MIMO User Rate Balancing In Multicell Networks with Per Cell Power Constraints

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***Abstract*—In this paper, we investigate the problem of user rate balancing for the downlink transmission of multiuser multicell Multiple-Input-Multiple-Output (MIMO) systems with per cell power constraints. Due to the multiple streams per user, user rate balancing involves both aspects of balancing and sum rate optimization. We exploit the rate Mean Squared Error (MSE) relation, formulate the balancing operation as constraints leading to Lagrangians in optimization duality, allowing to transform rate balancing into weighted MSE minimization with Perron Frobenius theory. The Lagrange multipliers for the multiple power constraints can be formulated as a single weighted power constraint in which the weighting can be optimized to lead to the satisfaction with equality of all power constraints. Actually, various problem formulations are possible, including single cell full power transmission leading to a dual norm optimization problem, and per cell rate balancing which breaks the balancing constraint between cells. Simulation results are provided to validate the proposed algorithms and demonstrate their performance improvement over e.g. unweighted MSE balancing.**

***Index Terms*—Inter-cell interference coordination (ICIC) and coordinated multi-point (CoMP); MIMO, multi-user MIMO, and massive MIMO**

I. Introduction

We consider a wireless network in which a set of base stations serve their users and all nodes are equipped with multiple antennas. The goal is to maximize the minimum rate among all users in the system subject to a total sum power constraint, to achieve network-wide fairness [1].

Actually, several works in the literature have studied the max-min/min-max fairness problems w.r.t. given utility functions. For instance, the max-min signal-to-interference-plus-noise ratio (SINR) problem is of particular interest because it is directly related to common performance measures like system capacity and bit error rates [2]. Maximizing the minimum user SINR in the uplink can be done straightforwardly since the beamformers can be optimized individually and SINRs are only coupled by the users' transmit powers. In contrast, downlink optimization is generally a nontrivial task because the user SINRs depend on all optimization variables and have to be optimized jointly. Downlink transmitter optimization for single antenna receivers with a constraint on the total transmit power is comprehensively studied in [3] and [4] where algorithmic solutions for maximizing the minimal user SINR are proposed. This SINR balancing technique has been extended to underlay cognitive radio networks with transmit power and interference constraints in [5], [6].

Another utility of interest for fairness optimization problems is the mean squared error (MSE). In fact, the min-max MSE optimization is based on the stream-wise MSE duality where it has been shown that the same MSE values are achievable in the downlink and the uplink with the same transmit power constraint. This MSE duality has been exploited to solve various minimum MSE (MMSE) based optimization problems [7], [8]. In [9], three levels of MSE dualities have been established between MIMO BC and MIMO MAC with the same transmit power constraint and these dualities have been exploited to reduce the computational complexity of the sum-MSE and weighted sum-MSE minimization problems (with fixed weights) in a MIMO Broadcast Channel (BC).

In this work, we focus on user rate balancing in a way to maximize the minimum per user (weighted) rate in the network. This balancing problem is studied in [10] without providing an explicit precoder design. As in [11], we provide here a solution via the relation between user rate (summed over its streams) and a weighted sum MSE. But also another ingredient is required: the exploitation of scale factor that can be freely chosen in the weights for the weighted rate balancing. User-wise rate balancing outperforms user-wise MSE balancing or streamwise rate (or MSE/SINR) balancing when the streams of any MIMO user are quite unbalanced. In [11] the problem is handled for BC (single cell) and is transformed into weighted MSE balancing using non-diagonal weight matrices. Here we consider a multicell case and solve the user rate balancing problem using diagonal weight matrices by diagonalizing the user signal error covariance matrices, which allows to link the per stream and per user power allocation problems.

II. System Model

We consider a MIMO system with C cells. Each cell c has one base station (BS) of M_c transmit antennas serving K_c users, with total number of users $\sum_c K_c = K$. We refer to the BS of user $k \in \{1,\ldots,K\}$ by b_k. Each user has N_k antennas. The channel between the kth user and the BS in cell j is denoted by $\boldsymbol{H}_{k,j}^{\mathrm{H}} \in \mathbb{C}^{M_j \times N_k}$. We consider zero-mean white Gaussian noise $\boldsymbol{n}_k \in \mathbb{C}^{N_k \times 1}$ with distribution $\mathcal{CN}(0, \sigma_n^2 \boldsymbol{I})$ at the kth user.

We assume independent unity-power transmit symbols $\mathbf{s}_c = [\mathbf{s}_{K_{1:c-1}+1}^{\mathrm{T}} \ldots \mathbf{s}_{K_{1:c}}^{\mathrm{T}}]^{\mathrm{T}}$, i.e., $\mathbb{E}[\mathbf{s}_c \mathbf{s}_c^{\mathrm{H}}] = \boldsymbol{I}$, where $\mathbf{s}_k \in \mathbb{C}^{d_k \times 1}$ is the data vector to be transmitted to the kth user, with d_k being the number of streams allowed by user k and $K_{1:c} = \sum_{i=1}^{c} K_i$. The latter is transmitted using the transmit filtering matrix $\boldsymbol{\mathcal{G}}_c = [\boldsymbol{\mathcal{G}}_{K_{1:c-1}+1} \ldots \boldsymbol{\mathcal{G}}_{K_{1:c}}] \in \mathbb{C}^{M_c \times N_c}$, with $\boldsymbol{\mathcal{G}}_k = p_k^{1/2} \boldsymbol{G}_k$, $\boldsymbol{G}_k$ being the beamforming matrix, p_k is non-negative downlink power allocation of user k and $N_c = \sum_{k:b_k=c} d_k$ is the total number of streams in cell c. Each cell is constrained with $P_{\max,c}$, i.e., the total transmit power in c is limitted such that $\sum_{k:b_k=c} p_k \leq P_{\max,c}$. The received signal at user k in cell b_k is

$$\boldsymbol{y}_k = \underbrace{\boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_k\boldsymbol{s}_k}_{\text{signal}} + \underbrace{\sum_{\substack{i\neq k\\ b_i=b_k}} \boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_i\boldsymbol{s}_i}_{\text{intracel interf.}} + \underbrace{\sum_{j\neq b_k}\sum_{i:b_i=j} \boldsymbol{H}_{k,j}\boldsymbol{\mathcal{G}}_i\boldsymbol{s}_i}_{\text{intercell interf.}} + \boldsymbol{n}_k \tag{1}$$

Similarly, the receive filtering matrix for each user k is defined as $\boldsymbol{\mathcal{F}}_k^{\mathrm{H}} = p_k^{-1/2}\boldsymbol{F}_k^{\mathrm{H}} \in \mathbb{C}^{d_k\times N_k}$, composed of beamforming matrix $\boldsymbol{F}_k^{\mathrm{H}} \in \mathbb{C}^{d_k\times N_k}$. The received filter output is $\hat{\boldsymbol{s}}_k = \boldsymbol{\mathcal{F}}_k^{\mathrm{H}}\boldsymbol{y}_k$.

III. Problem Formulation

In this work, we aim to solve the weighted user-rate max-min optimization problem under per cell total transmit power constraint, i.e., the user rate balancing problem expressed as follows

$$\max_{\boldsymbol{G},p}\min_k \; r_k/r_k^{\circ} \quad \text{s.t.} \sum_{k:b_k=c} p_k \leq P_{\max,c}, 1\leq c\leq C \tag{2}$$

where r_k is the kth user-rate

$$r_k = \ln\det\Big(\boldsymbol{I} + \boldsymbol{R}_{\overline{k}}^{-1}\boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_k\boldsymbol{\mathcal{G}}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}^{\mathrm{H}}\Big) = \ln\det\Big(\boldsymbol{R}_{\overline{k}}^{-1}\boldsymbol{R}_k\Big), \tag{3}$$

$$\boldsymbol{R}_{\overline{k}} = \sigma_n^2\boldsymbol{I} + \sum_{l\neq k}\boldsymbol{H}_{k,b_l}\boldsymbol{\mathcal{G}}_l\boldsymbol{\mathcal{G}}_l^{\mathrm{H}}\boldsymbol{H}_{k,b_l}^{\mathrm{H}}, \tag{4}$$

$$\boldsymbol{R}_k = \boldsymbol{R}_{\overline{k}} + \boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_k\boldsymbol{\mathcal{G}}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}^{\mathrm{H}}, \tag{5}$$

$\boldsymbol{R}_{\overline{k}}$ and $\boldsymbol{R}_k$ are the interference plus noise and total received signal covariances, and r_k° is the rate priority (weight) for user k. However, the problem presented in (2) is complex and can not be solved directly.

Lemma 1. *The rate of user k in (3) can also be represented as [12]*

$$r_k = \max_{\boldsymbol{W}_k,\boldsymbol{\mathcal{F}}_k}\big[\ln\det\big(\boldsymbol{W}_k\big) - \mathrm{tr}\big(\boldsymbol{W}_k\mathbf{E}_k\big) + d_k\big]. \tag{6}$$

where

$$\begin{aligned}\mathbf{E}_k &= \mathbb{E}\Big[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^{\mathrm{H}}\Big]\\ &= (\boldsymbol{I} - \boldsymbol{\mathcal{F}}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_k)(\boldsymbol{I} - \boldsymbol{\mathcal{F}}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_k)^{\mathrm{H}}\\ &\quad + \sum_{l\neq k}\boldsymbol{\mathcal{F}}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_l}\boldsymbol{\mathcal{G}}_l\boldsymbol{\mathcal{G}}_l^{\mathrm{H}}\boldsymbol{H}_{k,b_l}^{\mathrm{H}}\boldsymbol{\mathcal{F}}_k + \sigma_n^2\boldsymbol{\mathcal{F}}_k^{\mathrm{H}}\boldsymbol{\mathcal{F}}_k\end{aligned} \tag{7}$$

is the kth-user downlink MSE matrix between the decision variable $\hat{\mathbf{s}}_k$ and the transmit signal $\mathbf{s}_k$, and $\{\boldsymbol{W}_k\}_{1\leq k\leq K}$ are auxiliary weight matrix variables with optimal solution $\boldsymbol{W}_k^{\mathrm{opt}} = \big[\mathbf{E}_k\big]^{-1}$ and the optimal receivers are

$$\boldsymbol{\mathcal{F}}_k = \boldsymbol{R}_k^{-1}\boldsymbol{H}_{k,b_k}\boldsymbol{\mathcal{G}}_k. \tag{8}$$

Now consider both (2) and (6), and let us introduce $\xi_k = \ln\det\big(\boldsymbol{W}_k\big) + d_k - r_k^{\triangle}$, the WMSE requirement, with target rate $r_k^{\triangle}$. Assume that we shall be able to concoct an optimization algorithm that ensures that at all times and for all users the matrix-weighted MSE (WMSE) satisfies $\epsilon_{w,k} = \mathrm{tr}\big(\boldsymbol{W}_k\mathbf{E}_k\big) \leq d_k$ and $\ln\det\big(\boldsymbol{W}_k\big) \geq r_k^{\triangle}$ or hence $\xi_k \geq d_k$. This leads $\forall k$ to

$$\frac{\epsilon_{w,k}}{\xi_k} \leq 1 \iff \ln\det\big(\boldsymbol{W}_k\big) + d_k - \mathrm{tr}\big(\boldsymbol{W}_k\mathbf{E}_k\big) \geq r_k^{\triangle} \overset{(a)}{\Longrightarrow} r_k/r_k^{\triangle} \geq 1 \tag{9}$$

where (a) follows from (6). To get to (9), what we can exploit in (2) is a scale factor t that can be chosen freely in the rate weights r_k° in (2). We shall take $t = \min_k r_k/r_k^{\circ}$, which allows to transform the rate weights r_k° into target rates $r_k^{\triangle} = tr_k^{\circ}$, and at the same time allows to interpret the WMSE weights ξ_k as target WMSE values.

Doing so, the initial rate balancing optimization problem (2) can be transformed into a matrix-weighted MSE balancing problem expressed as follows

$$\min_{\boldsymbol{G},p,\boldsymbol{\mathcal{F}}}\max_k \; \epsilon_{w,k}/\xi_k \quad \text{s.t.} \sum_{k:b_k=c} p_k \leq P_{\max,c}, 1\leq c\leq C \tag{10}$$

which needs to be complemented with an outer loop in which $\boldsymbol{W}_k = \big(\mathbf{E}_k\big)^{-1}$, $t = \min_k r_k/r_k^{\circ}$, $r_k^{\triangle} = tr_k^{\circ}$ and $\xi_k = d_k + r_k - r_k^{\triangle}$ get updated. The problem in (10) is still difficult to be handled directly.

IV. The Weighted User-MSE Optimization

In this section, the problem (10) with respect to the matrix weighted user-MSE is studied. The per user matrix weighted MSE (WMSE) can be expressed as follows

$$\begin{aligned}\epsilon_{w,k} &= \mathrm{tr}\big(\boldsymbol{W}_k\mathbf{E}_k\big)\\ &= \mathrm{tr}\big(\boldsymbol{W}_k\big) - \mathrm{tr}\big(\boldsymbol{W}_k\boldsymbol{G}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}^{\mathrm{H}}\boldsymbol{F}_k\big) - \mathrm{tr}\big(\boldsymbol{W}_k\boldsymbol{F}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}\boldsymbol{G}_k\big)\\ &\quad + p_k^{-1}\sum_{l=1}^{K} p_l\mathrm{tr}\big(\boldsymbol{W}_k\boldsymbol{F}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_l}\boldsymbol{G}_l\boldsymbol{G}_l^{\mathrm{H}}\boldsymbol{H}_{k,b_l}^{\mathrm{H}}\boldsymbol{F}_k\big)\\ &\quad + \sigma_n^2 p_k^{-1}\mathrm{tr}\big(\boldsymbol{W}_k\boldsymbol{F}_k^{\mathrm{H}}\boldsymbol{F}_k\big).\end{aligned} \tag{11}$$

Define the diagonal matrix $\boldsymbol{D}$ of signal WMSE contributions

$$\begin{aligned}[\boldsymbol{D}]_{ii} &= \mathrm{tr}\big(\boldsymbol{W}_i\big) - \mathrm{tr}\big(\boldsymbol{W}_i\boldsymbol{G}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_i}^{\mathrm{H}}\boldsymbol{F}_i\big) - \mathrm{tr}\big(\boldsymbol{W}_i\boldsymbol{F}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_i}\boldsymbol{G}_i\big)\\ &\quad + \mathrm{tr}\big(\boldsymbol{W}_i\boldsymbol{F}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_i}\boldsymbol{G}_i\boldsymbol{G}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_i}^{\mathrm{H}}\boldsymbol{F}_i\big),\end{aligned} \tag{12}$$

and the matrix of weighted interference powers

$$[\boldsymbol{\Psi}]_{ij} = \begin{cases}\mathrm{tr}\big(\boldsymbol{W}_i\boldsymbol{F}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_j}\boldsymbol{G}_j\boldsymbol{G}_j^{\mathrm{H}}\boldsymbol{H}_{i,b_j}^{\mathrm{H}}\boldsymbol{F}_i\big), & i\neq j\\ 0, & i=j.\end{cases}$$

We can rewrite (11) as, with $\boldsymbol{p} = [p_1\cdots p_K]^{\mathrm{T}}$

$$\epsilon_{w,i} = [\boldsymbol{D}]_{ii} + p_i^{-1}[\boldsymbol{\Psi}\boldsymbol{p}]_i + \sigma_n^2 p_i^{-1}\mathrm{tr}\big(\boldsymbol{W}_i\boldsymbol{F}_i^{\mathrm{H}}\boldsymbol{F}_i\big) \tag{13}$$

Collecting all user WMSEs in a vector $\boldsymbol{\epsilon}_w = \mathrm{diag}(\epsilon_{w,1},\ldots,\epsilon_{w,K})$, we get

$$\boldsymbol{\epsilon}_w\boldsymbol{1}_K = \mathrm{diag}(\boldsymbol{p})^{-1}\left[(\boldsymbol{D}+\boldsymbol{\Psi})\mathrm{diag}(\boldsymbol{p})\boldsymbol{1}_K + \sigma_n^2\boldsymbol{A}\boldsymbol{1}_K\right] \tag{14}$$

where the diagonal matrix $\boldsymbol{A}$ is defined as

$$[\boldsymbol{A}]_{ii} = \mathrm{tr}\big(\boldsymbol{W}_i\boldsymbol{F}_i^{\mathrm{H}}\boldsymbol{F}_i\big).$$

By multiplying both sides of (14) with $\mathrm{diag}(\boldsymbol{p})$, we get

$$\boldsymbol{\epsilon}_w\boldsymbol{p} = (\boldsymbol{D}+\boldsymbol{\Psi})\boldsymbol{p} + \sigma_n^2\boldsymbol{A}\boldsymbol{1}_K. \tag{15}$$

Let $\boldsymbol{\xi} = \mathrm{diag}(\xi_1,\ldots,\xi_K)$, then

$$\boldsymbol{\xi}^{-1}\boldsymbol{\epsilon}_w\,\boldsymbol{p} = \boldsymbol{\xi}^{-1}(\boldsymbol{D}+\boldsymbol{\Psi})\boldsymbol{p} + \sigma_n^2\boldsymbol{\xi}^{-1}\boldsymbol{A}\boldsymbol{1}_K. \tag{16}$$

Actually, problem (10) always has a global minimizer $\boldsymbol{p}$ characterized by the equality $\boldsymbol{\xi}^{-1}\boldsymbol{\epsilon}_w(\boldsymbol{p}) = \Delta\boldsymbol{I}$, i.e.,

$$\Delta\,\boldsymbol{p} = \boldsymbol{\xi}^{-1}(\boldsymbol{D}+\boldsymbol{\Psi})\boldsymbol{p} + \sigma_n^2\boldsymbol{\xi}^{-1}\boldsymbol{A}\boldsymbol{1}_K. \tag{17}$$

Now, consider the following problem

$$\max_{\boldsymbol{G},p,\boldsymbol{\mathcal{F}}}\min_k \; r_k/r_k^{\circ} \quad \text{s.t.} \sum_{c=1}^{C}\theta_c\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} \leq \sum_{c=1}^{C}\theta_c P_{\max,c} \tag{18}$$

where $\boldsymbol{c}_c$ is a column vector with $\boldsymbol{c}_c(j) = 1$ for $K_{1:c-1}+1 \leq j \leq K_{1:c}$, and 0 elsewhere. This problem formulation is a relaxation of (2), and $\boldsymbol{\theta} = [\theta_1 \cdots \theta_C]^{\mathrm{T}}$can be interpreted as the weights on the individual power constraints in the relaxed problem. The power constraint in (18) can be interpreted as a single weighted power constraint

$$(\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}})\,\boldsymbol{p} \leq \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max} \tag{19}$$

with $\boldsymbol{C}_C = [\boldsymbol{c}_1 \cdots \boldsymbol{c}_C] \in \mathbb{R}_+^{K_{1:C}\times C}$ and $\boldsymbol{p}_{\max} = [P_{\max,1} \cdots P_{\max,C}]^{\mathrm{T}}$. Reparameterize $\boldsymbol{p} = \frac{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}}\boldsymbol{p}'}\boldsymbol{p}'$ where now $\boldsymbol{p}'$ is unconstrained, which allows us to write (17) as follows (rewriting $\boldsymbol{p}'$ as $\boldsymbol{p}$)

$$\Delta\,\boldsymbol{p} = \boldsymbol{\Lambda}\,\boldsymbol{p} \text{ with } \boldsymbol{\Lambda} = \boldsymbol{\xi}^{-1}(\boldsymbol{D}+\boldsymbol{\Psi}) + \frac{\sigma_n^2}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}\boldsymbol{\xi}^{-1}\boldsymbol{A}\,\mathbf{1}_K\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}}. \tag{20}$$

Now with (20), the WMSE balancing problem of (10) becomes

$$\min_{\boldsymbol{p}}\max_k \frac{\epsilon_{w,k}}{\xi_k} = \min_{\boldsymbol{p}}\max_k \frac{[\boldsymbol{\Lambda}\,\boldsymbol{p}]_k}{p_k} \tag{21}$$

According to the Collatz–Wielandt formula [13, Chapter 8], the above expression corresponds to the Perron-Frobenius (maximal) eigenvalue Δ of $\boldsymbol{\Lambda}$ and the optimal $\boldsymbol{p}$ is the corresponding Perron-Frobenius (right) eigenvector

$$\boldsymbol{\Lambda}\boldsymbol{p} = \Delta\,\boldsymbol{p}. \tag{22}$$

Note that this implies the equality $\boldsymbol{\xi}^{-1}\boldsymbol{\epsilon}_w = \Delta\,\boldsymbol{I}$ as announced in (17).

V. Algorithmic Solution via Lagrangian Duality

A. Algorithm

The max-min weighted user rate optimization problem (2) can be reformulated as

$$\min_{t,\boldsymbol{G},\boldsymbol{p}} -t$$
$$\text{s.t. } t\,r_k^{\circ} - r_k \leq 0,\ \boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c} \leq 0\,. \tag{23}$$

Introducing Lagrange multipliers to augment the cost function with the constraints leads to the Lagrangian

$$\max_{\lambda',\mu}\min_{t,\boldsymbol{G},\boldsymbol{p}} \mathcal{L}$$
$$\mathcal{L} = -t + \sum_k \lambda_k'(t\,r_k^{\circ} - r_k) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \tag{24}$$

Integrating the result (6), we get a modified Lagrangian

$$\max_{\lambda',\mu}\min_{t,\boldsymbol{G},\boldsymbol{p},\boldsymbol{\mathcal{F}},\boldsymbol{W}} \mathcal{L}$$
$$\mathcal{L} = -t + \sum_k \lambda_k'(\mathrm{tr}(\boldsymbol{W}_k\mathbf{E}_k) - \xi_k) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \tag{25}$$

From (18), we get $\mu_c = \mu\theta_c$. Introducing $\lambda_k = \lambda_k'\xi_k$, we can rewrite (with some abuse of notation since actually $\min_{\boldsymbol{W}}$ continues to apply to $\mathrm{tr}(\boldsymbol{W}_k\mathbf{E}_k) - \xi_k(\boldsymbol{W}_k)$)

$$\max_{\lambda,\mu}\min_{t,\boldsymbol{G},\boldsymbol{p},\boldsymbol{F},\boldsymbol{W}} \mathcal{L}$$
$$\mathcal{L} = -t + \sum_k \lambda_k(\frac{\mathrm{tr}(\boldsymbol{W}_k\mathbf{E}_k)}{\xi_k} - 1) + \mu\sum_c \theta_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \tag{26}$$

We shall solve this saddlepoint condition for $\mathcal{L}$ by alternating optimization. As far as the dependence on $\lambda, \boldsymbol{G}, \boldsymbol{p}, \boldsymbol{\mathcal{F}}$ is concerned, we have (omitting the power constraint)

$$\max_{\lambda}\min_{\boldsymbol{G},\boldsymbol{p},\boldsymbol{\mathcal{F}}} \sum_k \lambda_k\,\frac{\mathrm{tr}(\boldsymbol{W}_k\mathbf{E}_k)}{\xi_k} \tag{27}$$

which is of the form Weighted Sum MSE (WSMSE). Optimizing w.r.t. Rxs $\boldsymbol{\mathcal{F}}_k$ leads to the MMSE solution mentioned in Lemma 1. To optimizing w.r.t. the Txs $\boldsymbol{\mathcal{G}}_k$, we can follow the approach in [14], which is based on [15], but needs to be adapted to a weighted sum power constraint. We get a shorter derivation by following [16]. To that end, consider a reparameterization of the Tx filters to inherently satisfy the power constraint (see (19) where $p_i = \mathrm{tr}\{\boldsymbol{\mathcal{G}}_i^{\mathrm{H}}\boldsymbol{\mathcal{G}}_i\}$) :

$$\boldsymbol{\mathcal{G}}_k = \sqrt{\frac{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}{\sum_i \theta_{b_i}\mathrm{tr}\{\mathbf{G}_i^{\mathrm{H}}\mathbf{G}_i\}}}\,\mathbf{G}_k\,. \tag{28}$$

involving a unique system-wide scale factor (and note $\mathbf{G}_k \neq \boldsymbol{G}_k$). Introducing (28) directly into (27) does not lead to a quadratic criterion. However, reinterpreting the WSMSE (27) as a weighted sum rate via Lemma 1, we get

$$\mathrm{WSR} = \sum_k \frac{\lambda_k}{\xi_k}\,\ln\det(\boldsymbol{R}_{\overline{k}}^{-1}\boldsymbol{R}_k) \tag{29}$$

with $\boldsymbol{R}_{\overline{k}}$ as in (4) but with $\boldsymbol{\mathcal{G}}_i$ replaced by $\mathbf{G}_i$ and with the noise covariance term replaced by

$$\frac{\sum_i \theta_{b_i}\mathrm{tr}\{\mathbf{G}_i^{\mathrm{H}}\mathbf{G}_i\}}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}\Sigma_k \tag{30}$$

where in fact $\Sigma_k = \sigma_n^2\boldsymbol{I}$. Using $\partial\ln\det(\boldsymbol{A}) = \mathrm{tr}\{\boldsymbol{A}^{-1}\,\partial\boldsymbol{A}\}$ and e.g. $(\boldsymbol{R}_{\overline{i}}^{-1}\boldsymbol{R}_i)^{-1}\boldsymbol{R}_{\overline{i}}^{-1} = \boldsymbol{R}_i^{-1}$, we get

$$\begin{aligned}
&\tfrac{\partial\mathrm{WSR}}{\partial\mathbf{G}_k^*} = 0 = \tfrac{\lambda_k}{\xi_k}\boldsymbol{H}_{k,b_k}^{\mathrm{H}}\boldsymbol{R}_k^{-1}\boldsymbol{H}_{k,b_k}\mathbf{G}_k \\
&-\big(\textstyle\sum_{i\neq k}\tfrac{\lambda_i}{\xi_i}\boldsymbol{H}_{i,b_k}^{\mathrm{H}}\boldsymbol{R}_{\overline{i}}^{-1}\boldsymbol{H}_{i,b_k}\mathbf{G}_i\mathbf{G}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_k}^{\mathrm{H}}\boldsymbol{R}_i^{-1}\boldsymbol{H}_{i,b_k}\big)\,\mathbf{G}_k \\
&-\big(\textstyle\sum_{i=1}^{K}\tfrac{\lambda_i}{\xi_i}\mathrm{tr}\{\Sigma_i\boldsymbol{R}_{\overline{i}}^{-1}\boldsymbol{H}_{i,b_k}\mathbf{G}_i\mathbf{G}_i^{\mathrm{H}}\boldsymbol{H}_{i,b_k}^{\mathrm{H}}\boldsymbol{R}_i^{-1}\}\big)\,\theta_{b_k}\mathbf{G}_k
\end{aligned} \tag{31}$$

Now if we note that $\boldsymbol{\mathcal{F}}_i = \boldsymbol{R}_i^{-1}\boldsymbol{H}_{i,b_i}\mathbf{G}_i = \boldsymbol{R}_{\overline{i}}^{-1}\boldsymbol{H}_{i,b_i}\mathbf{G}_i\boldsymbol{E}_i$, $\boldsymbol{W}_i = \boldsymbol{E}_i^{-1}$ and $\boldsymbol{R}_k^{-1}\boldsymbol{H}_{k,b_k}\mathbf{G}_k = \boldsymbol{\mathcal{F}}_k = \boldsymbol{\mathcal{F}}_k\boldsymbol{W}_k\boldsymbol{E}_k = \boldsymbol{\mathcal{F}}_k\boldsymbol{W}_k(\boldsymbol{I} - \boldsymbol{\mathcal{F}}_k^{\mathrm{H}}\boldsymbol{H}_{k,b_k}\mathbf{G}_k)$, then (31) leads to

$$\begin{aligned}
\mathbf{G}_k = &\Big(\sum_{l=1}^{K}\boldsymbol{H}_{l,b_k}^{\mathrm{H}}\boldsymbol{\mathcal{F}}_l\boldsymbol{W}_l'\boldsymbol{\mathcal{F}}_l^{\mathrm{H}}\boldsymbol{H}_{l,b_k} + \sigma_n^2\theta_{b_k}\frac{\sum_l \mathrm{tr}(\boldsymbol{W}_l'\boldsymbol{\mathcal{F}}_l^{\mathrm{H}}\boldsymbol{\mathcal{F}}_l)}{\sum_c \theta_c P_{\max,c}}\boldsymbol{I}\Big)^{-1} \\
&\times \boldsymbol{H}_{k,b_k}^{\mathrm{H}}\boldsymbol{\mathcal{F}}_k\boldsymbol{W}_k'
\end{aligned}$$
$$\boldsymbol{\mathcal{G}}_k = \sqrt{p_k}\,\boldsymbol{G}_k\,, \boldsymbol{G}_k = \frac{1}{\sqrt{\mathrm{tr}\{\mathbf{G}_k^{\mathrm{H}}\mathbf{G}_k\}}}\,\mathbf{G}_k \tag{32}$$

where $\boldsymbol{W}_k' = \lambda_k/\xi_k\boldsymbol{W}_k$, and accounting for the fact that the user powers are actually optimized by the Perron-Frobenius theory. Note that this result for $\boldsymbol{G}_k$ would also be obtained by direct optimization of (26), but we needed the extra development above to get the expression for the Lagrange multiplier μ. The Perron-Frobenius theory also allows for the optimization of the Lagrange multipliers λ_k. With (21), we can reformulate (27) as

$$\Delta = \max_{\lambda:\sum_k\lambda_k=1}\min_{\boldsymbol{p}}\sum_k \lambda_k\,\frac{[\boldsymbol{\Lambda}\,\boldsymbol{p}]_k}{p_k} \tag{33}$$

which is the Donsker–Varadhan–Friedland formula [13, Chapter 8] for the Perron Frobenius eigenvalue of Λ. A related formula is the Rayleigh quotient

$$\Delta = \max_{\boldsymbol{q}}\min_{\boldsymbol{p}}\frac{\boldsymbol{q}^T\boldsymbol{\Lambda}\,\boldsymbol{p}}{\boldsymbol{q}^T\boldsymbol{p}} \tag{34}$$

TABLE I: User Rate Balancing Algorithm

1. initialize: $\boldsymbol{G}_k^{(0,0)} = (\boldsymbol{I}_{d_k} : \boldsymbol{0})^{\mathrm{T}}$, $\boldsymbol{p}_k^{(0,0)} = \boldsymbol{q}_k^{(0,0)} = \frac{P_{\max,c}}{K_c}$, $m = n = 0$ and fix $n_{\max}, m_{\max}$ and $r_k^{\circ(0)}$, initialize $\boldsymbol{W}_k^{(0)} = \boldsymbol{I}_{d_k}$ and $\xi_k^{(0)} = d_k$
2. initialize $\boldsymbol{F}_k^{(0,0)}$ in $\boldsymbol{\mathcal{F}}_k^{(0,0)} = p_k^{(0,0)-1/2}\boldsymbol{F}_k$ from (8)
3. **repeat**
   - 3.1. $m \leftarrow m+1$
   - 3.2. **repeat**
     $n \leftarrow n+1$
     - *i* update $\boldsymbol{G}_k$ in $\boldsymbol{\mathcal{G}}_k = p_k^{1/2}\boldsymbol{G}_k$ from (32)
     - *ii* update $\boldsymbol{F}_k$ in $\boldsymbol{\mathcal{F}}_k = p_k^{-1/2}\boldsymbol{F}_k$ from (8)
     - *iii* update $\boldsymbol{p}$ and $\boldsymbol{q}$ using Table II
   - 3.3 **until** required accuracy is reached or $n \geq n_{\max}$
   - 3.4 compute $\boldsymbol{E}_k^{(m)}$ and update $\boldsymbol{W}_k^{(m)} = (\boldsymbol{E}_k^{(m)})^{-1}$
   - 3.5 determine $t = \min_k \frac{r_k^{(m)}}{r_{k_c}^{\circ(m-1)}}$, $r_k^{\circ(m)} = t\, r_k^{\circ(m-1)}$, and $\xi_k^{(m)} = d_k + r_k^{(m)} - r_k^{\circ(m)}$
   - 3.6 set $n \leftarrow 0$ and set $(.)^{(n_{\max},m-1)} \rightarrow (.)^{(0,m)}$ in order to re-enter the inner loop
4. **until** required accuracy is reached or $m \geq m_{\max}$

TABLE II: Power Distribution Optimization

1. for given $\boldsymbol{\theta}^{(0)}$, $\alpha = \frac{\alpha_0}{\sum_c P_{\max,c}}$, $\mathcal{C} = \{\boldsymbol{\theta} \in \mathbb{R}^C_{\geq 0} : \boldsymbol{\theta} \ngeq 0, \boldsymbol{1}_C^{\mathrm{T}}\boldsymbol{\theta} = 1\}$
2. **repeat**
   - 2.1. compute $\boldsymbol{\Lambda}(\boldsymbol{\theta})$, update $\boldsymbol{p}$ and $\boldsymbol{q}$ as right and left Perron Frobenius eigen vectors of $\boldsymbol{\Lambda}$
   - 2.2. update $\boldsymbol{\theta}$ using the subgradient projection method, [2] :
     $$\boldsymbol{\theta}^{(i+1)} = \mathcal{P}_c\{\boldsymbol{\theta}^{(i)} - \alpha\hat{\boldsymbol{g}}(\boldsymbol{p}^{(i)})\},$$
     where $\hat{\boldsymbol{g}}(\boldsymbol{p}^{(i)}) = \boldsymbol{p}_{\max} - \boldsymbol{C}_C^{\mathrm{T}}\boldsymbol{p}^{(i)}$ and $\mathcal{P}_{\mathcal{C}}$ is the projection operator onto $\mathcal{C}$.
   - 2.3. $i \leftarrow i+1$
3. **until** required accuracy is reached

where $\boldsymbol{p}$, $\boldsymbol{q}$ are the right and left Perron Frobenius eigenvectors. Comparing (34) to (33), then apart from normalization factors, we get $\lambda_k/p_k = q_k$ or hence $\lambda_k = p_k q_k$.

The proposed optimization framework is summarized in Table I; Table II represents the power optimization algorithm ensuring the per cell power constraints. Superscripts refer to iteration numbers. The algorithm in Table I is based on a double loop. The inner loop solves the WMSE balancing problem in (10) whereas the outer loop iteratively transforms the WMSE balancing problem into the original rate balancing problem in (2).

B. Proof of Convergence

In case the rate weights r_k° would not satisfy $r_k \geq r_k^\circ$, this issue will be rectified by the scale factor t after one iteration (of the outer loop). It can be shown that $t = \min_k \frac{r_k^{(m)}}{r_k^{\circ(m-1)}} \geq 1$. By contradiction, if this was not the case, it can be shown to lead to $\frac{\mathrm{tr}\left(\boldsymbol{W}_k^{(m-1)}\mathbf{E}_k^{(m)}\right)}{\xi_k^{(m-1)}} > 1,\ \forall k$ and hence $\Delta^{(m)} > 1$. But we have

$$\begin{aligned}\Delta^{(m)} &= \frac{\mathrm{tr}\left(\boldsymbol{W}_k^{(m-1)}\mathbf{E}_k^{(m)}\right)}{\xi_k^{(m-1)}},\ \forall k, = \max_k \frac{\mathrm{tr}\left(\boldsymbol{W}_k^{(m-1)}\mathbf{E}_k^{(m)}\right)}{\xi_k^{(m-1)}} \\ &\overset{(a)}{<} \max_k \frac{\mathrm{tr}\left(\boldsymbol{W}_k^{(m-1)}\mathbf{E}_k^{(m-1)}\right)}{\xi_k^{(m-1)}} = \max_k \frac{d_k}{\xi_k^{(m-1)}} \overset{(b)}{<} 1\,.\end{aligned} \tag{35}$$

Let $\mathbf{E} = \{\mathbf{E}_k, k = 1, \ldots, K\}$ and $f^{(m)}(\mathbf{E}) = \max_k \frac{\mathrm{tr}\left(\boldsymbol{W}_k^{(m-1)}\mathbf{E}_k\right)}{\xi_k^{(m-1)}}$. Then (a) is due to the fact that the algorithm in fact performs alternating minimization of $f^{(m)}(\mathbf{E})$ w.r.t. $\{\boldsymbol{G}_c, \boldsymbol{F}_c\}$, $\tilde{\boldsymbol{q}}$ and hence will lead to $f^{(m)}(\mathbf{E}^{(m)}) < f^{(m)}(\mathbf{E}^{(m-1)})$. On the other hand, (b) is due to $\xi_k^{(m-1)} = d_k + r_k^{(m-1)} - r_k^{\circ(m-1)} > d_k$, for $m \geq 3$. Hence, $t \geq 1$. Of course, during the convergence $t > 1$. The increasing rate targets $\{r_k^{\circ(m)}\}$ constantly catch up with the increasing rates $\{r_k^{(m)}\}$. Now, the rates are upper bounded by the single user MIMO rates (using all power), and hence the rates will converge and the sequence t will converge to 1. That means that for at least one user k, $r_k^{(\infty)} = r_k^{\circ(\infty)}$. The question is whether this will be the case for all users, as is required for rate balancing. Now, the WMSE balancing leads at every outer iteration m to $\frac{\mathrm{tr}\left(\boldsymbol{W}_k^{(m-1)}\mathbf{E}_k^{(m)}\right)}{\xi_k^{(m-1)}} = \Delta^{(m)}, \forall k$. At convergence, this becomes $\frac{d_k}{\xi_k^{(\infty)}} = \Delta^{(\infty)}$ where $\xi_k^{(\infty)} = d_k + r_k^{(\infty)} - r_k^{\circ(\infty)}$. Hence, if we have convergence because for one user k_∞ we arrive at $r_{k_\infty}^{(\infty)} = r_{k_\infty}^{\circ(\infty)}$, then this implies $\Delta^{(\infty)} = 1$ which implies $r_k^{(\infty)} = r_k^{\circ(\infty)}, \forall k$. Hence, the rates will be maximized and balanced.

VI. Simulation results

In this section, we numerically evaluate the performance of the proposed algorithm. The simulations are carried out over normalized flat fading channels, i.e., each element is i.i.d. and normally distributed : starting from i.i.d. $[\boldsymbol{H}_{k,j}]_{mn} \sim \mathcal{CN}(0,1)$. For all simulations, we take 500 channel realisations and $n_{\max} = 20$. The algorithm converges after 4-5 (or 13-15) iterations of m at SNR = 15dB (or 30dB).

In Figure 1 (only), the singular values are modified to a geometric series α^i to control the MIMO channel conditioning, in particular $\alpha = 0.3$. Fig. 1 plots the minimum achieved per user rate using *i)* our max-min user rate approach for equal priorities with total sum-power constraint and per cell power constraints, and *ii)* the user MSE balancing approach [17] w.r.t. the Signal to Noise Ratio (SNR). We observe that our approach outperforms significantly the unweighted MSE balancing optimization. Furthermore, the gap between the achieved balanced rate using per cell power constraints and the one obtained with total sum-power constraint over cells is very tiny.

In Figure 2, which illustrates the difference between the per cell power constraint $P_{\max,c}$ and the total power allocated per cell, i.e., $\boldsymbol{c}_{K_c,c}^{\mathrm{T}}\boldsymbol{p}$, for per cell power contraints and total sum-power constraint, we observe that using Table II we ensure the per cell power constraint with equality, unlike the total sum-power constraint which verifies the total power over cells.

In Figure 3, we illustrate how rate is distributed among users according to their priorities represented by the rate targets r_k°. We can see that, using the min-max weighted MSE approach, the rate is equally distributed between the users with equal user priorities, i.e., $r_k^\circ = r_1^\circ\ \forall k$, whereas with different user priorities, the rate differs from one user to another accordingly.

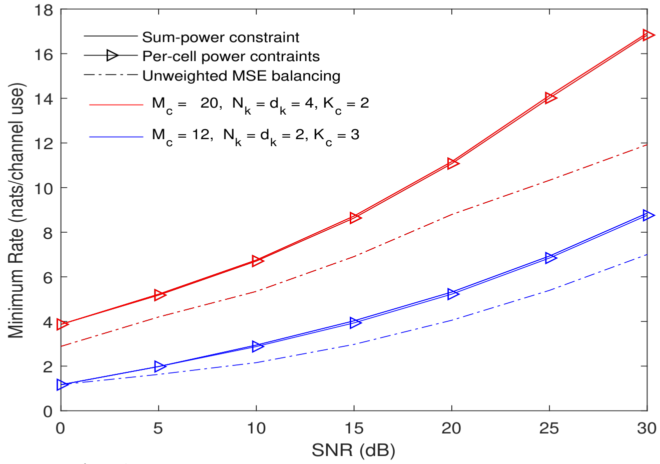

Fig. 1: Minimum rate in the system VS SNR, $C = 2$.

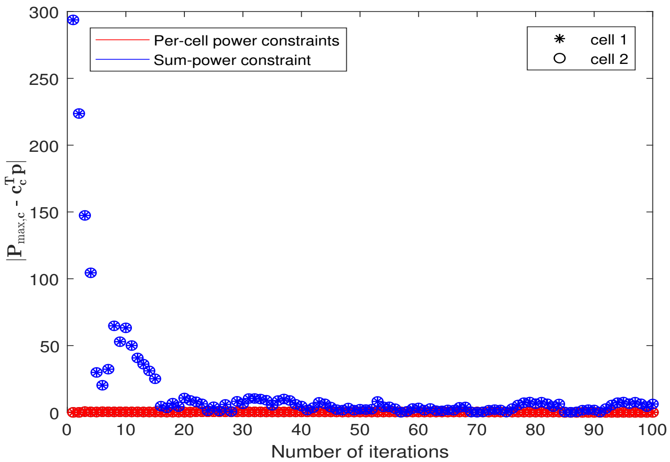

Fig. 2: Difference between the per cell total power and its respective power constraint VS number of iterations: SNR = 30 dB, $C = 2, K_c = 3, M_c = 20, N_k = d_k = 2, P_{\max,1} = P_{\max,2}$.

VII. Conclusions

In this work, we addressed the multiple streams per MIMO user case for which we considered user rate balancing, not stream rate balancing, in a multicell downlink channel. Actually, we optimized the rate distribution over the streams of a user, within the user rate balancing under per cell power constraints. We transformed the max-min rate optimization problem into a min-max weighted MSE optimization problem which itself was shown to be related to a weighted sum MSE minimization via Langrangian duality. Moreover, we reformulated the multiple power constraints as a single weighted constraint satisfying with equality of all power contraints. We provided a comparison between our weighted MSE balancing approach and the min-max unweighted MSE optimization. Simulation results showed that our solution maximizes the minimum rate.

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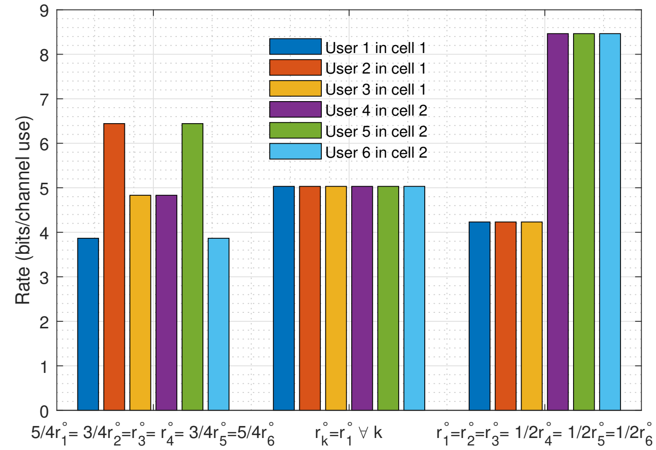

Fig. 3: Rate distribution among users: SNR= 15 dB, $C = 2, K_c = 3, M_c = 12, N_k = d_k = 2$.

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