Zero-forcing Electrical Filters for Direct Detection Optical Systems

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Abstract — **Intersymbol interference in direct detection optical systems can limit the channel spacing in frequency division multiplexing and prevent multilevel signaling. We investigate a zero-forcing electrical filter to cancel intersymbol interference and compare its performance with a matched filter and a rectangular response filter, for M-ary amplitude modulation.**

I. Model

In our model, the receiver front-end is composed of an optical filter, a photodetector and a low-pass electrical filter as shown in Figure 1. The photodiode is modelled as a square-law de-

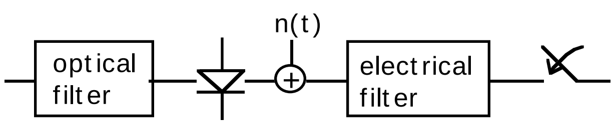

Figure 1: The receiver front-end includes an optical filter, a photodiode and a low-pass electrical filter followed by a baud rate sampler.

vice whose output is proportional to the magnitude square of the received signal envelope. The thermal noise $n(t)$ from the electronics is assumed to be the dominant noise and is modelled as additive white Gaussian noise.

II. Zero-forcing Filters

Without loss in generality, we design the filter for sampling time $t = 0$. The output of the electrical filter at this time is

$$\sum_i \sum_j a_i a_j \int p_{i,j}(\sigma)\, h(\sigma) d\sigma + N, \tag{1}$$

where $h(t)$ is the *time-reversed* impulse response of the electrical filter, $p_{i,j}(t) = \mathrm{Re}\{p(t-iT)p^*(t-jT)\}$, $p(t)$ is the complex envelope of the received optical pulse taking into account the transmit pulse and the channel response, $p^*(t)$ is the complex conjugate of $p(t)$, a_k (taken as real here) is the kth transmit amplitude, and N is the noise at the sampled output. We write the integral $\int p_{i,j}(t)h(t)dt$ as an inner product $\langle p_{i,j}, h\rangle$ where $\langle x, y\rangle \equiv \int x(t)y(t)dt$.

We define the *ISI space*, I, as the space spanned by $p_{i,j}(t)$'s without $p_{0,0}(t)$, the desired signal. The *signal space* is defined as the space spanned by I and $p_{0,0}(t)$. A filter $h(t)$ is a zero-forcing filter if the sampled output has no ISI, i.e. we want the output of $h(t)$ to depend only on $a_0 a_0$ at sampling time $t = 0$. A necessary and sufficient condition for $h(t)$ to be a zero-forcing filter is

$$\begin{aligned} &\langle p_{i,j}, h\rangle \neq 0, \text{ if } i = j = 0, \text{ and} \\ &\langle p_{i,j}, h\rangle = 0, \text{ otherwise.} \end{aligned} \tag{2}$$

For filters that satisfy (2), we are interested in the one that minimizes the noise variance when $\langle p_{0,0}, h\rangle$ is set equal to a constant. The time-reversed impulse response of this filter, if it exists, is proportional to the component of $p_{0,0}(t)$ orthogonal to I.

III. Fabry-Perot interferometer

We consider a Fabry-Perot filter as the optical demultiplexing filter [1]. The envelope of the impulse response is well approximated by $\frac{1}{\tau}e^{-t/\tau}$ for $t \geq 0$. The envelope of the transmit pulse is equal to 1 in $[0, \frac{\log_2 M}{R}]$ and 0 otherwise, where M is the number of signaling levels and R is the bit rate. The symbol set is equal to $\{\sqrt{\frac{i}{M-1}} \,|\, 0 \leq i \leq M-1\}$. We keep the bit rate fixed and plot the normalized eye opening for different τ', where $\tau' = \tau R$ is a parameter proportional to bandwidth efficiency in units of (b/s/Hz). The normalized eye opening is the vertical opening of an eye-diagram when the electrical filter has unit energy. We consider a filter matched to the post-detection electrical pulse $p_{0,0}(t)$ (single pulse transmitted), the minimum noise variance zero-forcing filter, and a filter with rectangular impulse response on $[0, \frac{\log_2 M}{R}]$. For the rectangular filter, there is no advantage to multilevel signaling, same as in [1]. On the other hand the zero-forcing filter performs much better and there is advantage to four level signaling for large values of τ'.

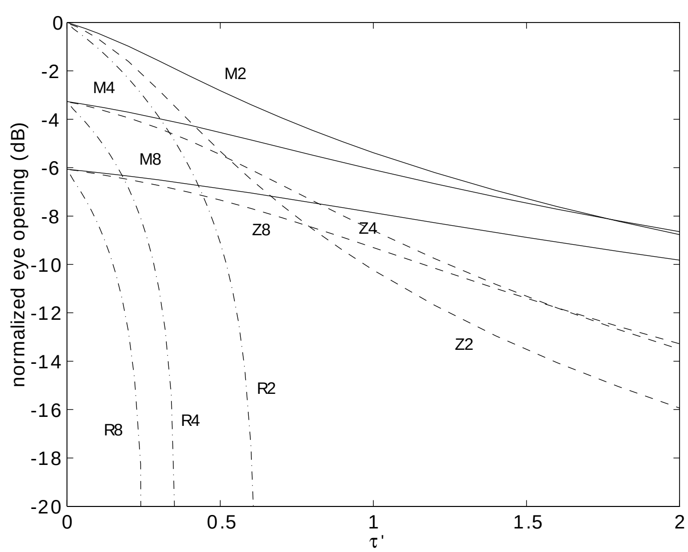

Figure 2: The optical filter is a Fabry-Perot interferometer. The curves are labelled as follows: (M) matched filter curve, (Z) zero-forcing filter, (R) rectangular filter, and (2,4,8) level signaling.

Acknowledgements

This work was supported by NSF grant 9206379-NCR.

References

[1] L. Cimini and G. Foschini. Can multilevel signaling improve the spectral efficiency of ask optical fdm systems? *IEEE Transactions on Communications*, 41(7):1084–1090, July 1993.