1

CHANNEL CONTROL AND MULTIPLE–ACCESS

R. Knopp, P.A. Humblet

Mobile Communications Dept.,
Institut Eurécom,
B.P. 193, Sophia Antipolis,
06904 FRANCE

ABSTRACT

This work considers fundamental performance limits for single and multiuser channels on time–varying (fading) channels. We consider systems which are burst–oriented along the lines of GSM and IS-54. We examine the benefits of exploiting channel feedback to vary transmit information rate/power in a single–user (or orthogonal multiuser) scenario. It is shown that significant increases in spectral efficiency can be expected over constant–rate systems. Even in the latter case, however, higher spectral efficiencies can be expected if high outage probabilities are tolerated. For the general multiuser case, channel feedback can be used to exploit *multiuser diversity*, which can be seen as a form of selection diversity. We show that the optimal multiple–access method in terms of spectral efficiency is channel–controlled TDMA/FDMA. We discuss some practical issues concerning channel–controlled multiple–access.

1 INTRODUCTION

In this work we examine the role of channel state feedback in mobile radio systems from the point of view of increasing spectral efficiency. We restrict our treatment here to a single–cell model, since there is significant practical insight to be gained without complicating the model to include inter–cell interference. Moreover, when considering a multi–cell environment, we must consider two possible scenarios. Firstly, we can consider the case when there is joint processing between receivers, which, in effect, reduces the problem to a single–cell situation with multiple independent receivers. This is treated in [1][2] for non–fading Gaussian channels. The other case of interest is when processing at the receivers is restricted to the signals of the users within the cell. This calls for the inter–cell interference to be modeled.

We are mostly interested in burst–oriented systems along the lines of GSM [3] or IS–54. Here, data is transmitted in bursts and processed respecting some time–constraint. We consider both fixed and variable–rate systems. Such systems were modeled and analysed by Ozarow *et al.* in [5]. We review the techniques from [5] necessary to demonstrate our results.

In mobile radio systems, the received signal strength is dependent on the physical locations of the receiver(s) and transmitter(s), and includes both short and long–term attenuation effects. These effects become time–varying when the receiver/transmitter are in motion, and result in signal fading. Long–term or slow–fading effects are attributed mostly to the path loss due to the separation between transmitters and receivers. Another long–term fading effect is shadowing [4]. The short–term or fast–fading effects are do to multipath propagation, which is arguably the most difficult aspect to handle in mobile radio systems.

Power control is necessary for combating these time–varying signal attenuations. By power control we mean the exploitation of channel knowledge at the transmission end in any form. We will show that such channel information can be used for variable–rate coding and dynamic channel–access.

2 SINGLE–USER BURST–ORIENTED COMMUNICATIONS

Consider the single–user information signal $x(t)$ transmitted over an arbitrary time–varying channel $h(t,\tau)$ and subject to additive white Gaussian noise, $z(t)$, with power spectral density $N_0/2$ upon reception. The received signal is given by

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t,\tau)d\tau + z(t). \tag{1.1}$$

This can also represent an orthogonal multiuser system. Following the approach in [5] we divide the information signal into bursts $x_n(t)$ which are time–limited to $[(n-1)(T+T_G)/2, (n+1)(T+T_G)/2]$, where $T_G > T_o - T$ is a guard–time between bursts during which $x_n(t) = 0$. It is inserted so that the received signal in a given burst does not contain information from previous bursts (i.e. T_G is chosen to be slightly larger than the maximum time–delay spread experienced in the system.) We also place the following average energy constraint on the information signal

$$\lim_{N\to\infty} \frac{1}{N(T+T_G)} \sum_{i=-N/2}^{N/2} \int_{(i-1)T}^{iT} \overline{x_i(t)^2}dt = S. \tag{1.2}$$

When processing is performed over a finite number, N, of such bursts and the receiver has complete knowledge of the channel, the maximum attainable information rate is given by

$$\begin{aligned} I_N &= \frac{1}{N(T+T_G)} \sum_{n=0}^{N-1} I(Y_n(t), H_n(t,\tau); X_n(t)) \\ &= \frac{1}{N(T+T_G)} \sum_{n=0}^{N-1} I(Y_n(t); X_n(t,\tau)|H_n(t)) + I(H_n(t); X_n(t)) \\ &= \frac{1}{N(T+T_G)} \sum_{n=0}^{N-1} I(Y_n(t); X_n(t)|H_n(t,\tau)) \end{aligned} \tag{1.3}$$

where $y_n(t)$ and $h_n(t,\tau)$ are the received signal and channel response for the n^{th} burst. Equation (1.3) follows from the assumption that the information process is statistically independent of the channel and that the received signal in a given burst is dependent only on the signal transmitted during that burst. Processing over several bursts is done to some extent in GSM and IS-54, where the data are interleaved over F bursts transmitted over F independent fading channels (via frequency–hopping in GSM and time–separation in IS-54). It is for this reason that we have included the subscript n in $h_n(t,\tau)$, since it may represent a different realization of a fading channel. Assuming an L–path fading channel, $h_n(t,\tau)$ is given by

$$h_n(t,\tau) = \sum_{l=0}^{L-1} c_{n,l}(t)\delta(t-\tau-d_{n,l}(t)), \tag{1.4}$$

where $c_{n,l}(t)$ and $d_{n,l}(t)$ are the gain and propagation delay for the l^{th} path in the n^{th} burst respectively. If the rate of channel variation is slow enough so that $h_n(t,\tau) \approx h_n(t-\tau)$, $t \in [(n-1)(T+T_G)/2, (n+1)(T+T_G/2)]$ (i.e. the $c_{n,l}(t)$ and $d_{n,l}(t)$ are constant during the burst), then we have that [5]

$$I_N \geq \frac{1}{N} \sum_{n=0}^{N-1} \int_{-W/2}^{W/2} \log\left(1 + \frac{S}{WN_0} S_n(f) \left|H_n(f)\right|^2\right) df, \tag{1.5}$$

with equality as $T \to \infty$. $H_n(f)$ is the Fourier transform of $h_n(t-\tau)$ and $S_n(f)$ is the power spectral density of the Gaussian information process during the n^{th} burst.

We restrict $|H_n(f)|^2$ to be an ergodic process for all f, which means that the slowly–varying statistics (i.e. path loss and shadowing) are made transparent to the receiver. This is made possible by multiplying the information signal by a slow power control signal inversely proportional to the path loss and shadowing prior to transmission. In other words, the *average* received signal power is constant for all time. This is quite different from perfect fast power–control which keeps the *instantaneous* received signal power constant. We also assume that $H_n(f)$ is identically distributed at all frequencies and that $\overline{H_n(f)} = 1$. Because of the ergodicity of $H_n(f)$, if we can process over infinitely many bursts the maximum achievable information rate is bounded below by

$$I_\infty = \overline{\int_{-W/2}^{W/2} \log\left(1 + \frac{S}{WN_0} S_n(f)|H_n(f)|^2\right) df}. \tag{1.6}$$

Considering first the case when $H_n(f)$ is not known to the transmitter, it follows straightforwardly from (1.6) and the fact that the statistics of $|H(f)|^2$ do not vary with f that $S_n(f) = 1$ maximizes I_∞. When $H_n(f)$ is known to the transmitter and processing over multiple bursts is not possible, I_∞ can be achieved by variable–rate coding. Over each burst, the transmitter would choose a rate R_n satisfying

$$R_n < \int_{-W/2}^{W/2} \log\left(1 + \frac{S}{WN_0} S_n(f)|H_n(f)|^2\right) df, \tag{1.7}$$

so that the average information rate would also be I_∞. If the transmitter wishes to optimize power allocation we may choose $S_n(f)$ to maximize I_∞ subject to the power constraint in (1.2), resulting in the capacity C_∞. Applying the Kuhn–Tucker conditions [6], the maximizing $S_n(f)$ is given by

$$S_n(f) = \left[B\frac{S}{WN_0} - \frac{1}{|H_n(f)|^2}\right] ind\left(|H_n(f)|^2 > \frac{WN_0}{SB}\right), \tag{1.8}$$

so that

$$C_\infty = \overline{W \log\left(B\frac{S}{WN_0}|H_n(f)|^2\right) ind\left(|H_n(f)|^2 > \frac{WN_0}{SB}\right)}, \tag{1.9}$$

where B is a constant chosen to satisfy the power constraint, and ind$(\cdot)$ is the indicator function. This is water–filling both in time(block) and frequency and suggests a time–varying OFDM scheme. In a given block, high–rate/high–power codes would be used on the strong carriers, and low–rate(or zero)/low–power codes on the weak carriers. Looking at a particular frequency, this is simply a fast power–control law which works in an opposite fashion to conventional power control, since more power is allocated when the channel is strong and less when it is weak.

In existing systems such as GSM or IS-54 which use a fixed information rate, the ergodicity of the channel cannot be exploited fully because of time–delay constraints. In GSM, for example, data is interleaved over 4 or 8 bursts which are transmitted on a different carrier (ideally) which, when combined with error–control coding, provides a certain amount of diversity. Nevertheless, the short decoding interval and small number of frequencies do not provide enough realizations of the fading channel to achieve information rates close to I_∞. As a result, a more useful benchmark for system performance limits is the information outage probability

$$P_{\text{out}}(R) = \Pr(W^{-1} I_N \leq R). \tag{1.10}$$

This indicates the probability that reliable communication of the N bursts is impossible at rate R. For frequency–flat or single–path systems, where the channel strength is a constant H for all f, we can write this as

$$P_{\text{out}}(R) = \Pr\left(H \leq \left(\frac{WN_0}{S}\right)\left(e^R - 1\right)\right), \tag{1.11}$$

by inverting the logarithm in (1.7). For systems employing variable–rate coding and possibly fast power control, it makes little sense to consider information outage probabilities. This is because the rate R_n will always be chosen such that reliable communication is possible. Practically speaking, it may be useless to code at an achievable rate during a deep fade, and communication could be stopped. The optimal power control law in (1.8) indicates this as well. This would be the only outage probability to consider but would not be a fair means for comparison. We can, however, compare the systems based on average throughput. For the variable–rate coding case, the average throughput is simply I_∞ or C_∞ depending whether or not fast power–control is performed. For the fixed–rate case, we define the average throughput as

$$T(R) = R\left(1 - P_{\text{out}}(R)\right), \tag{1.12}$$

which assumes that no information is conveyed when the achievable information rate falls below R. This is the case in GSM, for instance, where an error–detecting code is used to determine whether or not the burst has been received reliably. If not, the burst is rejected and speech information is obtained by extrapolating information from previous bursts. Under this assumption, the maximum throughput, $T_{\max}$ is attained by choosing the operating information rate $R_{\rm opt}$, which maximizes $T(R)$ for a given SNR.

In order to compare different systems, we choose a Rayleigh fading model for the path gains in (1.4). In this case the $|H(f)|^2$ are exponential with $p_H(\alpha) = e^{-\alpha}, \quad \alpha > 0$. This is, in some sense, a worst–case model for what is experienced in real systems. It is also convenient for analysis purposes. The corresponding mutual informations are given by

$$I_\infty = W e^{\frac{WN_0}{S}} \mathrm{Ei}\left(\frac{WN_0}{S}\right), \tag{1.13}$$

$$C_\infty = W \mathrm{Ei}\left(\frac{WN_0}{BS}\right), \tag{1.14}$$

and B is the solution to

$$Be^{-\frac{WN_0}{BS}} - \frac{WN_0}{S} \mathrm{Ei}\left(\frac{WN_0}{BS}\right) = 1, \tag{1.15}$$

where $\mathrm{Ei}(\cdot)$ is the first–order exponential integral. We note for large S/WN_0 that $B \approx 1$ and therefore that $I_\infty \approx C_\infty$. Thus, when ergodicity can be exploited, fast power–control, and transmitting only when channel conditions are favourable, have no advantage. We plot $W^{-1}I_\infty$ and $W^{-1}C_\infty$ in Fig. 1. We also show the spectral efficiency of a non–fading channel,

$$W^{-1}C_G = \log\left(1 + \frac{S}{WN_0}\right). \tag{1.16}$$

It is worthwhile noting that C_G is the capacity with perfect fast power–control (i.e. when the instantaneous received SNR is kept constant) and that it is not significantly higher than systems with a variable received SNR. Moreover, in Rayleigh fading, a perfect power–control law cannot satisfy the average–power constraint since the average transmit power is infinite. In practice, the transmit power is limited to some peak level, and when this is considered, it turns out that the capacity is significantly less than C_∞ [7].

When $N = 1$, the information outage probability for frequency–flat fading in (1.11) is given by

$$P_{\rm out}(R) = 1 - \exp\left(-\frac{WN_0}{S}\left(e^R - 1\right)\right). \tag{1.17}$$

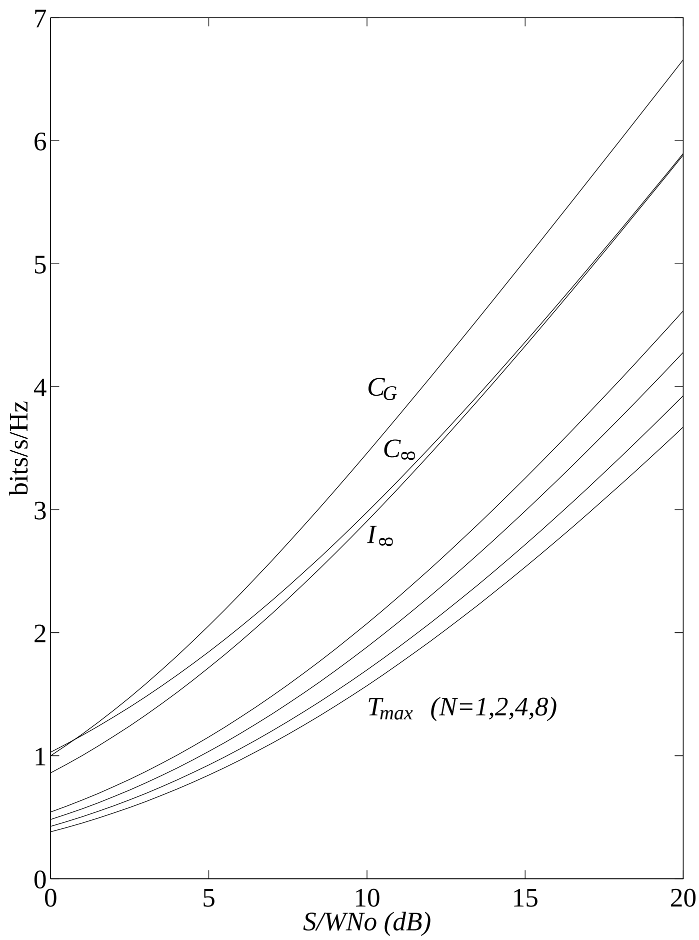

Figure 1: Information Capacity Comparison

It is easily shown that the optimal operating information rate is the solution to the following non–linear equation

$$R_{\rm opt} e^{R_{\rm opt}} = \frac{S}{WN_0}. \tag{1.18}$$

The maximum throughput, $T_{\max}$, when $R_{\rm opt}$ is chosen for each SNR, is also shown in Fig. 1. For the case when processing is performed over several bursts, the information outage probability can be calculated numerically. We have

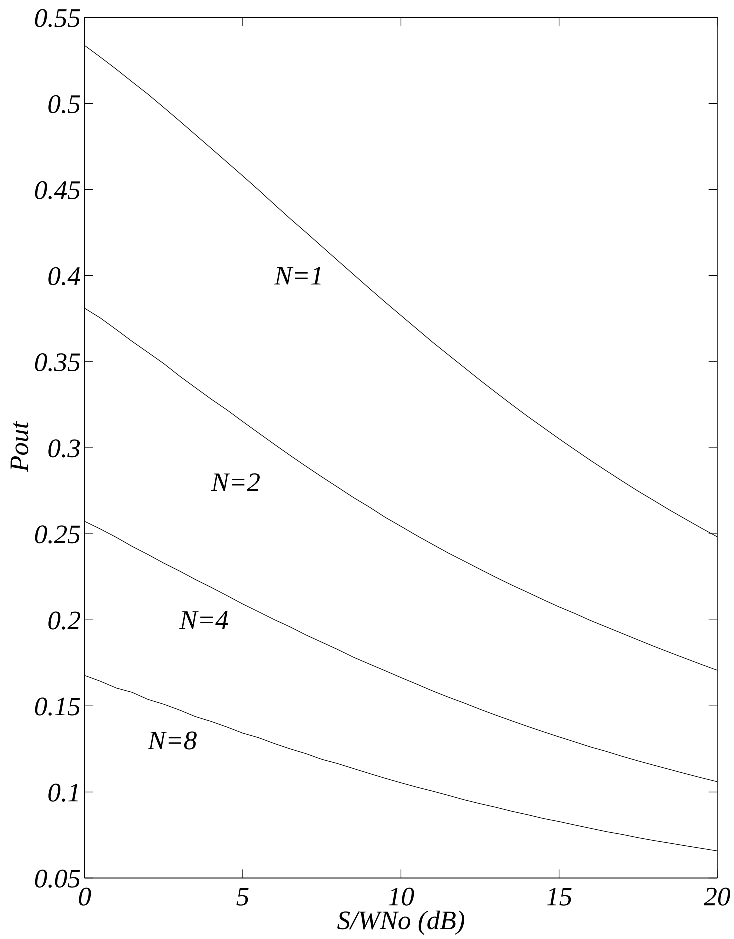

Figure 2: Optimal operating outage rates

performed the calculations for $N = 2, 4, 8$ which corresponds to ideal conditions in the IS–54, half–rate GSM, and full–rate GSM systems respectively. Although the spectral efficiency increases towards I_∞, it is clear that variable–rate coding with and without fast power–control, which achieve either I_∞ or C_∞, can have a significant advantage for practical SNR. In Fig. 2 we show the operating outage rates which achieve maximum throughput in constant–rate systems. Even here, we see that for maximum throughput, the channel is used effectively only when not in a deep fade since the outage probabilities are high (i.e. many bursts will be received in error).

We have not considered the information outage probability for frequency–selective channels. In [5] expressions for P_{out} on two–path Rayleigh fading channels are given. Although the analysis is more complicated than the single–path case, it is shown that at a given SNR P_{out} is significantly lower when multipath diversity can be exploited. This means that the average throughput also increases towards $W^{-1}I_{\infty}$.

3 MULTIUSER DIVERSITY

In this section we generalize the ideas of Section 2 for a single–cell multiuser system. Let us assume that we have K users sharing a fixed bandwidth KW, such that the bandwidth per user is W. Each user's signal, $x^{(i)}(t)$, is transmitted over a different channel with time–varying impulse response $h^{(i)}(t,\tau)$. The information signals are average–power limited as in (1.2). We have the following composite complex baseband signal at the basestation,

$$y(t) = \sum_{i=0}^{K-1} \int_{-T/2}^{T/2} x^{(i)}(\tau) h^{(i)}(t,\tau) d\tau + z(t). \tag{1.19}$$

We assume that the attenuations on different channels are independent processes. As in the single-user case, we consider the case when the channel attenuations are ergodic processes by assuming perfect slow power control. The achievable data rates in the n^{th} burst belong to the region inner–bounded by

$$\sum_{i\in\mathcal{U}} R_i^{(n)} < \int_{-KW/2}^{KW/2} \log\left(1 + \frac{2}{N_0}\sum_{i\in\mathcal{U}} S_{in}(f)\left|H_{in}(f)\right|^2\right) df, \tag{1.20}$$

where $S_{in}(f)$ and $H_{in}(f)$ are the information power spectrum and channel response for user i over the n^{th} burst respectively, and $\mathcal{U} \in \{0,1,\cdots,K-1\}$. Generalizing the results of the single–user case, the achievable rate region for systems where processing is carried out over an infinite number of bursts

$$\sum_{i\in\mathcal{U}} R_{i,\infty} < \overline{\int_{-KW/2}^{KW/2} \log\left(1 + \frac{2}{N_0}\sum_{i\in\mathcal{U}} S_{in}(f)\left|H_{in}(f)\right|^2\right) df}. \tag{1.21}$$

This region is that of SSMA (Spread-Spectrum Multiple Access) systems, since all the users transmit over the entire available bandwidth. It is shown in [8] that the achievable rate region for FDMA (or TDMA) is strictly enclosed by the region of an SSMA system if the channels are all statistically identical. Moreover, diversity techniques such as frequency–hopping or time–separation cannot enlarge the average achievable rate region. This implies that higher data rates are theoretically possible with SSMA than with systems like GSM or IS-54.

In the case of equal average received powers, the equal–rate point on the achievable rate region is achieved by the total sum–of–rates (i.e. the inequality in (1.21) corresponding to the case when $\mathcal{U} = \{0, 1, \cdots, K-1\}$). From this point we define the per user average achievable spectral efficiency as $W^{-1} I_\infty^K = (1/K) \sum_{i=0}^{K-1} R_{i,\infty}$. In [9], it is shown that the gain in $W^{-1} I_\infty^K$ for SSMA over FDMA/TDMA in Rayleigh fading tends to $\gamma / \ln 2 = .8327$ bits/s/Hz, where γ is Euler's constant, for a large number of users and high SNR. This means that K users transmitting over K independent fading channels can transmit more total information than a single user with K times as much power. Furthermore, as the number of users increases, $W^{-1} I_\infty^K$ increases towards $W^{-1} C_G$, the spectral efficiency of a non–fading Gaussian channel.

In order to operate on the boundary of the achievable rate region, we must employ a successive decoding strategy [10]. For each group of N bursts (N large) over which decoding is performed, we choose an order in which to decode the users' signals. Each $x^{(i)}(t)$ is decoded, treating the others as noise, regenerated and subtracted from $y(t)$. To operate at the equal–rate point, we must choose the ordering uniformly at random each time decoding is performed. This implies that the transmitters must adjust their code rates according to the ordering to be used by the decoder.

Unlike single–user channels, channel state feedback may be used to allocate users in time and frequency dynamically depending on the states of *all* the channels. This assumes, of course, a closed–loop system where the basestation controls all the users. We illustrate this with a simple example. Consider a two-user system in a flat fading environment using modified TDMA. As in ordinary TDMA only one users transmits at any given time, but now we use channel knowledge to allocate the channel to the user who has the stronger instantaneous received signal. In doing so, we have effectively made this equivalent to a single-user system with selection diversity, except that the diversity is not due to multiple antennas but to multiple users. In the remainder of this section we investigate the effect of channel knowledge at the transmission end more closely, and show that a generalization of this simple idea is optimal under certain conditions.

The capacity region for burst–oriented systems is achieved by a multiuser water–filling solution similar to that described in [11] for time–invariant Gaussian channels. We do not discuss this at any length here, except that, as in the single–user case, it is water–filling both in time(block) and frequency. We consider only the equal–rate point on the capacity region which, with equal average received powers, is achieved by the total sum–of–rates.

For a system with K users, the maximum sum-of-rates is achieved by having only the *strongest* user transmitting at any given frequency and time [9], and the corresponding power controllers are given by

$$S_{in}(f) = \left[B\frac{S}{KWN_0} - \frac{1}{|H_{in}(f)|^2} \right]^+, \quad |H_{in}(f)| \geq |H_{jn}(f)| \forall j \neq i \tag{1.22}$$

where B is chosen to satisfy the average–power constraint. This is a result of applying the Kuhn–Tucker conditions to the sum–of–rates inequality in (1.21).

We now give numerical results for the per–user spectral efficiencies for the various multiple–access schemes in Rayleigh fading assuming equal average received powers. The spectral efficiency per user for FDMA is simply $W^{-1}I_\infty$ in (1.13) (or $W^{-1}C_\infty$ if the users are aware of the states of only their own channels). For SSMA, it can be calculated numerically. For the optimal scheme, the spectral efficiency per user is given by

$$W^{-1}C_\infty^K = \sum_{i=1}^{K} (-1)^{i-1} \binom{K}{i} \mathrm{Ei}\left(\frac{iK}{B}\frac{WN_0}{S} \right) \quad \text{nats/s/Hz} \tag{1.23}$$

where B is the solution to

$$\sum_{i=1}^{K} (-1)^{i-1} \binom{K}{i} \left[B \exp\left(-\frac{iK}{B}\frac{WN_0}{S} \right) - \left(iK\frac{WN_0}{S} \right) \mathrm{Ei}\left(\frac{iK}{B}\frac{WN_0}{S} \right) \right] = K. \tag{1.24}$$

These are shown in Fig. 3.

As mentioned earlier, we see that spectral efficiency per user increases slightly with the number of users for SSMA. For the optimal scheme with multiuser diversity, a significant increase is achievable, which is not the case with the non-fading Gaussian channel or FDMA on a fading channel. This is not surprising, since with many users, the probability that one of the channels is good is high which means that the corresponding user can transmit at a high rate. Moreover, even in the two-user case a system exploiting multiuser diversity on Rayleigh

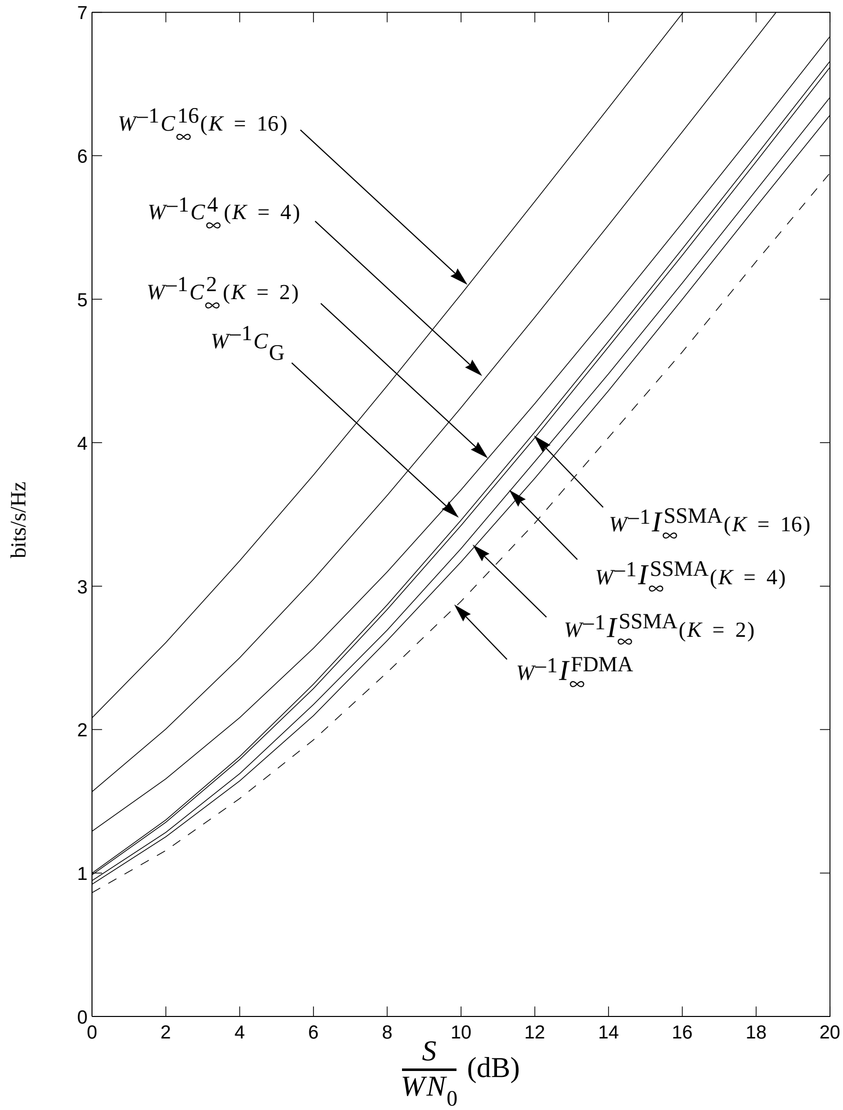

Figure 3: Spectral Efficiency Comparison

fading channel can theoretically achieve higher spectral efficiency than a system transmitting over the non-fading Gaussian channel.

Although not shown here, if we compute the maximum average throughput ($T_{\max}$) when the transmitters use a constant–code rate with multiuser diversity, we find that high spectral efficiency can be obtained without having to decode over many bursts. The calculation of the information outage probability is identical to the case of selection diversity discussed in [5].

4 DISCUSSION

Optimal bandwidth partitioning based on channel knowledge at the user terminals would be hard to achieve practically. A more realistic alternative would be to divide the entire bandwidth into equal size subbands and allocate a single user to each these subbands based on the instantaneous frequency responses of all the users. The DECT system [12, chap. 30] employs a technique along these lines. In this system, the available bandwidth is divided into several subbands, which we assume to be frequency flat. The users measure the strength of each subband (from the downlink which operates in time–division duplex with the uplink), and choose the best available subband in which to transmit. Our results suggest that the opposite should be done, namely that the basestation would instruct the user with the best channel response in a particular subband to use that subband alone. The subbands need not be adjacent, and, in fact, should not be. This would assure independence of the subband channel gains. We could envisage a variable–rate system where the total bandwidth per cell is divided into L independent subbands (i.e. with carrier spacings greater than the coherence bandwidth) of bandwidth KW/L where the basestation instructs the strongest user to transmit at a constant rate R in each subband. The instantaneous data rate per user would be R times an $(L,1/K)$ binomial random variable, so that the average data rate would be RL/K. The probability that a user transmits no information in any given burst would be $\left(\frac{K-1}{K}\right)^L \approx e^{-L/K}$, which can be made arbitrarily small by increasing L/K. We can interpret L as the fineness control of the variable–rate system.

In this paper, we have ignored the problem of how channel measurements can be made available to the users. In two–way systems the measurements could be made at the basestation and passed to the users via the downlink. The main problem, however, lies in obtaining the state of the channels when the users are not transmitting in certain parts of the spectrum. This would call for wideband pilot signals to be transmitted periodically if the channel changes slowly. For quickly–varying channels, low–power measurement signals (several dB below the information signals) could be overlaid at all times so that sudden changes could be tracked and accounted for.

It would also be interesting to investigate the effect that channel–controlled multiple–access schemes and variable–rate coding would have on cellular system capacity by considering adjacent–cell interference.

REFERENCES

[1] Wyner A.D., "Shannon–Theoretic Approach to a Gaussian Cellular Multiple Access Channel," IEEE Trans. on Information Theory, vol. 40, no. 6, pp. 1713–1727.

[2] Hanly S.H., *Information Capacity of Radio Networks,* PhD Thesis, Cambridge, UK, 1993.

[3] Steele R., *Mobile Radio Communications*, Pentech Press 1992, chap 8.

[4] Lee W.C.Y., *Mobile Communications Engineering,* McGraw-Hill, 1982.

[5] Ozarow L.H., Shamai S., Wyner A.D., "Information Theoretic Considerations for Cellular Mobile Radio," IEEE Trans. on Vehic. Tech., vol. 43, no. 2, May 1994, pp. 359-378.

[6] Gallager, R.G., *Information Theory and Reliable Communication,* Wiley & Sons, 1968.

[7] Goldsmith A., Varaiya P., "Increasing Spectral Efficiency Through Power Control," Proc. ICC '93, Geneva, Switzerland, pp. 600–604.

[8] Gallager R.G., "An Inequality on the Capacity Region of Multiaccess Multipath Channels," in Communica tions and Cryptography - Two Sides of One Tapestry -, Kluwer 1994, pp. 129-139.

[9] Knopp R., Humblet P.A., "Multiple Accessing over Frequency–Selective Fading Channels," Proc. PIMRC '95, Toronto, Canada, pp. 1326–1330.

[10] Cover T., Thomas J., *Elements of Information Theory,*, Wiley & Sons, 1991.

[11] Cheng R.S., Verdú S.,"Gaussian Multiaccess Channels with Capacity Region and Multiuser Water–Filling", IEEE Trans. on Information Theory, vol. 39, no. 3, May 1993.

[12] *The Mobile Communications Handbook,*, ed. J.D. Gibson, CRC Press, 1996.