Utility-based Opportunistic Scheduling under Multi-Connectivity with Limited Backhaul Capacity

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***Abstract*—Multi-connectivity is a 5G key enabler to fulfill multi-service quality-of-service (QoS) requirements. This work examines an opportunistic resource allocation problem under multi-connectivity and limited backhaul capacity in evolved LTE using two utility functions: proportional fairness (PF) and utility proportional fairness (UPF). We then propose an efficient algorithm to deal with the formulated NP-hard problem. Such algorithm guarantees to produce a solution with an explicit bound on the optimal solution. Finally, simulation results justify that UPF utility function with opportunistic scheduling in multi-connectivity can better satisfy QoS requirements.**

***Index Terms*—5G, Multi-connectivity, Backhaul, QoS.**

I. Introduction

To flexibly utilize available radio access technologies (RATs) and spectrum bands, simultaneous multiple connections per end-user perspective can be exploited. Such concept is termed as *multi-connectivity* [1] and is highlighted by 3GPP as one key enabler toward 5G [2]. In practice, the multi-connectivity deployment can mix-and-match with several 5G new radio techniques, e.g., millimeter wave [3], to exploit its benefit. The authors of [4] study the improvement of cell-edge user throughput as well as mobility robustness. In [5], the operating expenditure is minimized via jointly considering the user association and power allocation under dual connectivity. Our prior work in [6] stresses a better quality of service (QoS) satisfaction levering multiple connections. Also, it is necessary to consider the non-ideal backhaul condition [7], e.g., limited backhaul capacity, to make use of multiple connections.

Nevertheless, current studies of multi-connectivity mostly focus on exploiting its versatility, while less effort is made to allow the scheduling of proper transmission resources over several connections for each user, termed as *opportunistic scheduling* in [8]. Such opportunistic scheduling approach aims to take the channel quality information into account and can provide several advantages [9], like improving capacity, QoS satisfaction, and power efficiency. An example shown in [10] exploits channel variability to allow for better QoS awareness. In summary, the opportunistic scheduling approach is complementary to the multi-connectivity scheme and can further unleash its potential.

To this end, this work focuses on the *utility-based opportunistic scheduling* in a *multi-connectivity* deployment with *capacity-limited* backhaul network. Also, we propose an efficient algorithm for the formulated NP-hard optimization problem maximizing the network utility function. To the best of our knowledge, this is the first work exploring the opportunistic scheduling in multi-connectivity and providing an algorithm with an explicit bound on the optimal solution.

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II. System model

We consider an area served by a set of base stations (BSs) $\mathcal{B} = \{b_1, \cdots, b_{|\mathcal{B}|}\}$ connected to the core network (CN) via the backhaul network to serve a set of user equipment (UEs) $\mathcal{U} = \{u_1, \cdots, u_{|\mathcal{U}|}\}$ distributed in this area.

A. Air-interface model

A.1-Carrier frequency: Multi-connectivity deployments can be divided into: (a) Intra-frequency and (b) Inter-frequency [3]. The inter-frequency deployment stands for the case where a UE is connected through multiple carrier frequencies, either in a single or multi RATs. The intra-frequency one refers to transmissions over the same frequency from a single RAT, where the coordinated multipoint (CoMP) processing is required to mitigate interference. This work studies the inter-frequency single LTE RAT deployment and denotes the carrier frequency as f_j for the j-th BS. It is noted that the current setup can be extended also to scenarios where 5G NR small cells are collocated with an LTE macro cell in non-standalone operation mode [11].

A.2-Physical data rate: The physical data rate that the i-th UE can get from the j-th BS using the k-th sub-channel[1] in bps is $R_{j,k,i} = W_{j,k} \log_2 (1 + \text{SINR}_{j,k,i})$, where $W_{j,k}$ is the bandwidth of the k-th sub-channel at the j-th BS in Hz and

$$\text{SINR}_{j,k,i} = \frac{\text{RSRP}_{j,k,i}}{\sum\limits_{b_m \neq b_j,\ f_m = f_j} \text{RSRP}_{m,k,i} + W_{j,k} N_0} \quad (1)$$

is the signal-to-interference-plus-noise-ratio (SINR) of the received signal from the j-th BS to the i-th UE of the k-th sub-channel. The reference signal received power (RSRP) $\text{RSRP}_{j,k,i} = L_{j,i} s_{j,k,i} G_{j,i} P_{j,k,i}$ includes (a) large-scale fading as $L_{j,i}$ integrating pathloss and shadowing effects from the j-th BS to the i-th UE, (b) small-scale fading as $s_{j,k,i}$ comprising the multi-path effects from the j-th BS to the i-th UE over the k-th sub-channel, (c) combined antenna gain of the j-th BS and the i-th UE as $G_{j,i}$, and (d) transmission power from the j-th BS to the i-th UE of the k-th sub-channel as $P_{j,k,i}$. Note N_0 is the thermal noise density, and thus the product $W_{j,k} N_0$ shows the aggregated noise power.

B. Connection and Traffic model

B.1-Multi-connectivity: To identify the feasible connections between a single UE and mutliple BSs, the average SINR over all sub-channels is used as the decision criterion.

[1] The term sub-channel stands for the minimum radio resource that spans over time and frequency domains, a.k.a. physical resource block (PRB). It is noted that a sub-channel set $\mathcal{K}$ can be allocated by each BS.

Specifically, when the average SINR from the j-th BS to the i-th UE $\overline{\text{SINR}}_{j,i} = \frac{1}{|\mathcal{K}|}\sum_{k\in\mathcal{K}} \text{SINR}_{j,k,i}$ is above a threshold, $\overline{\text{SINR}}_{\text{th}}$, then the connection in between is feasible. Thus, a set $\mathcal{E} \triangleq \{(b_j, u_i) : \overline{\text{SINR}}_{j,i} > \overline{\text{SINR}}_{\text{th}}, \forall\, b_j \in \mathcal{B}, \forall\, u_i \in \mathcal{U}\}$ is formed to represent all feasible connections.

B.2-Backhaul network: For the connected backhaul link from the CN to the j-th BS, we denote its capacity as C_j in bps. Further, we consider a homogeneous backhaul capacity for different BSs in this work (i.e., $C_j = C, \forall j$) and leave the heterogeneous case (i.e., $C_i \neq C_j, \forall i \neq j$) for future work.

B.3-Requested traffic rate: The requested traffic rate $\hat{R}_i$ of the i-th UE is defined as the sum rate required by a number of user applications running on the UE side in bps. Such requested rate can be provided through multiple BSs and multiple sub-channels to fulfill multi-service QoS requirements.

III. Problem setup

A. Network utility function

We define $x_{j,k,i} \in \{0,1\}$ as the binary variable representing the allocation or not of the k-th sub-channel to the i-th UE from the j-th BS. Hence, $y_{j,k,i} = x_{j,k,i} \cdot R_{j,k,i}$ is the corresponding physical data rate.

First of all, we introduce two applied utility functions [12]; **a) Proportional Fairness (PF):** The utility function is defined as $\Phi(y) = \log(y)$ that adopts "proportional fair" allocation; **b) Utility Proportional Fairness (UPF):** Such utility function aims to achieve the UE's requested traffic rate as introduced in Sec.II-B.3 applying the family of the natural logarithm of the sigmoid utility function $\Phi(y) = \log\left(1/(1+e^{-\gamma(y-\beta)})\right)$, $\gamma > 0$, $y \geq 0$, where $\beta \equiv \hat{R}_i$. Then, we form two vectors as $\mathbf{x}_i = \left[x_{1,1,i},\ldots,x_{1,|\mathcal{K}|,i}, x_{2,1,i},\ldots,x_{2,|\mathcal{K}|,i},\ldots,x_{|\mathcal{B}|,1,i},\ldots,x_{|\mathcal{B}|,|\mathcal{K}|,i}\right]^{\text{T}}$ is the vector form of binary allocation variables and $\mathbf{R}_i = \left[R_{1,1,i},\ldots,R_{1,|\mathcal{K}|,i}, R_{2,1,i},\ldots,R_{2,|\mathcal{K}|,i},\ldots,R_{|\mathcal{B}|,1,i},\ldots,R_{|\mathcal{B}|,|\mathcal{K}|,i}\right]^{\text{T}}$ represents the vectorized physical data rate[2]. Finally, the network utility function of the i-th UE can be expressed as $U(\mathbf{x}_i) \triangleq \Phi\left(\mathbf{R}_i^{\text{T}} \cdot \mathbf{x}_i\right)$, where $\mathbf{R}_i^{\text{T}} \cdot \mathbf{x}_i$ stands for a user's aggregated data rate over its connected BSs and allocated sub-channels and $\Phi(\cdot)$ can be any of the utility functions as presented above.

B. Problem formulation

Optimization Problem: The problem falls into the category of the network utility maximization for resource allocation and is given as follows:

$$\max_{\mathbf{x}_i \in \{0,1\}^{|\mathcal{B}|\times|\mathcal{K}|}} \sum_{u_i\in\mathcal{U}} U(\mathbf{x}_i) \quad (2)$$

$$\text{s.t.} \quad \textbf{C1:} \sum_{u_i\in\mathcal{U}}\sum_{k\in\mathcal{K}} x_{j,k,i} \leq B_j, \forall\, b_j \in \mathcal{B}, \quad (3)$$

$$\textbf{C2:} \sum_{b_j\in\mathcal{B}}\sum_{k\in\mathcal{K}} x_{j,k,i} \leq M_i, \forall\, u_i \in \mathcal{U}, \quad (4)$$

$$\textbf{C3:} \sum_{u_i\in\mathcal{U}} x_{j,k,i} \leq 1, \forall\, b_j \in \mathcal{B}, k \in \mathcal{K}, \quad (5)$$

$$\textbf{C4:} \sum_{u_i\in\mathcal{U}}\sum_{k\in\mathcal{K}} x_{j,k,i} R_{j,k,i} \leq C_j, \forall\, b_j \in \mathcal{B}. \quad (6)$$

[2] The $(\cdot)^{\text{T}}$ denotes the transpose operator.

Algorithm 1: Opportunistic scheduling algorithm in multi-connectivity with complexity $\mathcal{O}\left(\left(|\mathcal{B}|\cdot|\mathcal{K}|\right)^2 \cdot |\mathcal{U}|\right)$

Input : $R_{j,k,i},\ \hat{R}_i,\ B_j,\ M_i,\ \forall\, b_j \in \mathcal{B},\ k \in \mathcal{K},\ u_i \in \mathcal{U}$
Output: $\mathcal{S}^\star$

- $\mathcal{S}_0 \leftarrow \emptyset$; $t \leftarrow 0$; /* Parameter initialization */
- $\mathcal{I} \leftarrow \mathcal{I}_1 \bigcap \mathcal{I}_2 \bigcap \mathcal{I}_3$; /* Get intersection of three constraints */
- **while** $t < \sum_{b_j\in\mathcal{B}} B_j$ **do**
  - $t \leftarrow t+1$;
  - $v \leftarrow \arg\max_{q\in\mathcal{I}\setminus\mathcal{S}_{t-1}} f(\mathcal{S}_{t-1}\cup\{q\})$; /* v gets the largest gain */
  - **if** $v \neq \emptyset$ **then**
    - $\mathcal{S}_t \leftarrow \mathcal{S}_{t-1} \bigcup v$; /* Include v in current solution $\mathcal{S}_t$ */
  - **else**
    - **break**;
- $\mathcal{S}^\star \leftarrow \mathcal{S}_t$; /* Output final solution $\mathcal{S}^\star$ */

Objective function: The objective is to allocate the radio resources $x_{j,k,i} \in \{0,1\}$ for each UE $u_i \in \mathcal{U}$ to maximize the sum of utility functions over all UEs. Both PF/UPF utility functions described in Sec.III-A can be applied. Such objective function considers the channel variation across sub-channels (hence the term *opportunistic*) and is different from the non-opportunistic scheme, where $\text{SINR}_{j,k,i} = \overline{\text{SINR}}_{j,i},\ \forall\, k \in \mathcal{K}$.

Constraints: There are four major constraints in the formulated utility maximization problem. **C1:** It ensures that the number of allocated sub-channels of BS b_j to all UEs will not exceed the total number of available sub-channels at b_j as B_j. **C2:** It assures the total number of allocated sub-channels to UE u_i among all connected BSs will not exceed the maximum allocatable sub-channels at u_i as M_i, which depends on the user capability defined as the UE-category in 3GPP TS36.306. **C3:** It ensures that each single sub-channel will be assigned up to one UE simultaneously implying sub-channel exclusivity. **C4:** It guarantees that the aggregated rate at each BS b_j will not exceed the provisioned backhaul capacity C_j.

C. Proposed algorithm

The proposed optimization problem is an NP-hard combinatorial problem. To the best of our knowledge, there is not such an algorithm to provide guarantee on the solution given by a bound due to the non-matroid nature of constraint **C4**, i.e., affine transformation with weights $R_{j,k,i}$ not equal to one [13]. In consequence, to provide an explicit bound, we initially exclude the constraint **C4** of Eq. (6) from the optimization process and we apply it afterwards as discussed below.

Lemma 1. *The formulated optimization problem is with submodular and monotone objective function (Eq.* (2)*) and three matroid constraints (Eq.* (3)*, Eq.* (4)*, and Eq.* (5)*).*

Proof. The proof is given in Appendix A. □

Hence, we propose a greedy algorithm in Alg. 1 with $f(\cdot)$ as the objective function and $\mathcal{I}_1$/$\mathcal{I}_2$/$\mathcal{I}_3$ as constraints detailed in Eq. (11) of Appendix A. Such algorithm can solve the problem with polynomial time complexity $\mathcal{O}\left(\left(|\mathcal{B}|\cdot|\mathcal{K}|\right)^2 \cdot |\mathcal{U}|\right)$ with an explicit bound on the optimal solution.

TABLE I: Simulation parameters

Parameter	Value
LTE mode	FDD, SISO, Band 1 (3×20 MHz bandwidth)
Total PRBs of each BS (B_j)	100 (20 MHz bandwidth)
Maximum allocated PRBs of UE (M_i)	100
Number of BSs ($\lvert\mathcal{B}\rvert$)	3
UE traffic model	Full buffer
SINR threshold (SINR_{th})	-12 dB
BS transmission power ($P_{j,k,i}$)	46 dBm
Thermal noise density (N_0)	-174 dBm/Hz
Gamma of UPF utility function (γ)	$10/\beta$
Requested traffic rate ($\hat{R}_i$) distribution	Uniformly distributed between 10 to 100 Mbps

Theorem 1. *Let OPT be the optimal solution of the formulated problem and $\mathcal{S}^\star$ be the output of Alg. 1. Then, it holds that $f(\mathcal{S}^\star) \geq \frac{1}{4}$ OPT, where $f(\cdot)$ is the objective function of Eq.* (2) *rewritten as a set function in Eq.* (10) *of Appendix A.*

Proof. Define $\mathcal{I} = \bigcap_{l=1}^{L} \mathcal{I}_l$, where $\mathcal{I}_l$ is matroid constraints. Even $\mathcal{I}$ is generally not a matroid, the output of greedy algorithm of Alg. 1 is guaranteed with $f(\mathcal{S}^\star) \geq \frac{1}{L+1}$ OPT shown in [13]. As $L = 3$, i.e., three matroid constraints, in the formulated problem, we prove Theorem 1. □

Subsequently, to deal with the backhaul capacity limitation, i.e., C_j, introduced in the pre-excluded constraint **C4**, we further scale the allocated data rate of the i-th UE as $R_i \triangleq \sum_{b_j\in\mathcal{B}} \sum_{k\in\mathcal{K}} (x_{j,k,i})^\star R_{j,k,i} \cdot \min\left(\frac{C_j}{\sum_{u_i\in\mathcal{U}}\sum_{k\in\mathcal{K}} (x_{j,k,i})^\star R_{j,k,i}}, 1\right)$, where $(x_{j,k,i})^\star$ are the outcomes of the proposed algorithm. In reality, such scaling can be applied at the level of BS or gateway. For instance, each BS can extend legacy rate adaptation approach to be aware of the backhaul condition, while the gateway can leverage the flow shaping approach to indirectly restrict the resource allocation. Last but not least, the continuous relaxation technique is not applied here, since it does not guarantee how far the solution is from the optimal one and violates discrete resource block assignment used in LTE system.

IV. Simulation Results

Two major scenarios are simulated[3] using self-developed MATLAB system-level simulator with the proposed algorithm: **Scenario A-Empty cell**: There is 1 BS that serves 0 UE, i.e, 0% traffic load, while the rest 2 BSs serve non-zero UEs. Such empty cell traffic load can characterize some real-world measurements. For instance, the *phantom cell* use case [14] aims to deploy small cells that serve fewer UEs than macro cell in order to boost user plane performance via exploiting underutilized radio resources. The $0/z/z$ notation represents such scenario where z is the number of non-zero served UEs per BS. **Scenario B-Loaded cell**: All cells serve non-zero UEs and we use $z/z/z$ notation to represent this scenario. The value of z is 2 in our simulations[4]. In both scenarios, backhaul capacity for all BSs is considered to be homogeneous. Then, our considered performance metrics include (i) user satisfaction ratio as the percentage of users that are satisfied with the allocated data rate, i.e., $P_i^{\text{sat}} \triangleq \text{Prob}\{R_i \geq \hat{R}_i\}$ and (ii) unsatisfied normalized error as the normalized Euclidean distance from the allocated rate to the requested traffic rate formulated as

$$E_i = \begin{cases} \left\| \frac{R_i - \hat{R}_i}{\hat{R}_i} \right\|, & \text{if } R_i < \hat{R}_i, \\ 0 & , \text{otherwise.} \end{cases} \quad (7)$$

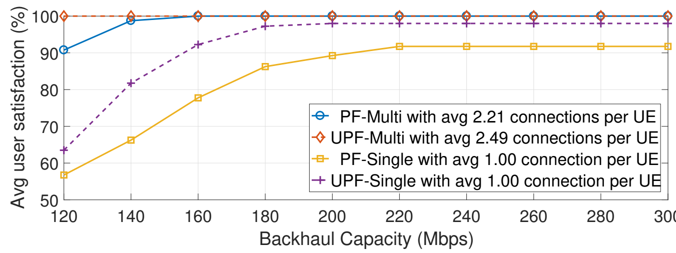

Fig. 1: User satisfaction ratio versus backhaul capacity $(0/z/z)$

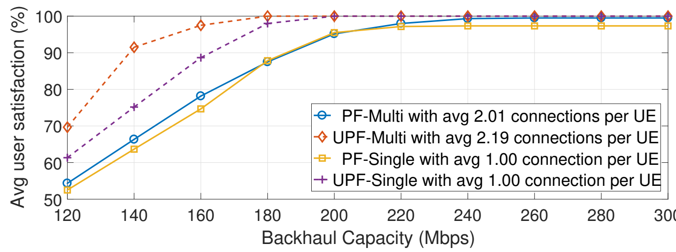

Fig. 2: User satisfaction ratio versus backhaul capacity $(z/z/z)$

In Fig. 1, empty cell scenario results are shown with several combinations of PF/UPF with single-/multi-connectivity. We can see that using UPF utility function can boost the satisfaction ratio even with single-connectivity; however, some QoS requirements can never be fulfilled even we continuously increase the backhaul capacity. It is due to the lack of utilizing unallocated resources at the empty cell, and thus multi-connectivity shows its advantage. Further, using UPF with multi-connectivity can better fulfill QoS requirements even with fewer backhaul capacity. Then, the results of loaded cell scenario are shown in Fig. 2. Using multiple connections is not enough to fulfill QoS requirements if only PF is adopted. Such observation is due to there are no unallocated resources compared to the empty cell case; hence, we need to efficiently reshuffle allocated resources via using UPF.

Finally, Fig. 3 shows the opportunistic scheduling benefit in loaded cell scenario with UPF and multi-connectivity compared to the non-opportunistic one. Note the non-opportunistic scheduling can optimally rely on the average SINR over all sub-channels, i.e., $\overline{\text{SINR}}_{j,i}$; however, its worst case is based on the minimum SINR among all sub-channels, i.e., $\min_{k\in\mathcal{K}}(\text{SINR}_{j,k,i})$. In comparison, the opportunistic scheduling can better utilize available sub-channels to provide a smaller unsatisfied normalized error compared to the best or the worst case of the non-opportunistic scheduling.

V. Conclusion and Future Work

In this work, we formulate the utility-based opportunistic resource allocation problem under multi-connectivity and limited backhaul capacity. The proposed algorithm can efficiently solve the problem with an explicit bound on the optimal solution. The simulation results justify the benefit of UPF utility function with opportunistic scheduling in terms of QoS satisfaction ratio and unsatisfied normalized error. As a future

[3] Simulation parameters applied to UEs, BSs and network planning are mostly taken from 3GPP (TR36.814, TR36.942, TR25.942) and NGMN documents, and some important parameters are listed in TABLE I. [4] Without loss of generality, we focus on the impact of zero and non-zero cell users and any variation of z can cover this case study.

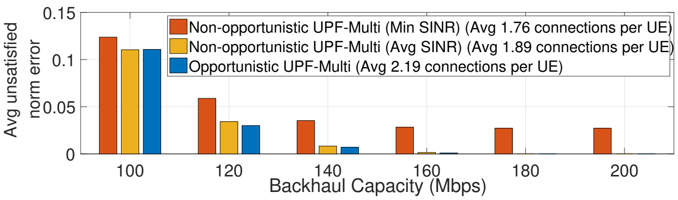

Fig. 3: Unsatisfied error versus backhaul capacity $(z/z/z)$

direction, the efforts will be given on the implementation applicability considering different RAN deployment flavors.

Appendix A
Proof of Lemma 1

We define the function $h(\mathbf{x}) : \{0,1\}^{|\mathcal{N}|} \to \mathbb{R}$ expressed as

$$h(\mathbf{x}) = g\left(\mathbf{a}^{\mathrm{T}} \cdot \mathbf{x}\right) = g\left(\sum_{n=1}^{|\mathcal{N}|} a_n x_n\right), \tag{8}$$

where $g(\cdot)$ is monotone and concave, $\mathbf{a} = \left[a_1, \cdots, a_{|\mathcal{N}|}\right]^{\mathrm{T}}$ and $a_n \in \mathbb{R}^+$. The above function is equivalent to the set function $h(\mathcal{W}) : 2^{\mathcal{N}} \to \mathbb{R}$, where $h(\mathcal{W}) = g\left(\sum_{n\in\mathcal{W}} a_n\right)$ and $\mathcal{W} = \{n \in \mathcal{N} : x_n = 1\}$.

Definition 1. *A set function is characterized as monotone iff* $h(\mathcal{Q}) \geq h(\mathcal{P})$ *for every* $\mathcal{P} \subseteq \mathcal{Q} \subseteq \mathcal{N}$.

The following holds with inequality for every $\mathcal{P} \subseteq \mathcal{Q} \subseteq \mathcal{N}$:

$$\sum_{n\in\mathcal{Q}} a_n \geq \sum_{n\in\mathcal{P}} a_n \Leftrightarrow g\left(\sum_{n\in\mathcal{Q}} a_n\right) \geq g\left(\sum_{n\in\mathcal{P}} a_n\right), \tag{9}$$

since g is monotone proving the monotonicity of h.

Proposition 1. *If* $a_n \in \mathbb{R}^+$ *and* $g(\cdot)$ *is concave, then the function* $h(\mathcal{W}) = g\left(\sum_{n\in\mathcal{W}} a_n\right)$ *is submodular.*

Proof. The proof is based on part (a) of Lemma 2.6.2. in [15]. □

Applying Proposition 1, we prove the submodularity of $h(\cdot)$.

The objective function in Eq. (2) can be rewritten as the set function $f(\mathcal{S}) : 2^{\mathcal{V}} \to \mathbb{R}$ that is expressed as

$$f(\mathcal{S}) = \sum_{u_i\in\mathcal{U}} \Phi\left(\sum_{b_j\in\mathcal{B}} \sum_{k\in\mathcal{K}} R_{j,k,i} \mathbf{1}_{\mathcal{S}}\left(x_{j,k,i}\right)\right), \tag{10}$$

where $\mathcal{V} = \mathcal{B} \times \mathcal{K} \times \mathcal{U}$, $\mathbf{1}_{\mathcal{S}}(\cdot)$ is the indicator function on the set $\mathcal{S} = \{(j,k,i) \in \mathcal{V} : x_{j,k,i} = 1\}$.

Proposition 2. $\Phi(\cdot)$ *utility functions are monotone and strictly concave.*

Proof. The PF utility function is the logarithmic function and thus is monotone and strictly concave. While the UPF takes the natural logarithm of sigmoid function proven as monotone and strictly concave following Lemma III.1. in [12]. □

As each term in the summation of objective function $f(\cdot)$ lying on the family of $h(\cdot)$ in Eq. (8), where $\Phi(\cdot) \equiv g(\cdot)$ is monotone and concave, $\mathcal{N} \equiv \mathcal{B} \times \mathcal{K}$, $a_n \equiv R_{j,k,i}$ and $x_n \equiv x_{j,k,i}$. Thus, we conclude that the objective function $f(\cdot)$ is monotone and submodular as it is the summation of monotone and submodular functions.

Finally, we define the three constraints in Eq. (3)-(5) equivalently to the sets $\mathcal{I}_l,\ l \in \{1,2,3\}$ in Eq. (11) as follows:

$$\mathcal{I}_1 = \{\mathcal{S} \subseteq 2^{\mathcal{V}} : |\mathcal{S} \cap 2^{\{j,\mathcal{K},\mathcal{U}\}}| \leq B_j, \forall\ b_j \in \mathcal{B}\}, \tag{11a}$$

$$\mathcal{I}_2 = \{\mathcal{S} \subseteq 2^{\mathcal{V}} : |\mathcal{S} \cap 2^{\{\mathcal{B},\mathcal{K},i\}}| \leq M_i, \forall\ u_i \in \mathcal{U}\}, \tag{11b}$$

$$\mathcal{I}_3 = \{\mathcal{S} \subseteq 2^{\mathcal{V}} : |\mathcal{S} \cap 2^{\{j,k,\mathcal{U}\}}| \leq 1, \forall\ b_j \in \mathcal{B},\ k \in \mathcal{K}\}. \tag{11c}$$

To prove these constraints are matroid (see definition in [13]) with $\mathcal{V}$ as ground set, two properties shall be fulfilled:

(a) For all sets $\mathcal{P}$ and $\mathcal{Q}$ with $\mathcal{P} \subseteq \mathcal{Q} \subseteq \mathcal{V}$, if $\mathcal{Q} \in \mathcal{I}_l$, i.e., the sub-channel allocation defined by $\mathcal{Q}$ does not violate the constraint $\mathcal{I}_l$, then $\mathcal{P} \in \mathcal{I}_l$ is held, i.e., the constraint $\mathcal{I}_l$ is not violated by sub-channel allocation in $\mathcal{P}$, since $\mathcal{P} \subseteq \mathcal{Q}$. All three aforementioned constraints possess such property.

(b) For all sets $\mathcal{P}, \mathcal{Q} \in \mathcal{I}_l$ and $|\mathcal{Q}| > |\mathcal{P}|$, i.e., more sub-channels are allocated in total defined by $\mathcal{Q}$, $\exists\ e \in \mathcal{Q} \backslash \mathcal{P}$ such that $\mathcal{P} \cup \{e\} \in \mathcal{I}_l$. Since not all sub-channels are allocated defined by $\mathcal{P}$ (otherwise, $\mathcal{Q}$ will either violate the constraint $\mathcal{I}_l$, i.e., $\mathcal{Q} \notin \mathcal{I}_l$, or has no more allocated sub-channels as $\mathcal{P}$, i.e., $|\mathcal{Q}| \leq |\mathcal{P}|$), there is at least one sub-channel that can be further allocated and will not violate the constraint $\mathcal{I}_l$. Hence, such property is held for all three constraints.

Thus, we conclude that $\mathcal{I}_l,\ \forall\ l \in \{1,2,3\}$ are matroid.

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