ROBUST MUSIC ESTIMATION UNDER ARRAY RESPONSE UNCERTAINTY

Ahmad Bazzi

CEVA
Les Bureaux Green Side 5, Bat 6
400, avenue Roumanille
06410 Biot, Sophia Antipolis, France
ahmad.bazzi@ceva-dsp.com

Dirk Slock[*]

EURECOM
Campus SophiaTech,
450 Route des Chappes
06410 Biot, France
slock@eurecom.fr

ABSTRACT

This paper addresses the problem of bearing estimation with partial knowledge on the array response. In a real life scenario, it may seem fictitious to assume a closed-form steering vector as a function of the direction of the received signal. Indeed, an antenna array is subject to many perturbations, such as calibration errors, imperfect positioning of array sensors, mutual coupling, and so on. We introduce a MUSIC estimator, hereby referred to as Robust-MUSIC, that is capable of estimating AoAs of multiple signals, when the antenna array's response is subject to uncertainties, due to the aforementioned reasons. Simulation results are also presented to demonstrate the robustness of the proposed MUSIC estimator.

Index terms— Robust-MUSIC, Parameter Estimation, Uncertainty, Calibration, Robustness

1. INTRODUCTION

The problem of estimating the directions of multiple sources is addressed. In fact, this problem appears in a majority of engineering applications such as navigation, tracking of objects, radar, sonar, and wireless communications [1]. Furthermore, numerous high-resolution and computationally efficient algorithms were implemented to solve this issue, such as: MUSIC [1], ESPRIT [2], and many others.

DoA methods require full knowledge of the array steering vector to yield reliable estimates. As a consequence, any deviation about the nominally assumed response would result in dramatic performance degradation. For example, mutual coupling causes inter-element interference in an array[5]. This leaves some uncertainty on the amplitude and phase of an element relative to its neighboring ones. Moreover, each receiver path associated with an antenna element may not be properly synchronized[6], hence leading to different phase shifts between antenna elements. Antenna elements are assumed to be perfectly placed relative to one another. In reality, this could hardly be realized[7]. All these errors, along with many others, make DoA estimation very challenging.

DoA estimation with "miscalibrated" arrays has been well studied, hence robust estimators are needed. Robust[1] DoA estimation techniques are, therefore, very attractive and motivating. Indeed, the class of robust DoA methods could be seen as a generalization of classical ones, thanks to their adaptability with a family of array responses. For example, a robust minimum variance (RMV) beamformer [3] could be viewed as a robust version of the traditional Capon estimator. Roughly speaking, RMV finds the best minimum variance DoA estimator, even when the array response is far from the actual one. The RMV uses sophisticated convex optimization theory [4] to derive the desired minimum variance estimator.

In this paper, we derive a robust MUSIC estimator. Our first contribution is the formulation of an optimization problem, whose optimal value turns out to be the classical MUSIC cost function. The second contribution is the derivation of a robust version of the MUSIC algorithm, hereby termed as Robust MUSIC. Said differently, we could generate MUSIC estimates, under array response uncertainty.

This paper is organized as follows: Section 2 presents the system model utilized throughout this paper. Section 3 derives the Robust MUSIC method via sophisticated convex optimization theory and appropriate problem formulation. Section 4 presents some computer simulations to compare the performance of the Robust MUSIC to other robust meth-

[*] EURECOM's research is partially supported by its industrial members: ORANGE, BMW, Symantec, SAP, Monaco Telecom, iABG, and by the French FUI projects MASS-START.

[1] Robust in a sense that one could reliably estimate AoAs, given some sort of uncertainty on the array response.

ods. Finally, we conclude the paper in Section 5.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^{\mathsf{T}}$ and $(.)^{\mathsf{H}}$ represent the transpose and transpose-conjugate operators. The matrix $\boldsymbol{I}$ is the identity matrix of appropriate dimensions. For any matrix $\boldsymbol{X}$, we refer to the $(i,j)^{th}$ entry found in the i^{th} row and j^{th} column as $[\boldsymbol{A}]_{i,j}$. The norm $\|\|$ is the Frobenius norm.

2. SYSTEM MODEL

Consider an array composed of N sensors with q narrowband sources, each at different bearings. Let $\theta_1 \dots \theta_q$ denote the Angles-of-Arrival (AoA) of each the sources, then the received signal could be modelled as

$$\boldsymbol{x}(t) = \sum_{k=1}^{q} \boldsymbol{a}(\theta_k) s_k(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{a}(\theta) \in \mathbb{C}^{N \times 1}$ represents the array response to a source arriving at θ. Moreover, the k^{th} transmit narrowband signal is denoted as $s_k(t)$. The vector $\boldsymbol{n}(t)$ denotes background noise. The problem here is to estimate the $\theta_1 \dots \theta_q$ given observations of the form of $\boldsymbol{x}(t)$ and uncertainty on $\boldsymbol{a}(\theta)$. Indeed, if $\boldsymbol{a}(\theta)$ is perfectly known, then one could estimate the AoAs using MUSIC through peak finding of the following cost

$$f_{\mathsf{MU}}(\theta) = \frac{1}{\left\|\hat{\boldsymbol{U}}_n^{\mathsf{H}} \boldsymbol{a}(\theta)\right\|_2^2} \tag{2}$$

where $\hat{\boldsymbol{U}}_n$ is the noise subspace extracted from the sample covariance matrix

$$\hat{\boldsymbol{R}} = \sum_{m=1}^{M} \boldsymbol{x}(t_m) \boldsymbol{x}^{\mathsf{H}}(t_m) \tag{3}$$

However, in many practical applications, it so happens that one can not assume perfect knowledge of $\boldsymbol{a}(\theta)$ due to imperfect antenna positioning, phase/gain mismatches per antenna, mutual coupling, etc. The work in [3] derives a minimum variance (Capon) beamformer under array uncertainty. This work derives a MUSIC beamformer under the same array uncertainty.

3. ROBUST MUSIC

Consider the following optimization problem

$$\begin{aligned} \underset{\boldsymbol{\omega}(\theta)}{\text{minimize}} \quad & \|\boldsymbol{\omega}(\theta) - \boldsymbol{a}(\theta)\|^2 \\ \text{subject to} \quad & \hat{\boldsymbol{U}}_n^{\mathsf{H}} \boldsymbol{\omega}(\theta) = 0 \end{aligned} \tag{4}$$

which is optimal at

$$\boldsymbol{\omega}_{\mathsf{MU}}(\theta) = \hat{\boldsymbol{U}}_s \hat{\boldsymbol{U}}_s^{\mathsf{H}} \boldsymbol{a}(\theta) \tag{5}$$

Plugging $\boldsymbol{\omega}_{\mathsf{MU}}(\theta)$ in the cost function of (4) gives the well-known MUSIC cost function in equation (2).

It is important to stress that MUSIC could also be viewed as a beamformer. Thanks to the optimal weighting vector $\boldsymbol{\omega}_{\mathsf{MU}}(\theta)$, we could define a measure that reports robustness of the associated beamformer against interferes. This measure is the signal-to-interference-noise ratio (SINR), given by

$$\mathsf{SINR}(\theta) = \frac{\left|\boldsymbol{\omega}^{\mathsf{H}}(\theta)\, \boldsymbol{a}(\theta)\right|^2}{\boldsymbol{\omega}^{\mathsf{H}}(\theta)\, \boldsymbol{R}_{i+n}\, \boldsymbol{\omega}(\theta)} \tag{6}$$

where $\boldsymbol{R}_{i+n}$ is the interference-plus-noise covariance matrix. Roughly speaking, one would aim at maximizing the numerator of (6), while fixing the denominator. Therefore, the projection $\left|\boldsymbol{\omega}_{\mathsf{MU}}^{\mathsf{H}}(\theta) \boldsymbol{a}(\theta)\right|^2 = \boldsymbol{a}(\theta) \hat{\boldsymbol{U}}_s \hat{\boldsymbol{U}}_s^{\mathsf{H}} \boldsymbol{a}(\theta)$ reports the output power of the MUSIC method.

In this work, we formulate a robust variation of (4), which we will refer to as Robust MUSIC

$$\begin{aligned} \underset{\boldsymbol{\omega}(\theta)}{\text{minimize}} \quad & \|\boldsymbol{\omega}(\theta) - \boldsymbol{a}(\theta)\|^2 \\ \text{subject to} \quad & \hat{\boldsymbol{U}}_n^{\mathsf{H}} \boldsymbol{\omega}(\theta) = 0 \\ & \boldsymbol{a}(\theta) \in \mathcal{E}(\boldsymbol{c}(\theta), \boldsymbol{P}) \end{aligned} \tag{7}$$

where $\mathcal{E}(\boldsymbol{c}(\theta), \boldsymbol{P})$ is an ellipsoid centered at $\boldsymbol{c}(\theta)$ and a symmetric positive semi-definite configuration matrix $\boldsymbol{P}$, which determines how far the ellipsoid extends in every direction from $\boldsymbol{c}(\theta)$ [4]. More specifically, the square root of the eigenvalues of $\boldsymbol{P}$ are the lengths of the semi-axis of the ellipsoid $\mathcal{E}(\boldsymbol{c}(\theta), \boldsymbol{P})$, defined as

$$\mathcal{E}(\boldsymbol{c}(\theta), \boldsymbol{P}) = \left\{ \boldsymbol{c}(\theta) + \boldsymbol{P}^{\frac{1}{2}} \boldsymbol{u} \;\middle|\; \|\boldsymbol{u}\| \leq 1 \right\} \tag{8}$$

Thanks to this representation, we can alternatively express (4) as

$$\begin{aligned} \underset{\boldsymbol{\omega}(\theta), \boldsymbol{u}}{\text{minimize}} \quad & \|\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta) - \boldsymbol{P}^{\frac{1}{2}} \boldsymbol{u}\|^2 \\ \text{subject to} \quad & \hat{\boldsymbol{U}}_n^{\mathsf{H}} \boldsymbol{\omega}(\theta) = 0 \\ & \|\boldsymbol{u}\|^2 \leq 1 \end{aligned} \tag{9}$$

Furthermore, the Lagrangian function of the problem (9) is

$$\begin{aligned} \mathcal{L}(\boldsymbol{\omega}(\theta), \boldsymbol{u}, \boldsymbol{\lambda}, \mu) = \|\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta) - \boldsymbol{P}^{\frac{1}{2}} \boldsymbol{u}\|^2 + \boldsymbol{\lambda}^{\mathsf{H}} \hat{\boldsymbol{U}}_n^{\mathsf{H}} \boldsymbol{\omega}(\theta) \\ + \mu(\|\boldsymbol{u}\|^2 - 1) \end{aligned} \tag{10}$$

We differentiate $\mathcal{L}$ with respect to $\boldsymbol{\omega}(\theta), \boldsymbol{u}, \boldsymbol{\lambda}$ and μ. Setting these partial derivatives equal to zero, we have, respectively,

$$2\boldsymbol{\omega}(\theta) - 2(\boldsymbol{c}(\theta) + \boldsymbol{P}^{\frac{1}{2}}\boldsymbol{u}) + \hat{\boldsymbol{U}}_n\boldsymbol{\lambda} = 0 \tag{11}$$

$$\boldsymbol{P}\boldsymbol{u} - \boldsymbol{P}^{\frac{1}{2}}(\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta)) + \mu\boldsymbol{u} = 0 \tag{12}$$

$$\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{\omega}(\theta) = 0 \tag{13}$$

$$\|\boldsymbol{u}\|^2 - 1 = 0 \tag{14}$$

Equation (12) gives us

$$\boldsymbol{u}_o = (\boldsymbol{P} + \mu_o\boldsymbol{I})^{-1}\boldsymbol{P}^{\frac{1}{2}}(\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta)) \tag{15}$$

where μ could be computed at the boundaries of the constraint $\|\boldsymbol{u}\|^2 \leq 1$, that is

$$\left\| \left(\boldsymbol{P} + \mu\boldsymbol{I}\right)^{-1}\boldsymbol{P}^{\frac{1}{2}}\left(\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta)\right) \right\|_2^2 = 1 \tag{16}$$

To facilitate the computation of μ, we use the eigenvalue decomposition $\boldsymbol{P} = \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathsf{H}}$, hence μ must satisfy

$$\sum_{n=1}^{N} \frac{\sigma_n\|\boldsymbol{V}^{\mathsf{H}}\left(\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta)\right)\|_2^2}{(\sigma_n + \mu)^2} = 1 \tag{17}$$

Replacing (15) in (11), we get

$$\boldsymbol{\omega}(\theta) = \boldsymbol{c}(\theta) - \frac{1}{2}\left(\boldsymbol{I} - \boldsymbol{P}^{\frac{1}{2}}(\boldsymbol{P} + \mu\boldsymbol{I})^{-1}\boldsymbol{P}^{\frac{1}{2}}\right)^{-1}\hat{\boldsymbol{U}}_n\boldsymbol{\lambda} \tag{18}$$

Using straightforward manipulations, we can re-write the above as

$$\boldsymbol{\omega}(\theta) = \boldsymbol{c}(\theta) - \frac{1}{2}\boldsymbol{P}_\mu^{-1}\hat{\boldsymbol{U}}_n\boldsymbol{\lambda} \tag{19}$$

where

$$\boldsymbol{P}_\mu = \boldsymbol{I} - \boldsymbol{P}^{\frac{1}{2}}(\boldsymbol{P} + \mu\boldsymbol{I})^{-1}\boldsymbol{P}^{\frac{1}{2}} = \boldsymbol{V}\boldsymbol{\Sigma}_\mu\boldsymbol{V}^{\mathsf{H}} \tag{20}$$

and $\boldsymbol{\Sigma}_\mu$ is a diagonal matrix given as

$$\left[\boldsymbol{\Sigma}_\mu\right]_{k,k} = \frac{\mu}{\mu + \sigma_k} \tag{21}$$

Forcing the orthogonality constraint, one obtains

$$\boldsymbol{\lambda} = 2\left(\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{P}_\mu^{-1}\hat{\boldsymbol{U}}_n\right)^{-1}\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{c}(\theta) \tag{22}$$

which when replaced back in (19) gives us

$$\boldsymbol{\omega}(\theta) = \boldsymbol{c}(\theta) - \boldsymbol{P}_\mu^{-1}\hat{\boldsymbol{U}}_n\left(\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{P}_\mu^{-1}\hat{\boldsymbol{U}}_n\right)^{-1}\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{c}(\theta) \tag{23}$$

Replacing (23) and (15) in the cost function and observing that

$$\|\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta) - \boldsymbol{P}^{\frac{1}{2}}\boldsymbol{u}\|^2 = \|\boldsymbol{P}_\mu\left(\boldsymbol{\omega}(\theta) - \boldsymbol{c}(\theta)\right)\|^2 \tag{24}$$

we arrive at the Robust MUSIC cost function, i.e.

$$f_{\mathsf{RMU}}(\theta) = \frac{1}{\|\hat{\boldsymbol{U}}_n\left(\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{P}_\mu^{-1}\hat{\boldsymbol{U}}_n\right)^{-1}\hat{\boldsymbol{U}}_n^{\mathsf{H}}\boldsymbol{c}(\theta)\|^2} \tag{25}$$

Notice the extreme case of perfect knowledge of the array response, i.e. $\boldsymbol{P} = \boldsymbol{0}$ or alternatively, $\boldsymbol{P}_\mu = \boldsymbol{0}$, thus Robust MUSIC boils down to the traditional MUSIC estimator, i.e. $f_{\mathsf{RMU}}(\theta) = f_{\mathsf{MU}}(\theta)$.

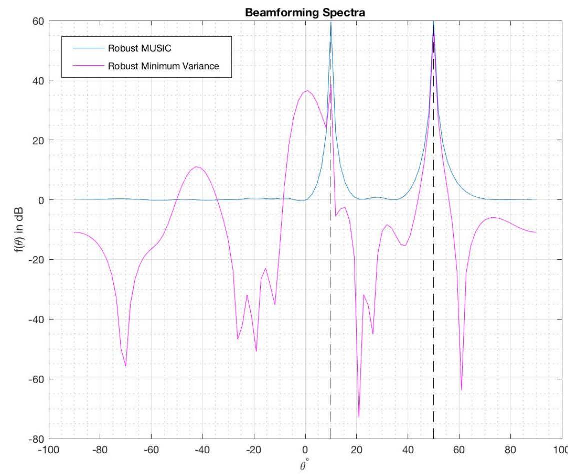

Figure 1: Direction-finding spectra of the Robust MUSIC vs Robust Minimum Variance

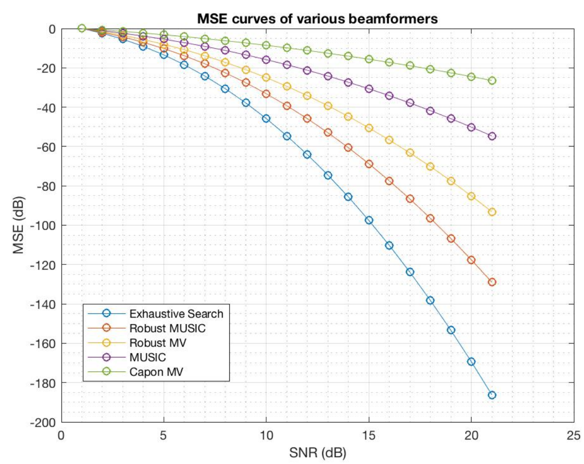

Figure 2: Performance analysis of various beamformers under ellipsoidal uncertainty

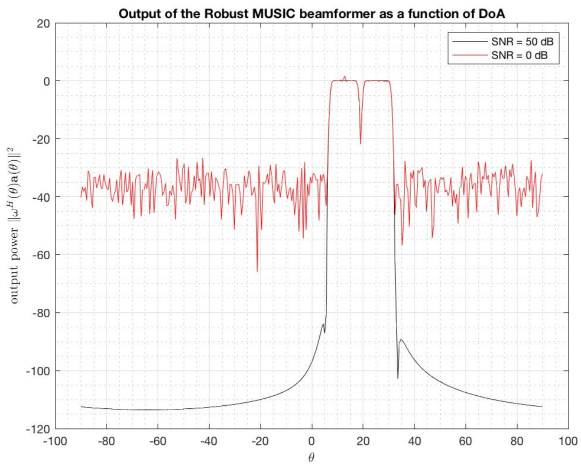

Figure 3: Output SINR of the Robust MUSIC as a function of DoA

4. COMPUTER SIMULATIONS

In the first experiment, we intend to compare the performance of the Robust minimum variance beamformer [3] and the Robust MUSIC derived herein. The simulation was carried out on an SNR of 10 dB. We assume sources that carry equal power, hence the SNR is defined as

$$\text{SNR} = \frac{\int_0^T |s(t)|^2 \, dt}{\sigma^2} \tag{26}$$

In all experiments, we consider a uniform linear antenna array composed of $N = 3$ elements, where its elements are placed at half a wavelength, i.e. the nominal array response takes a Vandermonde structure as

$$\boldsymbol{c}(\theta) = \begin{bmatrix} 1 & e^{-j\pi\sin(\theta)} & \dots & e^{-j\pi(N-1)\sin(\theta)} \end{bmatrix}^{\mathsf{T}} \tag{27}$$

In all experiments, the configuration matrix $\boldsymbol{P}$ is fixed to the following

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0.6 & 0.1 \\ 0.6 & 1 & 0.6 \\ 0.1 & 0.6 & 1 \end{bmatrix} \tag{28}$$

This means that we randomly pick any steering vector $\boldsymbol{a}(\theta)$ in $\mathcal{E}(\boldsymbol{c}(\theta), \boldsymbol{P})$ In the first experiment, the SNR was fixed to 5 dB and two sources were generated each at distinct angles $\theta_1 = 10°$ and $\theta_2 = 50°$. We use Gaussian sources $s_1(t)$ and $s_2(t)$ throughout this experiment. The number of time samples collected from $\boldsymbol{x}(t)$ is $T = 100$, which are further used to compute the sample covariance matrix $\hat{\boldsymbol{R}}$. According to Fig. 1, which plots the spectra of the Robust Minimum Variance derived in [3] and the Robust MUSIC derived herein, both seem to fully determine the proper DoAs; however, the noise level behaviour of the associated spectra behave differently. More specifically, the RMV spectra shows spurious peaks in the vicinity of $10°$, while the robust MUSIC seems to be "noise-flat". This is indeed dangerous as one might be mislead to a third source arriving at $\theta = 0°$, whereas no such source exists.

In the second experiment, we compare the normalized Mean-Squared-Error of multiple methods in the presence of array uncertainty. We run the simulations on 10^4 Monte Carlo simulations. The MSE is computed as

$$\text{MSE} = \frac{\sum_{m=1}^{M} \sum_{k=1}^{q} (\theta_k - \hat{\theta}_k^{(m)})^2}{M} \tag{29}$$

where $\hat{\theta}_k^{(m)}$ is the m^{th} DoA estimate of θ_k. As a benchmark, we run an exhaustive search on all joint parameters, that minimizes the deterministic likelihood probability density function, given all parameters of interest. One can clearly see the high performance of robust MUSIC when compared to existing methods. For instance, there is a 3dB gain of robust MUSIC when compared to RMV, when the target normalized MSE level is -80dB and a 3dB loss compared to the optimal exhaustive search bound.

In experiment three, we focus on SINR maximization applications. In other words, we aim at maximizing the output of the Robust MUSIC beamformer, as in the look direction, while keeping the interference-plus-noise level as low as possible. For the sake of demonstration, we focus on two sources with equal powers arriving at $\theta_1 = 12°$ and $\theta_2 = 25°$. We plot the output power of the beamformer as a function of DoA. The powers are normalized to the maximum power obtained by the spectrum. One could notice that an increase in 50dB SNR "pushes down" the noise level of the output beamformer power around the same order of SNR difference. However, in the region close to the look directions $12 < \theta < 25$, we can see much higher noise, which might be related to the configuration matrix $\boldsymbol{P}$. The reason is due to the particular drop at $\theta = 19°$ (around -22dB noise level).

5. CONCLUSIONS

In this paper, we have derived a robust version of the MUSIC method, through careful problem formulation, which enables us to impose some uncertainty on the array response. The Robust MUSIC could be argued to be computationally heavy due to the matrix inversion that is needed at each search point on the grid. On the other hand, computer simulations demonstrate the high performance and robustness to ellipsoidal noise about the nominal array response.

6. REFERENCES

[1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation,"*IEEE Transactions on Antennas and Propagation*,vol. 34, no. 3, pp. 276-280, Mar. 1986

[2] R. Roy and T. Kailath, "ESPRIT-Estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, Signal Processing*, vol.37, no. 7, pp. 984-995, July 1989

[3] R. G. Lorenz and S. P. Boyd, "Robust minimum variance beamforming." *IEEE transactions on signal processing* 53.5 (2005): 1684-1696.

[4] S. Boyd and L. Vandenberghe, "Convex optimization," *Cambridge university press*, 2004.

[5] A. Bazzi, D. Slock and L. Meilhac, "Online Angle of Arrival Estimation in the Presence of Mutual Coupling", *IEEE International Workshop on Statistical Signal Processing (SSP)*, 2016.

[6] Weiwei Hu, "DOA Estimation for UCA in the Presence of Gain-Phase Errors." *IEEE Communications Letters* 23.3: 446-449, 2019.

[7] M. Hawkes and A. Nehorai. "Effects of sensor placement on acoustic vector-sensor array performance." IEEE Journal of Oceanic Engineering 24.1 (1999): 33-40.

[8] B. Friedlander, "Antenna Array Manifolds for High-Resolution Direction Finding," *IEEE Transactions on Signal Processing*, VOL. 66, NO. 4, Feb. 15, 2018.

[9] A. Bazzi, D. TM Slock, and L. Meilhac. "A mutual coupling resilient algorithm for joint angle and delay estimation." *IEEE Global Conference on Signal and Information Processing* (GlobalSIP). IEEE, 2016.

[10] Z. Ye and C. Liu, "On the Resiliency of MUSIC Direction Finding Against Antenna Sensor Coupling," *IEEE Transactions on Antennas and Propagation*, VOL. 56, NO. 2, Feb. 2008.

[11] A. Bazzi, D. TM Slock, and L. Meilhac. "Performance Analysis of an AoA estimator in the presence of more mutual coupling parameters." *IEEE International Conference on Acoustics, Speech and Signal Processing* (ICASSP). IEEE, 2017.

[12] A. Lee Swindlehurst, "Robust Algorithms for Direction-Finding in the Presence of Model Errors", *Fifth ASSP Workshop on Spectrum Estimation and Modeling*, Oct. 1990.

[13] A. Bazzi, D. TM Slock, and L. Meilhac. "On Mutual Coupling for ULAs: Estimating AoAs in the presence of more coupling parameters." *IEEE International Conference on Acoustics, Speech and Signal Processing* (ICASSP). IEEE, 2017.

[14] B. Wang, Y. D. Zhang, and W. Wang, "Robust DOA Estimation in the Presence of Miscalibrated Sensor," *IEEE Signal Processing Letters*, VOL. 24, NO. 7, JULY 2017.

[15] A. Bazzi, D. TM Slock, and L. Meilhac. "On a mutual coupling agnostic maximum likelihood angle of arrival estimator by alternating projection." *IEEE Global Conference on Signal and Information Processing* (GlobalSIP). IEEE, 2016.

[16] L. Meilhac and A. Bazzi. "Pre-coding steering matrix for MU-MIMO communication systems." *U.S. Patent Application No.* 16/188,346, 2019.

[17] A. Bazzi, D. TM Slock, and L. Meilhac. "Online angle of arrival estimation in the presence of mutual coupling." *IEEE Statistical Signal Processing Workshop (SSP)*, IEEE, 2016.

[18] A. Bazzi, D. TM Slock, and L. Meilhac. "Blind on board wideband antenna RF calibration for multi-antenna satellites." *IEEE International Conference on Acoustics, Speech and Signal Processing* (ICASSP). IEEE, 2017.

[19] A. Bazzi, "Parameter estimation techniques for indoor localisation via WiFi". Diss. 2017.