for user k. It can be written as,

$$\gamma_k = \frac{\left| f_k H_{k,b_k}\, V^{b_k}\, g_k \right|^2}{\sum_{i=1, i \neq k}^{K} \left| f_k H_{k,b_i}\, V^{b_i}\, g_i \right|^2 + ||f_k||^2} \tag{6}$$

Solving weighted sum rate is equivalent to solving the weighted sum MSE problem from [8,9]. The augumented cost function for the WSMSE could be written as

$$\begin{aligned} WSMSE(V,g,f,w) &= \textstyle\sum_{k=1}^{K} u_k \left(w_k\, e_k\left(f_k, V, g\right) - ln\left(w_k\right) \right) \\ &+ \textstyle\sum_{c=1}^{C} \lambda_c \left[\mathrm{tr}\left(V^{c\,H} V^c \sum_{i:b_i=c} g_i\, g_i^H \right) - P_c \right] \end{aligned} \tag{7}$$

Where e_k is the MSE, w_k is the MSE weight. Let us define the transmit SNR as $\rho_c = P_c$. The MSE is

$$\begin{aligned} e_k(f_k, V, g) &= E\left[(s_k - f_k^H y_k)(s_k - f_k^H y_k)^H \right] \\ &= 1 - f_k^H\, H_{k,b_k}\, V^{b_k}\, g_k - g_k^H\, V^{b_k\, H}\, H_{k,b_k}^H\, f_k + \\ &\textstyle\sum_{i=1}^{K} f_k^H\, H_{k,b_i}\, V^{b_i}\, g_i\, g_i^H\, V^{b_i\, H}\, H_{k,b_k}^H\, f_k + \|f_k\|^2 \end{aligned} \tag{8}$$

assuming $\mathrm{E}\left[|s_k|^2 \right] = 1$. As in [9,11], performing alternating optimization leads to solving simple quadratic or convex functions:

$$\min_{w_k} WSMSE \implies w_k = e_k^{-1} = 1 + \gamma_k \tag{9}$$

$$\min_{f_k} WSMSE \implies \tag{10}$$

$$f_k = (\sum_{i=1}^{K} H_{k,b_i} V^{b_i} g_i g_i^H V^{b_i\,H} H_{k,b_i}^H + 1)^{-1} H_{k,b_k} V^{b_k} g_k$$

$$\min_{g_k} WSMSE \implies g_k = \tag{11}$$

$$(\sum_{i=1}^{K} \frac{u_i}{e_i} V^{b_k\,H} H_{i,b_k}^H f_i f_i^H H_{i,b_k} V^{b_k} + \lambda_{b_k} V^{b_k\,H} V^{b_k})^{-1}$$

$$V^{b_k\,H} H_{k,b_k}^H f_k \frac{u_k}{e_k}$$

where an analytical expression can be obtained for the Lagrange multipliers, viz.

$$\lambda_c = \frac{1}{P_c} \sum_{i:b_i=c} \frac{u_i}{e_i} ||f_i||^2 \,, \tag{12}$$

and the beamformers are actually rescaled to make sure that the transmit power constraints are satisfied:

$$g_k \leftarrow \xi_{b_k} g_k \;,\; \xi_c = \sqrt{P_c / \sum_{i:b_i=c} ||V^c g_i||^2} \,. \tag{13}$$

So the algorithm performs alternating optimization between the MSE weights, the Rx MMSE filters, and the digital and possibly the analog beamformers for which we shall discuss the optimization now.

A. Design of the Analog Beamformer with Perfect CSIT

Given g, f, and w, the analog beamformer V^c can be found by performing alternating optimization elementwise. Accounting of the unit modulus constraints of the entries of V^c can be done by parameterizing as

$$\left|V_{m,n}^c\right| = 1 \Rightarrow V_{m,n}^c = e^{j\,\theta_{m,n}^c} \,. \tag{14}$$

Now, as shown in the Appendix, the WSMSE can be written as

$$WSMSE = 2\Re\{e^{j\theta_{m,n}^c} a_{m,n}^c\} + \text{"terms not containing } \theta_{m,n}^c\text{"} \tag{15}$$

where the scalar $a_{m,n}^c$ is defined in (28). Then the minimization of the WSMSE w.r.t. $\theta_{m,n}^c$ yields

$$\theta_{m,n}^c = \pi - \angle a_{m,n}^c \,. \tag{16}$$

Note that one phase factor in V^c is undetermined, hence e.g. $\theta_{1,1}^c = 0$. Then (16) can be iterated for the other $m = 1,...,N_t^c$, $n = 1,...,M^c$. The complete derivation is given in the Appendix. The steps of the complete iterative algorithm are given in the table Algorithm 1. There the V^c are initialized from the M^c dominating generalized eigen vectors of the "signal" channel covariances $\sum_{k:b_k=c} \Theta_k^c$ and the "interference" channel covariances $\sum_{i:b_i \neq c} \Theta_i^c$ where $\Theta_k^c = \mathrm{E}[H_{k,c}^H H_{k,c}]$. Also, the operation $e^{j\angle A}$ for a matrix A takes the elementwise phasors. Various variations on the alternating optimization updating schedules are possible. For instance, the elements $\theta_{m,n}^c$ could be updated only once in every sweep of updates of all quantities (as suggested in the table), or these elements could be iterated separately until convergence before updating again the other quantities.

B. Mixed Time Scale Adaptation

In this paper, we consider two variants of the WSMSE iterative algorithm shown. In the first variant, Fast Time Scale Adaptation, the WSR is maximized straightforwardly using Algorithm 1, using (perfect) instantaneous CSIT for the computation of both analog and digital beamformers V and g. This adaptation is repeated whenever the instantaneous CSIT (the channels $H_{k,c}$) changes.

In the second variant, Mixed Time Scale Adaptation, the overall Fast Time Scale Adaptation just mentioned gets executed only from time to time, whenever the slow CSIT, here captured by the channel covariance matrices Θ_k^c, changes. In between those slow CSIT updates, the digital beamformers g and all auxiliary quantities, but not the analog beamformers, get updated whenever the fast CSIT changes. This can be done using Algorithm 1, in which step 5), the update of the analog beamformers, gets skipped. Hence, the values of the analog beamformers are frozen over the slow fading coherence time, whereas only the digital parts get updated at the fast fading rate. Whenever the slow fading CSIT is considered to have changed, all quantities are updated using the instantaneous CSIT available at such time instant. No dynamics of the fast or slow fading processes get exploited. When an update gets performed, all quantities to be updated get recomputed from scratch. The information in the previous updates gets ignored, except for providing the initialization values. The initialization mentioned in Algorithm 1 in fact gets performed only once, at the very first initialization of the whole process.

Algorithm 1 WSMSE Iterative algorithm

Given: $P_c, H_{k,c}, u_k \forall k, c$.
Initialization: $V^c = e^{j\angle V_{1:M^c}(\sum_{k:b_k=c} \Theta_k^c, \sum_{i:b_i \neq c} \Theta_i^c)}$
The f_k are taken as the dominant left singular vector of H_{k,b_k}.
The g_k are taken as the MMSEZF precoders for the effective channels $f_k H_{k,c} V^c$.
Initialize SINR $\gamma_k^{(0)}$ from (6).
Iteration (j)

1) Update $\forall k$, $e_k^{(j)}$, $w_k^{(j)}$ from (9)
2) Update $\forall k$, $f_k^{(j)}$ from (10)
3) Update $\forall c$, $\lambda_c^{(j)}$ from (12)
4) Update $\forall k$, $g_k^{(j)}$ from (11),(13)
5) Update $\forall c$, $\forall (m,n)$, $V_{m,n}^{c(j)}$ from (16)
6) Compute $\forall k$, $\gamma_k^{(j)}$, from (6)
7) Check for convergence of the WSR, if not go to step 1)

IV. Simulation Results

In this section we evaluate the proposed algorithm using simulation results, which will be limited to a single cell MISO system. We used a simple channel model similar to [4], that is based on a uniform linear array (ULA). Assume that there are L multi-path components between a user and the base station. The channel between base station and k^{th} user can be written as $h_k = \sum_{i=1}^{L} \alpha_{k,i}\, a_t\left(\phi_{k,l}\right)$. Assuming $\lambda/2$ as the antenna spacing, the antenna array response is written as

$$a_t\left(\phi\right) = \left[1 \; e^{j\pi \sin(\phi)} \; \ldots \; e^{j\pi (N_t - 1)\sin(\phi)}\right]^T . \tag{17}$$

The complex path amplitudes are modeled as Rayleigh fading $\alpha_{k,l} \sim \mathcal{CN}(0, \sigma^2_{\alpha_l})$, where $\sigma^2_{\alpha_l}$ is assumed to be from an exponential distribution with parameter 1. The $\phi_{k,l}$ are taken from a laplacian distribution with angular spread as 10 degrees. These angle values and path powers are fixed for all channel realizations (slow fading). In each channel realization, what changes is the complex path gains $\alpha_{k,l}$ (fast fading).

Notations used for the figures : oCSIT refers to outdate CSIT, iCSIT means instantaneous CSIT and CoCSIT implie covariance CSIT. V Fixed in the figures refers to the cas where V is fixed to be the M dominant eigen vectors of th sum of the users channel covariance matrix. In all the figure the simulations are done for $SNR = 20dB$, $L = 6$. Sinc single cell, $C = 1$ and the number of users is denoted by U

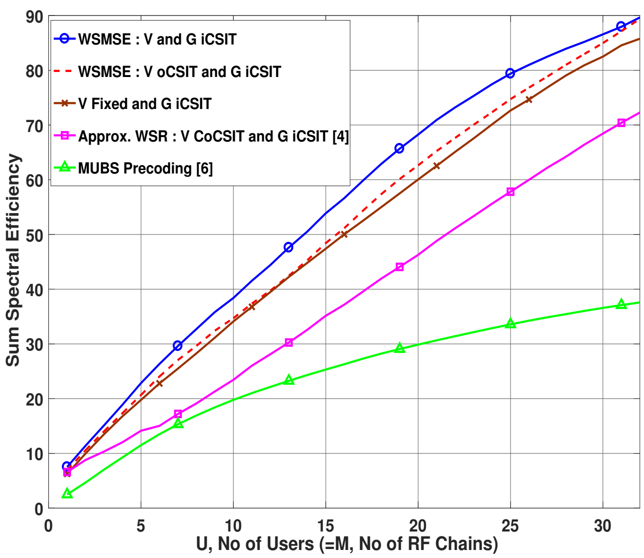

Fig. 1. Sum Spectral Efficiency vs No of Users U = M , M is no of RF Chains. $N_t = 32$, $SNR = 20\,dB$ and $L = 6$ paths.

The sum spectral efficiency (sum of the rates of the U users) is plotted versus the number of users and compared with the sub-optimal algorithms proposed in [4] and [6]. In [4], V is computed using covariance CSIT and the g are updated with instantaneous CSIT. The interesting part about our work is in showing that the analog beamformer need not be adapted at the fast fading rate. In order for the comparison to be fair, the proposed update of the analog beamformer with outdated CSIT and digital beamformer with instantaneous CSIT is compared with prior works which use covariance CSIT for V and instantaneous CSIT for g. It is evident from all the figures that our approach based on Mixed Time Scale WSMSE Adaptation outperforms those of [4] and [6]. In Figure 1 $N_t = 32$ and the number of users is equal to the number of RF chains ($U = M$). Simulations are done for the two variants of CSIT. In the first case, V and g are iteratively updated w.r.t the instantaneous channel. In the second case, we consider two realizations of the channels $(h_k(1), h_k(2))$. For $h_k(1)$, V and g are updated as per Algorithm 1. For $h_k(2)$, the result obtained with $h_k(1)$ is used for V and is not updated, whereas the g are updated. So this is the case of outdated CSIT for V. The results are averaged over 40 such pairs of channel realizations, each time with the same channel covariances. From Figure 1 we can see that in the second case with outdated CSIT for V, performance is slightly degraged but still much better than the suboptimal algorithms of [4] and [6].

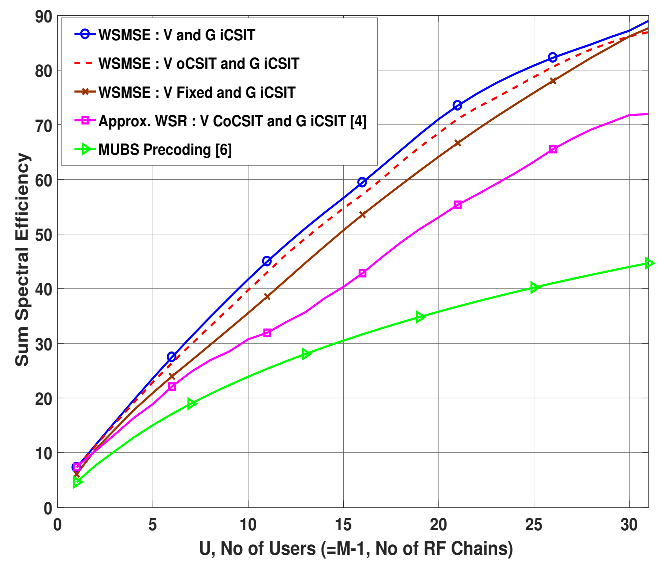

Fig. 2. Sum Spectral Efficiency vs No of Users, U = M -1, M is no of RF Chains. $N_t = 32$, $SNR = 20\,dB$ and $L = 6$ paths.

In Figures 2 and 3, we repeat the same simulation as described above with $U = M-1$ and $U = M-4$ respectively. It can be seen by comparing Figures 1 and 2 that when there is an excess of RF chains over the number of users, the spectral efficiency increases.

In Figure 4 we have considered the case where $N_t = 64$ and $U = M$. In Figures 5 and 6, we consider $N_t = 64$ antennas with $U = M - 1$ and $U = M - 4$ respectively.

V. Conclusion

In this paper, we derived and presented an optimal beamforming algorithm for the hybrid beamforming scenario in a multi-user multi-cell MIMO system. The weighted sum rate maximization problem is considered in a weighted sum MSE formulation and an iterative algorithm is obtained which jointly optimizes both analog and digital beamformers. The derived results are with the instantaneous channel information for both digital and analog beamformers. The simulations show that the spectral efficiency obtained by updating the analog

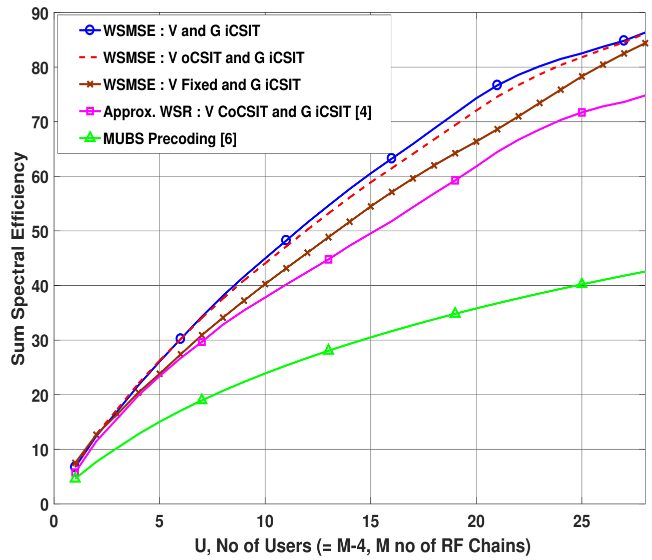

Fig. 3. Sum Spectral Efficiency vs No of Users, U= M -4, M is no of RF Chains. $N_t = 32$, $SNR = 20\,dB$ and $L = 6$ paths.

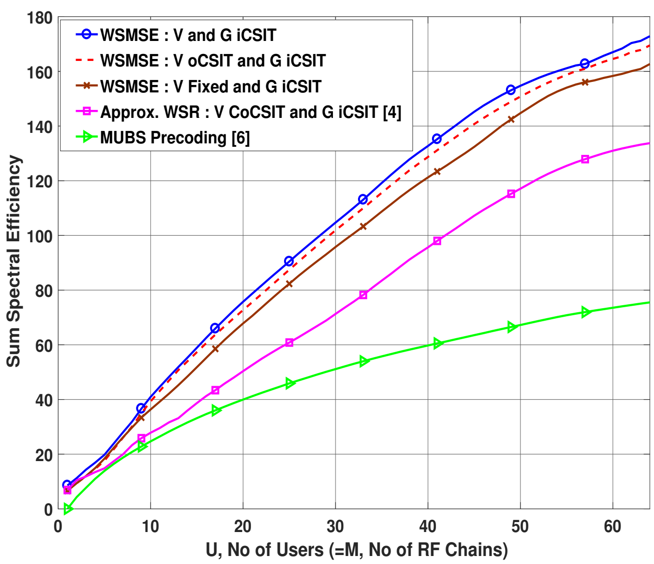

Fig. 4. Sum Spectral Efficiency vs No of Users, U = M, M is no of RF Chains. $N_t = 64$, $SNR = 20\,dB$ and $L = 6$ paths.

beamformer only at the slow fading rate, hence with outdated instantaneous CSIT, is still close to optimal and is much better than the existing suboptimal algorithms which suboptimally exploit covariance CSIT for the analog beamformer.

Appendix A
Derivation of (15)

Consider the terms involving V in the WSMSE.

$$\sum_{k=1}^{K} \left[u_k\, w_k\, e_k(f_k, V, g) \,+\, \lambda_{b_k} \, |V^{b_k}\, g_k|^2 \right] \tag{18}$$

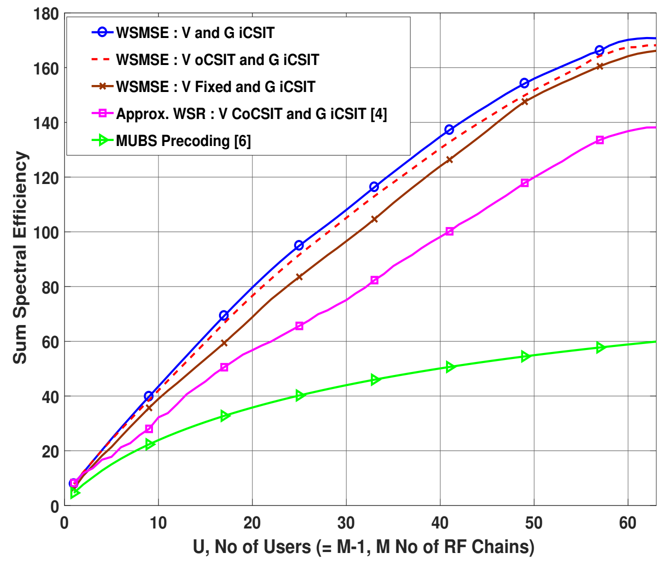

Fig. 5. Sum Spectral Efficiency vs No of Users, U = M-1, M is no of RF Chains. $N_t = 64$, $SNR = 20\,dB$ and $L = 6$ paths.

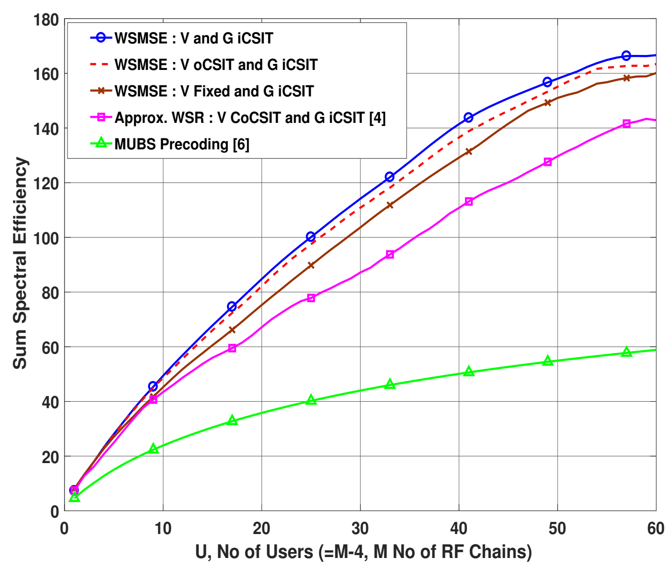

Fig. 6. Sum Spectral Efficiency vs No of Users, U = M-4, M is no of RF Chains. $N_t = 64$, $SNR = 20\,dB$ and $L = 6$ paths.

where $e_k(f_k, V, g)$ is specified in (8). Now rewrite each term as a function of $V^{b_k}_{m,n}$. Consider e.g. the generic term

$$\begin{aligned} & f_k^H\, H_{k,b_i}\, V^{b_i}\, g_i = \textstyle\sum_{j,l} (f_k^H\, H_{k,b_i})_j\, V^{b_i}_{j,l}\, g_{i,l} \\ & = V^{b_i}_{m,n}\, g_{i,n} (f_k^H\, H_{k,b_i})_m \,+ C^{k,i}_{m,n} \\ & \text{where } C^{k,i}_{m,n} = \textstyle\sum_{(j,l)\neq(m,n)} (f_k^H\, H_{k,b_i})_j\, V^{b_i}_{j,l}\, g_{i,l}. \end{aligned} \tag{19}$$

where $(f_k^H\, H_{k,b_i})_j$ denotes entry j of the row vector $f_k^H\, H_{k,b_i}$. $g_{i,n}$ denotes the n^{th} term of the vector g_i. Substituting $V^{b_i}_{m,n} =$

$e^{j\theta^{b_i}_{m,n}}$ now yields

$$\begin{aligned} &\left|f_k^H\, H_{k,b_i}\, V^{b_i}\, g_i\right|^2 = e^{j\theta^{b_i}_{m,n}}\, g_{i,n}(f_k^H\, H_{k,b_i})_m\, C^{k,i\,*}_{m,n} \\ &+e^{-j\theta^{b_i}_{m,n}}\, g^*_{i,n}(f_k^H\, H_{k,b_i})^*_m\, C^{k,i}_{m,n} + \text{"terms"} \end{aligned} \tag{20}$$

where "terms" denotes terms that do not depend on $\theta^{b_i}_{m,n}$. Expanding the MSE $e_k(f_k, V, g)$ gives,

$$\begin{aligned} &e_k(f_k, V, g) = -e^{j\theta^{b_k}_{m,n}}\, g_{k,n}(f_k^H\, H_{k,b_k})_m \\ &-e^{-j\theta^{b_k}_{m,n}}\, g^*_{k,n}(f_k^H\, H_{k,b_k})^*_m \\ &+\textstyle\sum_{i=1}^K [e^{j\theta^{b_i}_{m,n}}\, g_{i,n}(f_k^H\, H_{k,b_i})_m\, C^{k,i\,*}_{m,n} \\ &+e^{-j\theta^{b_i}_{m,n}}\, g^*_{i,n}(f_k^H\, H_{k,b_i})^*_m\, C^{k,i}_{m,n}] + \text{"terms"} \end{aligned} \tag{21}$$

where "terms" denote terms that do not depend on $\theta^{b_k}_{m,n}$. Let us define the following quantities

$$\begin{aligned} \alpha^k_{m,n} &= g_{k,n}(f_k^H\, H_{k,b_k})_m \\ \beta^{k,i}_{m,n} &= g_{i,n}(f_k^H\, H_{k,b_i})_m\, C^{k,i\,*}_{m,n} \end{aligned} \tag{22}$$

then we can rewrite (21) as

$$\begin{aligned} &e_k(f_k, V, g) = -e^{j\theta^{b_k}_{m,n}}\, \alpha^k_{m,n} - e^{-j\theta^{b_k}_{m,n}}\, \alpha^{k*}_{m,n} \\ &+\textstyle\sum_{i=1}^K [e^{j\theta^{b_i}_{m,n}}\, \beta^{k,i}_{m,n} + e^{-j\theta^{b_i}_{m,n}}\, \beta^{k,i*}_{m,n}] + \text{"terms"} \\ &= -e^{j\theta^{b_k}_{m,n}}\, \alpha^k_{m,n} - e^{-j\theta^{b_k}_{m,n}}\, \alpha^{k*}_{m,n} \\ &+e^{j\theta^{b_k}_{m,n}}\, \beta^k_{m,n} + e^{-j\theta^{b_k}_{m,n}}\, \beta^{k*}_{m,n} + \text{"terms"} \end{aligned} \tag{23}$$

where $\beta^k_{m,n} = \sum_{i:b_i=b_k} \beta^{k,i}_{m,n}$. Now consider the power constraint term

$$\begin{aligned} &||V^{b_k}\, g_k||^2 = \textstyle\sum_{l=1}^N \left[\sum_{i=1}^M V^{b_k}_{l,i}\, g_{k,i}\right] \left[\sum_{j=1}^M V^{b_k\,*}_{l,j}\, g^*_{k,j}\right] \\ &= V^{b_k}_{m,n}\, g_{k,n} \textstyle\sum_{i=1, i\neq n}^{M_{b_k}} V^{b_k\,*}_{m,i}\, g^*_{k,n} + \\ &V^{b_k\,*}_{m,n}\, g^*_{k,n} \textstyle\sum_{j=1, j\neq n}^{M_{b_k}} V^{b_k}_{m,j}\, g_{k,n} + \text{"terms"}. \end{aligned} \tag{24}$$

Defining $\xi^k_{m,n}$ as

$$\xi^k_{m,n} = \lambda_{b_k}\, g_{k,n} \sum_{i=1, i\neq n}^{M_{b_k}} V^{b_k\,*}_{m,i}\, g^*_{k,n}\,. \tag{25}$$

Then we can write

$$\lambda_{b_k}\, ||V^{b_k}\, g_k||^2 = \xi^k_{m,n}\, e^{j\theta^{b_k}_{m,n}} + \xi^{k\,*}_{m,n}\, e^{-j\theta^{b_k}_{m,n}} + \text{"terms"}. \tag{26}$$

Now summing the terms (23) and (26) over all k, we get

$$\begin{aligned} &\textstyle\sum_{k=1}^K \left[u_k\, w_k\, e_k(f_k, V, g) \,+\, \lambda_{b_k}\, \left||V^{b_k}\, g_k\right||^2\right] = \\ &\textstyle\sum_{k:b_k=c} [e^{j\theta^c_{m,n}}(u_k w_k(\beta^k_{m,n} - \alpha^k_{m,n}) + \xi^k_{m,n}) \\ &+e^{-j\theta^c_{m,n}}(u_k w_k(\beta^{k*}_{m,n} - \alpha^{k*}_{m,n}) + \xi^{k\,*}_{m,n})] + \text{"terms"} \end{aligned} \tag{27}$$

where "terms" denote terms that do not depend on $\theta^c_{m,n}$. Defining the terms $a^c_{m,n}$ as

$$a^c_{m,n} = \sum_{k:b_k=c} [u_k w_k(\beta^k_{m,n} - \alpha^k_{m,n}) + \xi^k_{m,n}] \tag{28}$$

we can rewrite the term of interest in the cost function (27) as

$$e^{j\theta^c_{m,n}}\, a^c_{m,n} + e^{-j\theta^c_{m,n}}\, a^{c*}_{m,n} = 2\Re\{e^{j\theta^c_{m,n}}\, a^c_{m,n}\} \tag{29}$$

where $a^c_{m,n} = |a^c_{m,n}|\, e^{j\angle a^c_{m,n}}$. To minimize (29), $e^{j\theta^c_{m,n}}\, a^c_{m,n}$ has to made real and negative, hence

$$\theta^c_{m,n} = \pi - \angle a^c_{m,n}\,. \tag{30}$$

This completes the derivation for θ.

Acknowledgments

EURECOM's research is partially supported by its industrial members: ORANGE, BMW, ST Microelectronics, Symantec, SAP, Monaco Telecom, iABG, and by the projects DUPLEX (French ANR) and MASS-START (French FUI).

References

[1] X. Zhang, A. Molisch, and S. Kung, "Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection," *IEEE Transactions on Signal Processing*, vol. 53, no. 11, pp. 4091-4103, 2005.

[2] A. Adhikary, J. Nam, J.-Y. Ahn, and G. Caire, "Joint spatial division and multiplexing: the large-scale array regime," *IEEE Trans. on Inform. Theory*, vol. 59, no. 10, pp. 6441-6463, 2013.

[3] A. Alkhateeb, O. El Ayach, G. Leus, and R. Heath, "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831-846, Oct. 2014.

[4] Sungwoo park, Jeonghun Park, Ali Yazdan, Robert W. Heath, "Exploiting Spatial Channel Covariance for Hybrid Precoding in Massive MIMO Systems", *IEEE Transactions on Signal Processing*, vol.65, no.14, July 15, 2017.

[5] F. Sohrabi and W.Yu, "Hybrid digital and analog beamforming design for large scale antenna arrays", *IEEE J.Sel.Topics Signal Process.* vol. 10, no.3, pp.501–513, Apr. 2016

[6] L.Liang, Y.Dai, W.Xu, and X.Dong, "How to approach zero-forcing under RF chain limitations in large mmWave multiuser systems ?" in *Proc. IEEE/CIC Int. Conf. Commun. China*, Oct.2014, pp.518–522.

[7] R.Mendez-Rial, Cristian Rusu, N.Gonzalez-Prelcic, Ahmed Alkhateeb and Robert W. Heath, "Hybrid MIMO Architectures for Millimeter Wave Communications: Phase Shifters or Switches ?", *IEEE Access*, vol.4, pp. 247–267, Jan. 2016.

[8] Q.Shi, M. Razaviyayn, Z.-Q. Luo and Chen He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," in *IEEE Transactions on Signal Processing*, vol.59, no.9, Sept.2011.

[9] S.S. Christensen, R. Agarwal, E. de Carvalho, and J. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Trans. on Wireless Comm's*, Dec. 2008.

[10] S. Wagner and D.T.M. Slock, "Weighted Sum Rate Maximization of Correlated MISO Broadcast Channels under Linear Precoding : a Large System Analysis," in *IEEE Int'l Workshop on Sig. Proc. Advances in Wireless Comm's (SPAWC)*, San Fransisco, CA, USA, 2011.

[11] F. Negro, I. Ghauri, Dirk T.M. Slock, "Sum Rate Maximization in the Noisy MIMO Interfering Broadcast Channel with Partial CSIT via the Expected Weighted MSE", in *Proc. Int'l Symposium on Wireless Communication Systems, ISWCS*, pp. 576-580, 2012.