Large Multi-Antenna Stochastic Geometry

Christo Kurisummoottil Thomas, Dirk Slock

Communication Systems Department, EURECOM

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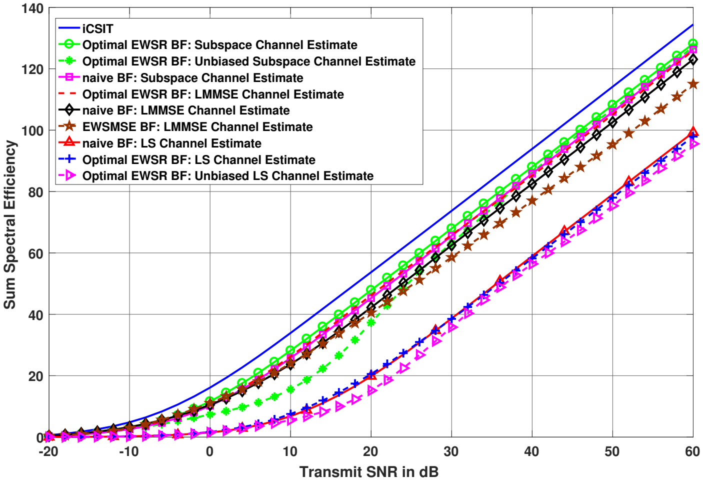
Outline

Fake News

- Original title:
  Optimizing CSIT for Multi-User MIMO - Beyond LMMSE



Issues

- MaMIMO systems may benefit from exploiting channel covariance information (eg coCSIT ZF)
- Large System Analysis of MaMIMO involves covariance matrices of all channels [isit'17]: a mess, no insights
- Use Stochastic Geometry extended to multi-antenna systems to randomize covariance subspaces, simplifying large system results.
- utility exploiting partial CSIT: naive, Expected Weighted Sum MSE, MaMIMO limit of Expected Weighted Sum Rate
- which channel estimate: least-squares, LMMSE, subspace projected LS

Motivation

- In stochastic geometry based methods, the location of the users is assumed to be random, leading to randomized channel attenuations.
- The multipath propagation for the various users leads to randomized angles of arrival at the BS which can be translated into spatial channel response contributions that depend on the antenna array response
- In Massive MIMO systems, the channel covariance matrix tends to be low rank (few significant scattering clusters).
- Exploiting these, we propose to model the user channel subspaces as isotropically randomly oriented. This allows us to assume the eigen vectors of the channel covariance matrix to be Haar distributed, and this identically and independently for all users.
- Large system analysis gives analytical insights into the performance of the various BFs.

State of the Art

- In [Wagner:TIT12], the authors obtain deterministic (instead of channel realization dependent) expressions for various scalar quantities.
- Helps to evaluate the system performance without resorting to heavy monte-carlo simulations.
- In [Lakshminarayana:TIT15] or [Muller:TSP15], MISO IBC is considered with perfect CSIT and weighted Regularized Zero-Forcing (R-ZF) BF, with two optimized weight levels, for intracell or intercell interference.
- Large system analysis under partial CSIT for optimal BF [Tabikh:ISIT17], but analysis quite cumbersome.
- In [Thomas:SAM18], we applied large system analysis to reduced order ZF beamforming for the simplified case of differently attenuated channel covariance matrices of the users.

Outline

1. Introduction

2. Asymptotic Analysis for Massive MISO
   - Partial CSIT Model
   - Optimal EWSR BF
   - Large System Analysis for Optimal Beamforming

3. Numerical Results, Conclusion and Future Work

Channel and CSIT Model

- We start from a deterministic Least-Squares (LS) channel estimate
$$\widehat{\mathbf{h}}_{LS} = \mathbf{h} + \widetilde{\mathbf{h}}, \;\; \widetilde{\mathbf{h}} \sim \mathcal{CN}(0, \widetilde{\sigma}^2 \boldsymbol{I})$$
- Exploitation of channel covariance information: LMMSE, LS and Subspace channel estimators considered.
- Each MISO channel h follows Karhunen-Loeve representation,
$$\mathbf{h} = \mathbf{C}\,\mathbf{c}, \;\; \mathbf{c} = \mathbf{D}^{1/2}\mathbf{c}',$$
where $\mathbf{c}' \sim \mathcal{CN}(0, \boldsymbol{I}_L)$ and $\mathbf{D}$ is diagonal. Here $\boldsymbol{C}$ is the $M \times L$ eigen vector matrix of the reduced rank channel covariance $R_{\mathbf{hh}} = \boldsymbol{CDC}^H$.
- The total sum rank across all users $N_p = \sum_{k=1}^{K} L_{k,c}$ is assumed to be less than M_c, where $L_{k,c}$ is the channel rank between user k and BS c.

Different Channel Estimates

- In the MaMIMO limit, BF design with partial CSIT will depend on the quantities $\mathbf{S} = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\left(\mathbf{h}\mathbf{h}^H\right) = \widehat{\mathbf{h}}\,\widehat{\mathbf{h}}^H + \widetilde{\boldsymbol{\Theta}}$.
  - LS Channel Estimate: $\widehat{\mathbf{h}}_{LS} = \mathbf{h} + \widetilde{\mathbf{h}}$ where $\mathbf{h}$ and $\widetilde{\mathbf{h}}$ are independent. In the LS case, $\widetilde{\boldsymbol{\Theta}} = \widetilde{\sigma}^2 \boldsymbol{I}$.
  - LMMSE Channel Estimate: $\mathbf{h} = \widehat{\mathbf{h}} + \widetilde{\mathbf{h}}$ in which $\widehat{\mathbf{h}}$ and $\widetilde{\mathbf{h}}$ are decorrelated and hence independent in the Gaussian case. In the LMMSE case, $\widetilde{\boldsymbol{\Theta}} = C_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}}$ is the posterior covariance. $\mathbf{S} = \mathbf{C}\mathbf{W}\mathbf{C}^H$, where $\mathbf{W} = \widehat{\mathbf{D}}^{1/2}\widehat{\mathbf{c}}\widehat{\mathbf{c}}^H\widehat{\mathbf{D}}^{1/2} + \widetilde{\mathbf{D}}$.
  - Subspace Projection based Channel Estimate: Effect of limiting channel estimation error to the covariance subspace (without the LMMSE weighting, this is a simplification of the LMMSE estimate).

$$\widehat{\mathbf{h}}_S = \boldsymbol{P}_C \widehat{\mathbf{h}}_{LS} = \mathbf{h} + \boldsymbol{P}_C \widetilde{\mathbf{h}}_{LS}\,,\ C_{\widetilde{\mathbf{h}}_S \widetilde{\mathbf{h}}_S} = \widetilde{\sigma}^2 \boldsymbol{P}_C,$$

where $\boldsymbol{P}_C = \boldsymbol{C}(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H$ represents the projection onto the covariance subspace.

Optimality of LMMSE

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  - Gaussian LMMSE $\widehat{\mathbf{h}} = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\mathbf{h}$

    Nonlinear MMSE estimate : $\mathbf{S} = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\left(\mathbf{h}\mathbf{h}^H\right) = \widehat{\mathbf{h}}\,\widehat{\mathbf{h}}^H + C_{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}}$

    (Bayesian) unbiased: $\mathrm{E}_{\widehat{\mathbf{h}}}\mathbf{S} = \mathrm{E}\,\mathbf{h}\mathbf{h}^H = \boldsymbol{CDC}^H$

    Unbiased and MMSE $\Rightarrow$ minimum variance.

    Also unbiased minimum variance estimate for $|\mathbf{g}^H\mathbf{h}|^2 = (\mathbf{g}^T \otimes \mathbf{g}^H)vec(\mathbf{h}\mathbf{h}^H)$

BF with Partial CSIT: Expected WSR

- Rx signal at user k,

$$y_k = \mathbf{h}_{k,b_k}^H \mathbf{g}_k x_k + \sum_{i=1,\neq k}^{K} \mathbf{h}_{k,b_i}^H \mathbf{g}_i x_i + v_k .$$

- MI Gaussian signaling, Expected Weighted Sum Rate (EWSR):

$$EWSR = \mathrm{E}_{\mathbf{h}} \sum_{k=1}^{K} u_k \ \ln\Big(1 + \frac{|\mathbf{h}_{k,b_k}^H \mathbf{g}_k|^2}{1 + \sum_{i=1,\neq k}^{K} |\mathbf{h}_{k,b_i}^H \mathbf{g}_i|^2}\Big)$$

$$= \mathrm{E}_{\mathbf{h}} \sum_{k=1}^{K} u_k \left(\ln(s_k) - \ln(s_{\overline{k}})\right)$$

$$s_{\overline{k}} = 1 + \sum_{i=1,\neq k}^{K} |\mathbf{h}_{k,b_i}^H \mathbf{g}_i|^2 , \ s_k = s_{\overline{k}} + |\mathbf{h}_{k,b_k}^H \mathbf{g}_k|^2$$

BF with Partial CSIT: Expected WSR (2)

- Naive Massive MISO ($K \to \infty$): $s_k, s_{\overline{k}} \to \mathrm{E}_{\mathbf{h}} s_k,\ \mathrm{E}_{\mathbf{h}} s_{\overline{k}}$. ESEI-WSR.
- More precisely: upper bound:

$$EWSR = \mathrm{E}_{\mathbf{h}} \sum_{k=1}^{K} u_k \ \ln(1 + \frac{|\mathbf{h}^H_{k,b_k}\, \mathbf{g}_k|^2}{s_{\overline{k}}})$$
$$\approx \mathrm{E}_{\mathbf{h}} \sum_{k=1}^{K} u_k \ \ln(1 + \frac{|\mathbf{h}^H_{k,b_k}\, \mathbf{g}_k|^2}{\mathrm{E}_{\mathbf{h}}\, s_{\overline{k}}})$$
$$\leq \sum_{k=1}^{K} u_k \ \ln(1 + \frac{\mathrm{E}_{\mathbf{h}}|\mathbf{h}^H_{k,b_k}\, \mathbf{g}_k|^2}{\mathrm{E}_{\mathbf{h}}\, s_{\overline{k}}}) = ESEI - WSR$$

- $\mathrm{E}_{\mathbf{h}}|\mathbf{h}^H_{k,b_k}\, \mathbf{g}_i|^2 = \mathbf{g}_i^H(\mathrm{E}_{\mathbf{h}} \mathbf{h}_{k,b_k} \mathbf{h}^H_{k,b_k})\mathbf{g}_i$

ESEI-WSR/EWSR gap

- We will need: $\mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}|\mathbf{h}_{k,b_k}^H\,\mathbf{g}_i|^2 = \mathbf{g}_i^H\mathbf{S}_{k,b_k}\mathbf{g}_i,\ r_{\overline{k}} = \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\,s_{\overline{k}}$
-

$$gap = \ln(1+\frac{\mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}|\mathbf{h}_{k,b_k}^H\,\mathbf{g}_k|^2}{r_{\overline{k}}}) - \mathrm{E}_{\mathbf{h}|\widehat{\mathbf{h}}}\ln(1+\frac{|\mathbf{h}_{k,b_k}^H\,\mathbf{g}_k|^2}{r_{\overline{k}}}) \leq \gamma$$

  $gap(snr)$ increases monotonically, $gap(0)=0$, $gap(\infty)\leq\gamma=0,577$
- [Gopala:icassp18]:

$$gap = \gamma - gap_c$$

  where gap_c is the gap fom EWSR to EWSR in which $\widehat{\mathbf{h}}$ gets replaced by $a\widehat{\mathbf{h}}$ where $a\sim\mathcal{CN}(0,1)$: degrading Rice to Rayleigh with same channel correlation matrix.

Different BFs with Partial CSIT

In the case of partial CSIT we get for the Rx signal,

$$y_k = \widehat{\mathbf{h}}^H_{k,b_k} \mathbf{g}_k x_k + \underbrace{\widetilde{\mathbf{h}}^H_{k,b_k} \mathbf{g}_k x_k}_{\text{sig. ch. error}} + \sum_{i=1,\neq k}^{K} (\widehat{\mathbf{h}}^H_{k,b_i} \mathbf{g}_i x_i + \underbrace{\widetilde{\mathbf{h}}^H_{k,b_i} \mathbf{g}_i x_i}_{\text{interf. ch. error}}) + v_k \,.$$

- Naive EWSR BF : just replace $\mathbf{h}$ by $\widehat{\mathbf{h}}$ in a perfect CSIT approach. Ignore $\widetilde{\mathbf{h}}$ everywhere.
- Optimal ESEI-WSR BF: accounts for covariance CSIT in the signal and interference terms.
- EWSMSE BF (Expected Weighted Sum MSE) [NegroSlock:ISWCS2012]: accounts for covariance CSIT in the interference terms, but also associates the signal $\widetilde{\mathbf{h}}$ term with the interference !

Max EWSR BF in the MaMISO Limit

- $EWSR = \mathrm{E}_{\widehat{\mathbf{h}}} \max_{\mathbf{g}} EWSR(\mathbf{g})$, $EWSR(\mathbf{g}) = \sum_{k=1}^{K} u_k \, \mathrm{E}_{\mathbf{h}|\widehat{h}} \ln(s_k/s_{\overline{k}}) \stackrel{(a)}{=} \sum_{k=1}^{K} u_k \, \ln((\mathrm{E}_{\mathbf{h}|\widehat{h}} s_k)/(\mathrm{E}_{\mathbf{h}|\widehat{h}} s_{\overline{k}}))$, $(a) \implies$ the MaMISO limit which is ESEI-WSR (Expected Signal Expected Interference WSR), $s_{\overline{k}}$ is the interference plus noise power and s_k is the signal plus interference plus noise power. Note that we define $r_k = E_{\mathbf{h}|\widehat{h}}(s_k), r_{\overline{k}} = E_{\mathbf{h}|\widehat{h}}(s_{\overline{k}})$.

- Optimizing this leads to the generalized eigen vector solution along with ILA-WF user powers,

$$\mathbf{g}'_k \propto [\sum_{i \neq k} \beta_i \mathbf{S}_{i,b_k} + \mu_{b_k} \mathbf{I}]^{-1} \mathbf{S}_{k,b_k} \mathbf{g}'_k, \quad \beta_k = u_k(\frac{1}{r_{\overline{k}}} - \frac{1}{r_k}),$$

$$\mathbf{g}_k = \mathbf{g}'_k p_k^{1/2}.$$

Max EWSR BF in the MaMISO Limit Continued ...

- ILA-WF for user powers,

$$p_k = \left(\frac{u_k}{\sigma_k^{(2)} + \mu_{b_k}} - \frac{1}{\sigma_k^{(1)}} \right)^+ ,$$

where $(x)^+ = \max\{0, x\}$ and the Lagrange multipliers μ_c are adjusted (e.g. by bisection) to satisfy the power constraints. Also, $\sigma_k^{(1)} = r_{\bar{k}}^{-1} \mathbf{g}_k'^H \mathbf{S}_{k,b_k} \mathbf{g}_k'$ (direct link power) $\sigma_k^{(2)} = \mathbf{g}_k'^H \sum_{i \neq k} \beta_i \mathbf{S}_{i,b_k} \mathbf{g}_k'$ (cross links leakage power).

- Under stochastic geometry MaMIMO regime, optimal EWSR BF $\Longrightarrow$

$$\mathbf{g}_k' \propto [\sum_{i \neq k} \beta_i \mathbf{S}_{i,b_k} + \mu_{b_k} \mathbf{I}]^{-1} \mathbf{C}_{k,b_k} \boldsymbol{V}_{max}(\mathbf{W}_{k,b_k}),$$

Stochastic Geometry MaMIMO Regime

- An appropriate model for the covariance subspaces $\boldsymbol{C}$ (independent across different user channels) is a Haar distribution (randomly oriented semi-unitary matrices).
- However, as we shall consider that the rank L remains finite, whereas M grows unboundedly, for the large system analysis we may equivalently consider the elements of $\boldsymbol{C}$ as i.i.d. with zero mean and variance $1/M$ so that the expected squared norm (and asymptotically the actual squared norm) of the columns of $\boldsymbol{C}$ is normalized to 1.
- From [Wagner:TIT12] that any terms of the form $\frac{1}{N}\text{tr}\{(\mathbf{A}_N - z\boldsymbol{I}_N)^{-1}\}$, where $\mathbf{A}_N$ is the sum of independent rank one matrices with covariance matrix $\boldsymbol{\Theta}_i$ is equal to the unique positive solution of $e_j = \frac{1}{N}\text{tr}\{(\sum_{i=1}^{K} \frac{\boldsymbol{\Theta}_i}{1+e_i} - z\boldsymbol{I}_N)^{-1}\}$.

Stochastic Geometry MaMIMO Regime Continued...

Considering the eigen decomposition of $\mathbf{W}_{i,b_k} = \boldsymbol{V}_{i,b_k}\boldsymbol{\Lambda}_{i,b_k}\boldsymbol{V}_{i,b_k}^H$, where $\boldsymbol{\Lambda}_{i,b_k} = \text{diag}(\zeta_{i,b_k}^{(1)}, ..., \zeta_{i,b_k}^{(L_{i,b_k})})$. Further terms of the following form in SINR,

$$\mathbf{C}_{k,b_k}^H[\sum_{i\neq k}\beta_i\boldsymbol{C}_{i,b_k}\mathbf{W}_{i,b_k}\boldsymbol{C}_{i,b_k}^H + \mu_{b_k}\mathbf{I}]^{-1}\boldsymbol{C}_{k,b_k} =$$
$$\frac{1}{M_{b_k}}\text{tr}\{[\sum_{i\neq k}\beta_i\boldsymbol{C}_{i,b_k}\mathbf{W}_{i,b_k}\boldsymbol{C}_{i,b_k}^H + \mu_{b_k}\mathbf{I}]^{-1}\}\boldsymbol{I}_{L_{k,b_k}} = e_{b_k}.$$

Using large system analysis results [Wagner:TIT12] simplifies it to the scalar, e_{b_k}, $\Longrightarrow$

$$e_{b_k} = \left(\frac{1}{M_{b_k}}\sum_{i=1}^{K}\sum_{r=1}^{L_{i,b_k}}\frac{\beta_i\zeta_{i,b_k}^{(r)}}{1+\beta_i\zeta_{i,b_k}^{(r)}e_{b_k}} + \mu_{b_k}\right)^{-1},$$

Also, we define, $\mathbf{B}_{k,b_i} = \text{diag}(1+\beta_k\zeta_{k,b_i}^{(1)}e_{b_i}, ..., 1+\beta_k\zeta_{k,b_i}^{(L)}e_{b_i})$, $\widetilde{\mathbf{E}}_k = e_{b_k}^{-1}\widetilde{\boldsymbol{D}}_{k,b_k} + \boldsymbol{I}$, e'_{b_k} is the derivative of e_{b_k} w.r.t μ_{b_k}.

Stochastic Geometry MaMIMO Regime Continued...

For the LMMSE case,

$$\mathbf{W} = \boldsymbol{F}(\widehat{\mathbf{c}}\widehat{\mathbf{c}}^H + \widetilde{\sigma}^2\boldsymbol{I})\boldsymbol{F}^H + (\boldsymbol{I}-\boldsymbol{F})\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{F})^H$$

where $\boldsymbol{F} = (\boldsymbol{I} + \widetilde{\sigma}^2\boldsymbol{D}^{-1})^{-1}$, $\widehat{\mathbf{c}} = \boldsymbol{C}^H\widehat{\mathbf{h}}_{LS}$, $\boldsymbol{F}\widehat{\mathbf{c}} = \widehat{\mathbf{c}}_{LMMSE}$ or $\widehat{\mathbf{c}}_L$ for short. So, $\mathbf{W} = \widehat{\mathbf{c}}_L\widehat{\mathbf{c}}_L^H + \widetilde{\sigma}^2\boldsymbol{D}(\widetilde{\sigma}^2\boldsymbol{I} + \boldsymbol{D})^{-1}$. At high SNR, up to first order in $\widetilde{\sigma}^2$, $\mathbf{W} = \widehat{\mathbf{c}}_L\widehat{\mathbf{c}}_L^H + \widetilde{\sigma}^2\boldsymbol{I}$ where the first term contains first-order terms also. At low SNR, $\boldsymbol{F} \approx \widetilde{\sigma}^{-2}\boldsymbol{D}$ and

$$\mathbf{W} = \widehat{\mathbf{c}}_L\widehat{\mathbf{c}}_L^H + (\boldsymbol{I} + \widetilde{\sigma}^{-2}\boldsymbol{D})^{-1}\boldsymbol{D} \approx \boldsymbol{D}\ .$$

For any SNR, these two extremes can be connected by the following approximation:

$$\begin{aligned}\boldsymbol{\Lambda} &= \widetilde{\sigma}^2\boldsymbol{D}(\widetilde{\sigma}^2\boldsymbol{I} + \boldsymbol{D})^{-1} + \|\widehat{\mathbf{c}}_L\|^2\boldsymbol{e}_1\boldsymbol{e}_1^H \\ &= \widetilde{\sigma}^2\boldsymbol{D}(\widetilde{\sigma}^2\boldsymbol{I} + \boldsymbol{D})^{-1} + \mathrm{tr}\{\boldsymbol{D}(\boldsymbol{I} + \widetilde{\sigma}^2\boldsymbol{D}^{-1})^{-1}\}\,\boldsymbol{e}_1\boldsymbol{e}_1^H\end{aligned}$$

where the last equality is due to the LLN.

Large System Results

- In the large system limit ($M, K \to \infty$, with $\frac{K}{M} = c < 1$), the quantities $r_{\overline{k}} - \overline{r}_{\overline{k}} \xrightarrow[a.s]{M_{b_k}\to\infty} 0$ and $r_k - \overline{r}_k \xrightarrow[a.s]{M_{b_k}\to\infty} 0$. By applying the continuous mapping theorem, it follows from the almost sure convergence of r_k and $r_{\overline{k}}$ that, $R_k - \overline{R}_k \xrightarrow[a.s]{M_{b_k}\to\infty} 0$, where R_k is the rate of user k, with $\overline{R}_k = \ln(\frac{\overline{r}_k}{\overline{r}_{\overline{k}}})$. The deterministic limits are obtained as,

$$\overline{r}_{\overline{k}} = 1 + \Upsilon_{\overline{k}},\ \overline{r}_k = 1 + \Upsilon_{\overline{k}} + p_k \frac{e^2_{b_k} \lambda_{max}(\mathbf{W}_{k,b_k})}{e'_{b_k}}, \textbf{where},$$

$$\Upsilon_{\overline{k}} = \sum_{\substack{i=1,\\ i\neq k}}^{K} p_i \frac{1}{M_{b_i}} [\sum_{r=1}^{L_{k,b_i}} \frac{\zeta^{(r)}_{k,b_i}}{(1+\beta_k \zeta^{(r)}_{k,b_i} e_{b_i})^2}],$$

$$\overline{\beta}_k = u_k \left(\frac{1}{\overline{r}_{\overline{k}}} - \frac{1}{\overline{r}_k} \right).$$

Rate Expressions (At High SNR)

$$\overline{R}_k^{OptBF} = \ln(1 + p_k \lambda_{max}(\mathbf{W}_{k,b_k})),$$

$$\overline{R}_k^{EWSMSE} = \ln\left(1 + \frac{p_k \lambda_{max}(\mathbf{W}_{k,b_k}(\boldsymbol{I} - \widetilde{\mathbf{E}}_k^{-1}))}{1 + \sum_{i=1, i \neq k}^{K} p_i \frac{\mathrm{tr}\{\boldsymbol{\Lambda}_{k,b_i}\mathbf{B}_{k,b_i}^{-2}\}}{L_{i,b_i} M_{b_i}} \frac{1}{(1 - 2\frac{\mathrm{tr}\{\widetilde{\mathbf{E}}_i^{-1}\}}{L_{i,b_i}} + \frac{\mathrm{tr}\{\widetilde{\mathbf{E}}_i^{-2}\}}{L_{i,b_i}})}}\right).$$

Conclusion:

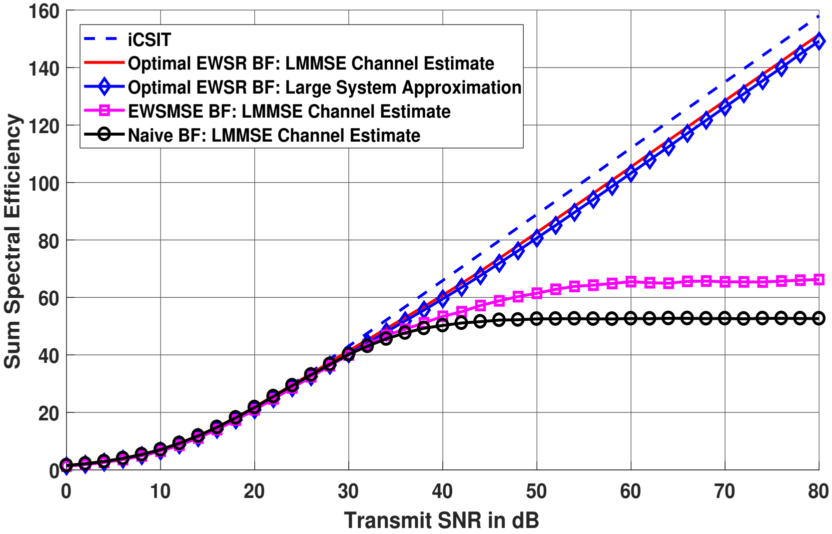
- The suboptimal EWSMSE BF doesn't ZF at high SNR and the interference power also increases with SNR $\rightarrow \infty \implies$ large gap in performance between the optimal BF and EWSMSE BF.
- We consider the scenario of the channel estimation error remaining finite with SNR (otherwise the channel estimate becomes perfect at high SNR and thus all the BFs converg to the same solution).

Outline

Simulation Setup

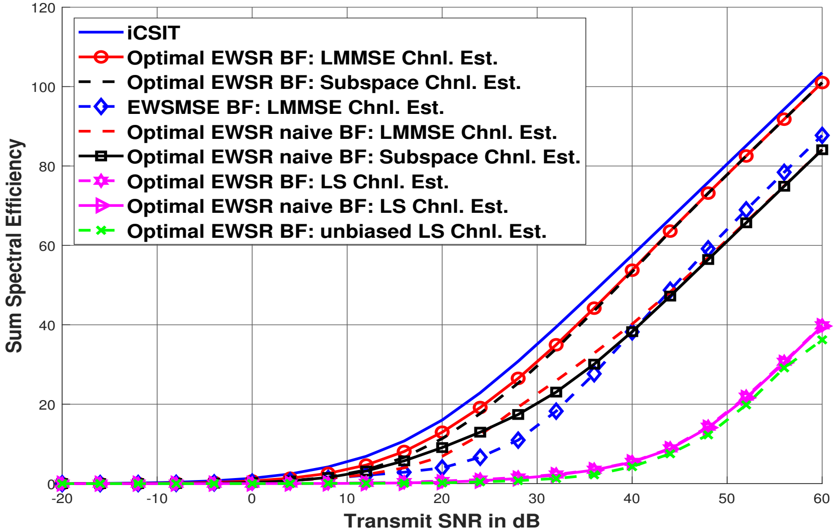
- C, number of cells. K_c, number of (single-antenna) users in cell c and $K = \sum_c K_c$. M, number of transmit antennas in each cell.
- We consider a path-wise or low rank channel model, with $L =$ number of paths $=$ channel covariance rank. d : scale factor in the LS channel estimation error variance $\widetilde{\sigma}^2 = d/SNR$.
- The elements of $\boldsymbol{C}$ are generated i.i.d complex Gaussian with variance $1/(2M)$.
- Notations: in the figures, iCSIT refers to the optimal BF design for the instantaneous CSIT case [Christensen:TWC2008].

Spectral Efficiency Results



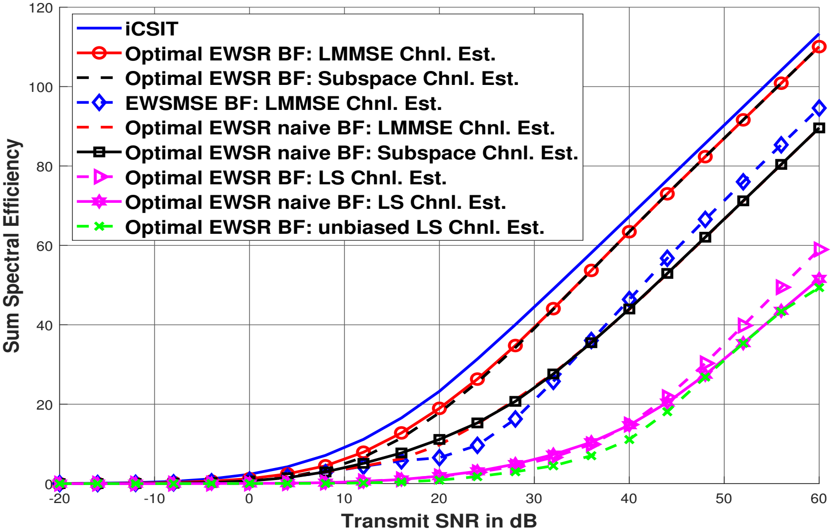
EWSR for $C = 1$ cell, $K = 10$ users, $M = 64$, $L = 4$, $\tilde{\sigma}^2 = 0.1$.

Spectral Efficiency Results



EWSR for $C = 1$ cells, $K_1 = K = 10$ users, $M = 64$, $L = 2$, $\tilde{\sigma}^2 = 1/SNR$.

Spectral Efficiency Results



EWSR for $C = 2$ cells, $K_1 = K_2 = 5$ users, $M = 32$, $L = 2$, $\tilde{\sigma}^2 = 1/SNR$.

Conclusion and Future Work

- We introduced a stochastic geometry inspired randomization of the channel covariance eigen spaces and analyzed the large system behavior with an optimal precoder under partial CSIT.
- Moreover, we show the improvement in performance by using an LMMSE channel estimate compared to just having LS estimates, and by furthermore properly exploiting all covariance information.
- Simulation results indicated that large system approximations are very accurate even for small system dimensions and reveal the deterministic dependence of the system performance on several important system parameters, such as the channel attenuation, signal powers and SNR.