3D Self-Positioning Algorithm based on Joint DDoA-ToA

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***Abstract*—Thanks to the recent development of antenna design, localization based on the Direction of Arrival (DoA) of the incident signals is becoming more and more promising approach. This paper presents a direction-based positioning algorithm which can be implemented at the mobile device in 3D schemes. As the mobile orientation is unclear and inconstant, an algorithm utilizing the Direction Difference of Arrival (DDoA) among the incident signals is considered. Because of the complexity in computation, we use the Time of Arrival (ToA) as the additional data for position estimation. As a result, the joint DDoA-ToA method is studied. Moreover, an iterative Maximum Likelihood (ML) estimator for position estimation is presented to rectify the final estimates. Simulation results show significant performance improvement compared to other methods.**

***Index Terms* - positioning, ToA, Time of Arrival, DDoA, Direction Difference of Arrival, 3D localization, Maximum Likelihood.**

I. INTRODUCTION

In localization, there are generally two positioning approaches [1]:

- **Self-Positioning:** A signal is transmitted from a base station at a known position and received by a mobile unit whose position has to be determined. The position is calculated at the device to localize itself.
- **Network-Positioning:** A signal is transmitted by a mobile unit whose position has to be determined and received from an base station at a known position. The position is calculated at a unit in the network of base stations.

In direction-based localization, the positioning algorithms are different between self-positioning and network-positioning. In network-positioning, the signals' DoAs are easily estimated and obtained, since the orientation of all the base stations are fixed. Nevertheless, in self-positioning, these DoAs cannot be directly obtained, because the orientation of mobile device is unclear and inconstant. Therefore, a localizing algorithm based on DoAs is not feasible. Consequently, an algorithm using Direction Difference of Arrival (DDoA) is proposed. In this technique, only the difference in directions of arrival of incident waves from a pair of base stations is required (Fig. 1).

Several papers illustrates their researches and results in self-localizing problems. The authors of [2] gives a solution for direction-based self-positioning problem when the tilt of receiver is already given. However, the tilt of the receiver is only expressed by its normal vector, which cannot represent the whole orientation for a solid subject. In [3], the authors propose an position algorithm using the DDoAs by a gradient iterative procedure, but they do not demonstrate how to get the initial point for the procedure, as well as what to do if the procedure does not converge. In [4], a DDoA positioning algorithm is studied. The sensor determines its position by the Visible Light Communications (VLC) emitted from the light-emitting diodes (LEDs) around. Nevertheless, the authors assume that all the LEDs are collinear and the z-coordinate of the sensor is always lowers than the common z-coordinate. These assumptions reduce the complexity for the problem, but also lose the generality for the solution.

Our previous work [5] analyzed a direction-based positioning algorithm at mobile devices in 2D schemes. To make the localization solution realistic, problems in 3D schemes must be considered. However, it is not as straightforward to estimate the position based on DDoA in 3D as it is in 2D. An additional time estimation is hence utilized to support the direction-based solution.

ToA-based localization is already well analyzed in [6]. In this paper, we investigate a hybrid DDoA ToA positioning algorithm. Firstly, the Weighted Least Squares (WLS) method is studied. Afterwards, we propose a Maximum Likelihood (ML) estimator with iterative procedure to robustify the position estimation. Simulation results show substantial improvement in performances compared to the methods using only ToA, in certain conditions.

II. TOA-BASED LOCALIZATION

In this section, we summarize the ToA-based algorithm proposed by [6].

Let t_i be the true propagation time of the signal from the i-th base station to the mobile deivce and $\hat{t}_i$ be its estimate, with i from 1 to N (N is the number of base stations). We have the relation

$$\hat{t}_i = t_i + \tilde{t}_i \tag{1}$$

where $\tilde{t}_i$ is the ToA estimation error and assumed to be Gaussian distributed with zero-mean and variance of $\sigma^2_{ToA,i}$.

As a result, the distance is estimated based on multiplying $\hat{t}_i$ by c (c is the speed of light), denoted by $\hat{d}_i$ is modeled as:

$$\hat{d}_i = d_i + \tilde{d}_i \tag{2}$$

where $d_i = ct_i$ is the true distance and $\tilde{d}_i = c\tilde{t}_i$ is the distance estimation error based on the i-th ToA with zero-mean and variance of $\sigma_i^2 = c^2\sigma_{\mathrm{ToA},i}^2$.

A. Equations

(x, y, z) are the coordinates of the undefined mobile station and $(x_i,\ y_i,\ z_i)$ are the coordinate of the i-th base station, i from 1 to N.

We have the following equation demonstrating the sphere whose center is the i-th base station and radius is d_i, where d_i is the distance from the i-th base station to the mobile device:

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = d_i^2 \tag{3}$$

B. Maximum Likelihood Estimator

In matrix formulation, we denote

$$\hat{\boldsymbol{d}} = \begin{bmatrix} \hat{d}_1 & \hat{d}_2 & \dots & \hat{d}_N \end{bmatrix}^T$$

$$\boldsymbol{f}_d(\boldsymbol{x}) = \begin{bmatrix} \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} \\ \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} \\ \dots \\ \sqrt{(x_N - x)^2 + (y_N - y)^2 + (z_N - z)^2} \end{bmatrix}$$

The measurement vector $\hat{\boldsymbol{d}}$ is Gaussian distributed with mean vector of $\boldsymbol{f}_d$ and covariance matrix $\boldsymbol{C}_d$, we have the probability density function (pdf) [7]:

$$\mathrm{p}(\hat{\boldsymbol{d}}|\boldsymbol{x}) = \frac{(2\pi)^{-N/2}}{|\boldsymbol{C}_d|^{1/2}} \exp\left[\frac{-1}{2}\left(\hat{\boldsymbol{d}} - \boldsymbol{f}_d\right)^T \boldsymbol{C}_d^{-1}\left(\hat{\boldsymbol{d}} - \boldsymbol{f}_d\right)\right] \tag{4}$$

where $\boldsymbol{x} = [x \;\; y \;\; z]$ is the coordinate vector of the mobile device and $\boldsymbol{C}_d = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2)$ is the covariance matrix of ToA-based distance measurements.

Maximizing the pdf in (4) is equivalent to

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \left(\hat{\boldsymbol{d}} - \boldsymbol{f}_d(\boldsymbol{x})\right)^T \boldsymbol{C}_d^{-1}\left(\hat{\boldsymbol{d}} - \boldsymbol{f}_d(\boldsymbol{x})\right) \tag{5}$$

Iterative procedures are applied to obtain the optimized $\hat{\boldsymbol{x}}$. The initialization is the result of Weighted Least Squares (WLS) estimation [6].

III. DDoA-based localization

A. Problem Formulation

Direction-based localization at the mobile device is much more complicated than the one at the base stations, because of the ambiguity of orientation of the mobile device. As a result, the DoA of an incident wave is really challenging to be expressed by azimuth and elevation angles in the true Cartesian coordinate system.

Fig. 2 shows the DoA expression in the true Cartesian coordinate system. The true azimuth angle and elevation angle of the incident wave from the i-th base station are ψ_i and ϑ_i, respectively. However, at the relative coordinate system with regard to the mobile device, the relative azimuth angle

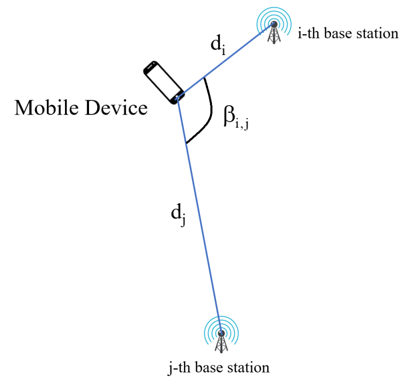


Fig. 1. Localization at mobile device with Direction Difference of Arrival (DDoA)

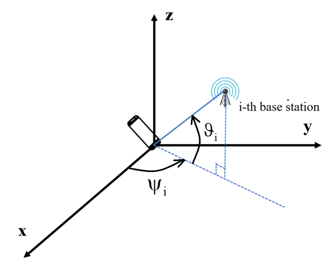


Fig. 2. Azimuth angle and elevation angle of the incident signal from the i-th base station in true Cartesian coordinate system

is φ_i and and elevation angle is θ_i (Fig. 3). As the tilt of the mobile device is undefined, it is likely impossible to compute (ψ_i, ϑ_i) from (φ_i, θ_i). It is essential to find a solution which can estimate the mobile position by (φ_i, θ_i).

We define $\beta_{i,j}$ as the Direction Difference of Arrival (DDoA) between incident waves from i-th base station and j-th base station (Fig. 1), where d_i and d_j are the distance from the mobile device to i-th base station and j-th base station.

B. Computing DDoA from the related DoAs

To calculate $\beta_{i,j}$, we use scalar product of $\overrightarrow{d_i}$ and $\overrightarrow{d_j}$, the vector demonstrating the incident signal from i-th and j-th base station, respectively.

In the reference frame attached to the mobile device, we have the coordinates of the following vectors:

$$\overrightarrow{d_i}^{(r)} = (d_i \cos\theta_i \cos\varphi_i,\ d_i \cos\theta_i \sin\varphi_i,\ d_i \sin\theta_i)$$
$$\overrightarrow{d_j}^{(r)} = (d_j \cos\theta_j \cos\varphi_j,\ d_j \cos\theta_j \sin\varphi_j,\ d_j \sin\theta_j)$$

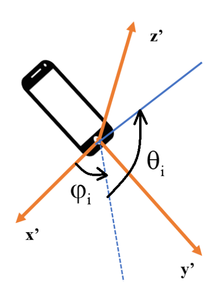


Fig. 3. Azimuth angle and elevation angle of the incident signal from the i-th base station in relative Cartesian coordinate system

Thus, their scalar product is

$$\begin{aligned}
&\overrightarrow{d_i}^{(r)} \bullet \overrightarrow{d_j}^{(r)} \\
&= d_i d_j (\cos\theta_i \cos\varphi_i \cos\theta_j \cos\varphi_j \\
&+ \cos\theta_i \sin\varphi_i \cos\theta_j \sin\varphi_j + \sin\theta_i \sin\theta_j) \\
&= d_i d_j (\cos\theta_i \cos\theta_j \cos(\varphi_j - \varphi_i) + \sin\theta_i \sin\theta_j)
\end{aligned} \tag{6}$$

where d_i and d_j are the length of two vectors $\overrightarrow{d_i}$ and $\overrightarrow{d_j}$, respectively.

The definition of scalar product of two vectors:

$$\overrightarrow{d_i}^{(r)} \bullet \overrightarrow{d_j}^{(r)} = d_i . d_j . \cos\beta_{i,j} \tag{7}$$

Hence, the relation between the DDoA$\beta_{i,j}$ and the DoAs $(\varphi_i, \theta_i), (\varphi_j, \theta_j)$ is

$$\cos\beta_{i,j} = \cos\theta_i \cos\theta_j \cos(\varphi_j - \varphi_j) + \sin\theta_i \sin\theta_j \tag{8}$$

In [8], the authors prove that the DDoA is unchanged when the mobile device rotates, no matter which coordinate system is chosen.

In practice, the estimates of φ_i and θ_i can be expressed by:

$$\hat{\varphi}_i = \varphi_i + \tilde{\varphi}_i \tag{9}$$

$$\hat{\theta}_i = \theta_i + \tilde{\theta}_i \tag{10}$$

The authors of [9] illustrates that when there is Gaussian noise in received signal, $\tilde{\varphi}_i$ and $\tilde{\theta}_i$ are asymptotically independently Gaussian distributed with zero-mean. As a result, we can assume that $\tilde{\varphi}_i$ and $\tilde{\theta}_i$ are independently Gaussian distributed with zero-mean. Their variances are ν_i^2 and μ_i^2, correspondingly.

We denote $\gamma_{i,j} = \cos\beta_{i,j}$. Appendix A illustrates the espected value and variance of $\gamma_{i,j}$. When the estimation errors are small enough, it is considered that $\gamma_{i,j}$ is Gaussian distributed.

C. Estimating position by Weighted Least Squares method

In the true Cartesian coordinate system, we have the coordinates of the vectors $\overrightarrow{d_i}$ and $\overrightarrow{d_j}$ as follows:

$\overrightarrow{d_i} = (x_i - x,\ y_i - y,\ z_i - z)$

$\overrightarrow{d_j} = (x_j - x,\ y_j - y,\ z_j - z)$

The DDoA between the incident waves from i-th base station and j-th base station does not depend on the orientation of the mobile device.

Taking the scalar product of $\overrightarrow{d_i}$ and $\overrightarrow{d_j}$:

$$\begin{aligned}
&\overrightarrow{d_i} \bullet \overrightarrow{d_j} = \\
&x^2 - (x_i + x_j)x + x_i x_j + y^2 - (y_i + y_j)y + y_i y_j + z^2 - (z_i + z_j)z + z_i z_j
\end{aligned} \tag{11}$$

Meanwhile, the definition of scalar product is

$\overrightarrow{d_i} \bullet \overrightarrow{d_j} = d_i d_j \cos\beta_{i,j}$

Thus

$$\begin{aligned}
&x^2 - (x_i + x_j)x + x_i x_j + y^2 - (y_i + y_j)y + y_i y_j + z^2 - (z_i + z_j)z + z_i z_j \\
&= d_i d_j \cos\beta_{i,j}
\end{aligned} \tag{12}$$

Replacing $d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$ and the similar to d_j, we get

$$\begin{aligned}
&x^2 - (x_i + x_j)x + x_i x_j + y^2 - (y_i + y_j)y + y_i y_j + z^2 - (z_i + z_j)z + z_i z_j = \\
&\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}\sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2}\cos\beta_{i,j}
\end{aligned} \tag{13}$$

It is really complicated to solve such an equation like (13) to estimate the mobile position (x, y, z). Consequently, ToA estimation is added to simplify the computation. When d_i and d_j are already estimated by the related ToA, the equation (12) can be utilized for estimating position.

In matrix formulation, we denote

$$\boldsymbol{A} = \begin{bmatrix} -(x_1 + x_2) & -(y_1 + y_2) & -(z_1 + z_2) & 1 \\ -(x_1 + x_3) & -(y_1 + y_3) & -(z_1 + z_3) & 1 \\ \dots & \dots & \dots & \dots \\ -(x_1 + x_N) & -(y_1 + y_N) & -(z_1 + z_N) & 1 \end{bmatrix}$$

$$\boldsymbol{\omega} = \begin{bmatrix} x \\ y \\ z \\ x^2 + y^2 + z^2 \end{bmatrix};\ \hat{\boldsymbol{b}} = \begin{bmatrix} \hat{d}_1 \hat{d}_2 \hat{\gamma}_{1,2} \\ \hat{d}_1 \hat{d}_3 \hat{\gamma}_{1,3} \\ \dots \\ \hat{d}_1 \hat{d}_N \hat{\gamma}_{1,N} \end{bmatrix}$$

where $\hat{d}_i = c\hat{t}_i$ and $\hat{\gamma}_{i,j}$ is estimated by (33).

We have

$$\begin{aligned}
&\hat{d}_i \hat{d}_j \hat{\gamma}_{i,j} = (d_i + \tilde{d}_i)(d_j + \tilde{d}_j)(\gamma_{i,j} + \tilde{\gamma}_{i,j}) \\
&\approx d_i d_j \gamma_{i,j} + \tilde{d}_i d_j \gamma_{i,j} + d_i \tilde{d}_j \gamma_{i,j} + d_i d_j \tilde{\gamma}_{i,j}
\end{aligned} \tag{14}$$

As a result, $\hat{\boldsymbol{b}} = \boldsymbol{b} + \tilde{\boldsymbol{b}}$ where

$$\boldsymbol{b} = \begin{bmatrix} d_1 d_2 \gamma_{1,2} \\ d_1 d_3 \gamma_{1,3} \\ \dots \\ d_1 d_N \gamma_{1,N} \end{bmatrix};$$

$$\tilde{\boldsymbol{b}} = \begin{bmatrix} \tilde{d}_1 d_2 \gamma_{1,2} + d_1 \tilde{d}_2 \gamma_{1,2} + d_1 d_2 \tilde{\gamma}_{1,2} \\ \tilde{d}_1 d_3 \gamma_{1,3} + d_1 \tilde{d}_3 \gamma_{1,3} + d_1 d_3 \tilde{\gamma}_{1,3} \\ \dots \\ \tilde{d}_1 N \gamma_{1,N} + d_1 \tilde{d}_N \gamma_{1,N} + d_1 d_N \tilde{\gamma}_{1,N} \end{bmatrix}$$

The Weighted Least Square (WLS) cost function

$$J_{WLS} = (\boldsymbol{A}\boldsymbol{\omega} - \boldsymbol{b})^T \boldsymbol{W} (\boldsymbol{A}\boldsymbol{\omega} - \boldsymbol{b}) \tag{15}$$

where $\boldsymbol{W}$ is a symmetric weighting matrix.

We choose $\boldsymbol{W} = \left[\mathrm{E}\left(\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^T\right)\right]^{-1}$ with $\mathrm{E}\left(\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^T\right)$ is expressed by (40).

The WLS estimate of $\boldsymbol{\omega}$ is

$$\hat{\boldsymbol{\omega}} = (\boldsymbol{A}^T \boldsymbol{W} \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{W} \hat{\boldsymbol{b}} \tag{16}$$

The estimated coordinate vector of the mobile device comprises the 3 first elements of $\hat{\boldsymbol{\omega}}$:

$$\hat{\boldsymbol{x}} = \left[[\hat{\boldsymbol{\omega}}]_1 \quad [\hat{\boldsymbol{\omega}}]_2 \quad [\hat{\boldsymbol{\omega}}]_3\right]^T \tag{17}$$

D. Iterative Maximum Likelihood Procedure

We propose an iterative Maximum Likelihood estimator, which uses the positioning result in (17) as its initialization, to optimize the estimation.

Linking the distance to the coordinates, we have:

$$d_i(\boldsymbol{x}) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{18}$$

From the equation (12), we have the relation between the DDoA and the coordinates:

$$\gamma_{i,j}(\boldsymbol{x}) = \cos\beta_{i,j}(\boldsymbol{x}) = \frac{x^2-(x_i+x_j)x+x_ix_j+y^2-(y_i+y_j)y+y_iy_j+z^2-(z_i+z_j)z+z_iz_j}{\sqrt{(x_i-x)^2+(y_i-y)^2+(z_i-z)^2}\sqrt{(x_j-x)^2+(y_j-y)^2+(z_j-z)^2}} \tag{19}$$

with i from 1 to $N-1$ and j from $i+1$ to N.

Then, we denote

$$\hat{\boldsymbol{d}} = \begin{bmatrix} \hat{d}_1 & \hat{d}_2 & \dots & \hat{d}_N \end{bmatrix}^T \tag{20}$$

$$\hat{\boldsymbol{\gamma}} = \begin{bmatrix} \hat{\gamma}_1 & \hat{d}_2 & \dots & \hat{d}_N \end{bmatrix}^T \tag{21}$$

$$\hat{\boldsymbol{r}} = \begin{bmatrix} \hat{\boldsymbol{d}} & \hat{\boldsymbol{\gamma}} \end{bmatrix}^T \tag{22}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2} \\ \sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2} \\ \dots \\ \sqrt{(x-x_N)^2+(y-y_N)^2+(z-z_N)^2} \\ \gamma_{1,1}(\boldsymbol{x}) \\ \gamma_{1,2}(\boldsymbol{x}) \\ \dots \\ \gamma_{1,N}(\boldsymbol{x}) \end{bmatrix} \tag{23}$$

where $\gamma_{i,j}(\boldsymbol{x})$ is defined in (19).

We have $\mathrm{cov}(\hat{\boldsymbol{d}}) = \boldsymbol{C}_{\mathrm{d}}$ and

$$\boldsymbol{C}_\gamma = \mathrm{cov}(\hat{\boldsymbol{\gamma}}) = \begin{bmatrix} s^2_{1,2} & s^2_{1,2,3} & \dots & s^2_{1,2,N} \\ s^2_{1,2,3} & s^2_{1,3} & \dots & s^2_{1,3,N} \\ \dots & \dots & \dots & \dots \\ s^2_{1,2,N} & s^2_{1,3,N} & \dots & s^2_{1,N} \end{bmatrix} \tag{24}$$

where $s^2_{i,j}$ and $s^2_{i,j,l}$ are expressed in the Appendix A.

The covariance matrix of $\hat{\boldsymbol{r}}$

$$\boldsymbol{C}_r = \mathrm{cov}(\hat{\boldsymbol{r}}) = \begin{bmatrix} \boldsymbol{C}_{\mathrm{d}} & \boldsymbol{0}_{N\times(N-1)} \\ \boldsymbol{0}_{(N-1)\times N} & \boldsymbol{C}_\gamma \end{bmatrix} \tag{25}$$

where $\boldsymbol{0}_{a\times b}$ is the null matrix of the size $a\times b$.

Similar to the section II-B, the measurement vector $\hat{\boldsymbol{r}}$ is Gaussian distributed with mean vector of $\boldsymbol{f}$ and covariance matrix $\boldsymbol{C}_r$, we have the probability density function (pdf):

$$\mathrm{p}(\hat{\boldsymbol{r}}|\boldsymbol{x}) = \frac{(2\pi)^{-N/2}}{|\boldsymbol{C}_r|^{1/2}} \exp\left[\frac{-1}{2}(\hat{\boldsymbol{r}}-\boldsymbol{f}_r)^T\boldsymbol{C}_r^{-1}(\hat{\boldsymbol{r}}-\boldsymbol{f}_r)\right] \tag{26}$$

Maximizing the pdf in (26) is equivalent to finding

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}}(\hat{\boldsymbol{r}}-\boldsymbol{f}(\boldsymbol{x}))^T\boldsymbol{C}_r(\hat{\boldsymbol{r}}-\boldsymbol{f}(\boldsymbol{x})) \tag{27}$$

which we shall perform alternatingly. We consider Gauss-Newton procedure [10] for $\hat{\boldsymbol{x}}$. At iteration $(u+1)$:

$$\hat{\boldsymbol{x}}^{(u+1)} = \hat{\boldsymbol{x}}^{(u)} + \left(\boldsymbol{G}^T\boldsymbol{C}_r\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\boldsymbol{C}_r\left(\hat{\boldsymbol{r}}-\boldsymbol{f}(\hat{\boldsymbol{x}}^{(u)}\right) \tag{28}$$

where $\boldsymbol{G}$ is the Jacobian matrix of $\boldsymbol{f}(\boldsymbol{x})$

$$\boldsymbol{G} = \boldsymbol{G}(\hat{\boldsymbol{x}}^{(u)}),\ \boldsymbol{G}(\boldsymbol{x}) = \frac{\partial\boldsymbol{f}(\boldsymbol{x})}{\partial\boldsymbol{x}^T}. \tag{29}$$

A procedure is expected to terminate when $\|\hat{\boldsymbol{x}}^{(u+1)} - \hat{\boldsymbol{x}}^{(u)}\|_2 < \varepsilon$, for the stopping criterion ε sufficiently small. Then, the final position of the procedure is considered to be the coordinates of the mobile device in the xyz space.

However, iterative procedures do not always converge. In [5], we show that there are **three** possible outcomes for an iterative procedure: **Convergence, Divergence** and **Oscillation**.

If a procedure is diverging or oscillating, we will take its initialization as the estimated mobile position. As for a converging procedure, the final position is selected as estimate.

In a nutshell, the Algorithm 1 is proposed for the Gauss-Newton iterative procedure of Maximum Likelihood estimator.

Algorithm 1: Proposed Maximum Likelihood estimtor with Gauss-Newton procedure

```
Take all the estimated ToAs and then compute the
  corresponding d̂_1, d̂_2, ..., d̂_N.
Take the measured Direction of Arrival: azimuth φ̂_i and
  elevation θ̂_i.
Compute γ̂_{i,j} by (33).
Assign u = 1 and ε sufficiently small.
Compute the estimation x̂ by (17) as the first estimated
  coordinates of the mobile device.
repeat
    Compute the following estimated coordinates x̂^(u+1) of
      the mobile device by (28).
    u = u + 1;
until ||x̂^(u+1) − x̂^(u)||_2 < ε or u > 1000 or ||x̂^(u+1)||_2 = ±∞;
if u > 1000 or ||x̂^(u+1)||_2 = ±∞ then
    x̂^(1) is the estimated position of the mobile device;
else
    x̂^(u) is the estimated position of the mobile device;
```

IV. Simulation Results

A. Cramer-Rao Bound (CRB)

The Cramer-Rao Bound (CRB) is computed for the quaility evaluation of the algorithm. The Fisher Information Matrix (FIM) is calculated by

$$\boldsymbol{I}(\boldsymbol{x}) = \boldsymbol{G}^T(\boldsymbol{x})\boldsymbol{C}^{-1}\boldsymbol{G}(\boldsymbol{x}) \tag{30}$$

The CRB is the trace of the inverse of FIM:

$$\mathrm{CRB} = \mathrm{tr}(\boldsymbol{I}^{-1}) \tag{31}$$

B. Simulation Setup

To compare the quality among of algorithms and CRLB, we use Root Mean Square Position Error (RMSE) which is defined by

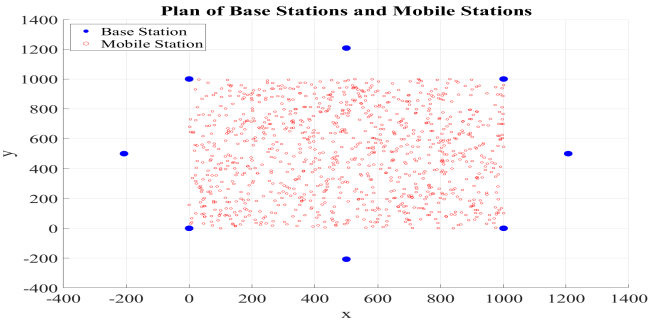
$$\mathrm{RMSE} = \sqrt{\mathrm{E}(\|\hat{\boldsymbol{x}}-\bar{\boldsymbol{x}}\|^2)} \tag{32}$$

where $\bar{\boldsymbol{x}}$ is the true position of the mobile device and $\hat{\boldsymbol{x}}$ is its estimate.

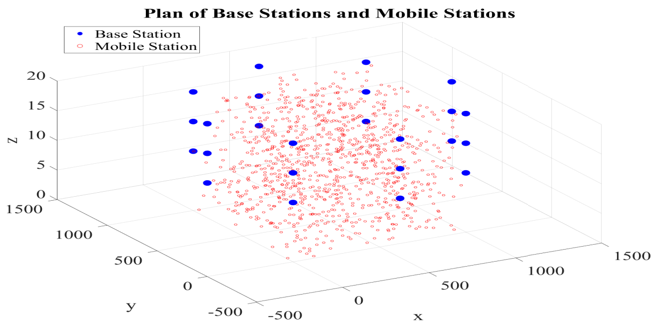
We consider a space of size 1000m x 1000m with the height of 20m. The network of 8 Base stations (numbered from 1 to 8) forms the circumscribed circle of this square. Each base station has 3 antenna arrays, which transmit signals, at the altitudes of 10m, 15m and 20m.

RMSE averaging is over 1000 mobile positions picked randomly in the space concerned. (Fig. 4).

Stopping criterion is $\varepsilon = 0.01$



(a) View from top



(b) View from one side

Fig. 4. Map of base stations and random positions of the mobile device

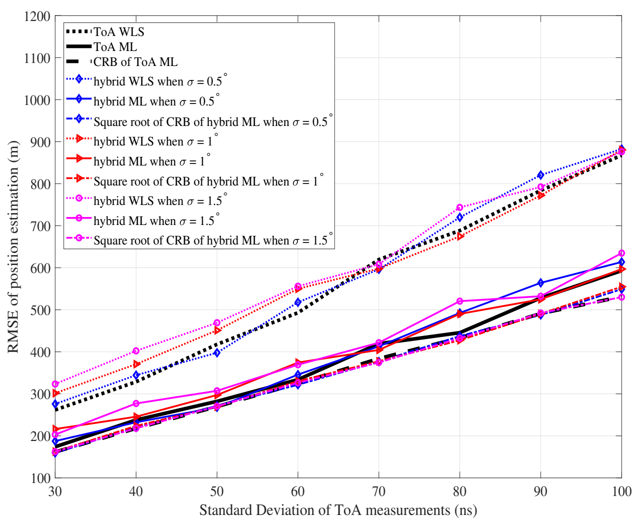


Fig. 5. Localization at mobile device with hybrid ToA-DDoA

C. Results

Instead of comparing the MSEs to the CRB, we compare their square roots: The Root Mean Square Error (RMSE) = $\sqrt{\text{MSE}}$ and square root of CRB ($\sqrt{\text{CRB}}$).

In the simulations, we assume that all the estimations of azimuth and elevation angles have the same standard deviation: $\mu_1 = \mu_2 = \cdots = \mu_N = \nu_1 = \nu_2 = \cdots = \nu_N = \sigma$. The standard deviations of all ToA measurements are also assumed to have the same value: $\sigma_{ToA,1} = \sigma_{ToA,2} = \cdots = \sigma_{ToA,N} = \sigma_{ToA}$.

We compare the RMSE of the four different scenarios, where in each scenario, the standard deviation of ToA measurements (σ_{ToA}) varies from 2.5 ns to 30 ns; the Weighted Least Squares (WLS) method and the Maximum Likelihood (ML) estimator are applied for localization (Fig. 5).

(a) ToA-based positioning algorithm given in [6].
(b) Hybrid positioning algorithm when $\sigma = 0.5^\circ$.
(c) Hybrid positioning algorithm when $\sigma = 1^\circ$.
(d) Hybrid positioning algorithm when $\sigma = 1.5^\circ$.

From Fig. 5, it is clear that the ML estimator considerably enhance the accuracy of WLS estimations. In addition, the advantage of the proposed hybrid positioning algorithm depends on the accuracy of DoA estimations. The more precise the DoA estimation is, the more useful the hybrid algorithm is, especially when the standard deviation of ToA estimation increases.

V. Conclusions

This paper develops a direction-based positioning algorithm at the mobile device. DDoA is the main element used to localize the mobile device because of its indistinct orientation. ToA is utilized as an additional element to reduce the computing complication. An iterative Maximum Likelihood procedure is also proposed to enhance the accuracy of localization.

However, all results presented are obtained by simulations. It would be of interest to evaluate the proposed positioning algorithm on measurements also.

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APPENDIX

A. Computations of expected value and variance of $\gamma_{i,j}$

In [11], it is proved that if $x \sim \mathcal{N}(x_0, \varsigma^2)$ then

$\mathrm{E}(\sin x) = e^{-\varsigma^2/2}\sin x_0; \quad \mathrm{E}(\cos x) = e^{-\varsigma^2/2}\cos x_0; \quad \mathrm{var}(\sin x) = \mathrm{var}(\cos x) = \frac{1}{2}\left(1 - e^{-2\varsigma^2}\right)$

$\mathrm{E}(\sin^2 x) = \frac{1}{2} - \frac{1}{2}e^{-2\varsigma^2} + e^{-2\varsigma^2}\sin^2 x_0; \quad \mathrm{E}(\cos^2 x) = \frac{1}{2} - \frac{1}{2}e^{-2\varsigma^2} + e^{-2\varsigma^2}\cos^2 x_0$

(8) shows the equation of $\gamma_{i,j} = \cos\beta_{i,j}$ in terms of the related DoAs: $\varphi_i, \varphi_j, \theta_i, \theta_j$. Therefore, the estimated value of the $\gamma_{i,j}$ is:

$$\begin{aligned}\hat{\gamma}_{i,j} &= \mathrm{E}(\gamma_{i,j}) = \mathrm{E}(\cos\beta_{i,j}) = \left(e^{-\mu_i^2/2}\cos\theta_i\right)\left(e^{-\mu_j^2/2}\cos\theta_j\right)\left(e^{-\left(\nu_i^2+\nu_j^2\right)/2}\cos(\varphi_i - \varphi_j)\right) + \left(e^{-\mu_i^2/2}\sin\theta_i\right)\left(e^{-\mu_j^2/2}\sin\theta_j\right) \\ &= e^{-\left(\mu_i^2+\mu_j^2\right)/2}(\cos\theta_i\cos\theta_j)\, e^{-\left(\nu_i^2+\nu_j^2\right)/2}\cos(\varphi_i - \varphi_j) + e^{-\left(\mu_i^2+\mu_j^2\right)/2}\sin\theta_i\sin\theta_j\end{aligned} \tag{33}$$

In addition, the variance of $\gamma_{i,j}$ is:

$$s_{i,j}^2 = \mathrm{var}(\gamma_{i,j}) = \mathrm{var}(\cos\beta_{i,j}) = \mathrm{E}(\cos^2\beta_{i,j}) - (\mathrm{E}(\cos\beta_{i,j}))^2 \tag{34}$$

where $\mathrm{E}(\cos\beta_{i,j})$ is expressed in (33) and

$$\cos^2\beta_{i,j} = (\cos\theta_i\cos\theta_j\cos(\varphi_i - \varphi_j) + \sin\theta_i\sin\theta_j)^2 = \cos^2\theta_i\cos^2\theta_j\cos^2(\varphi_i - \varphi_j) + \sin^2\theta_i\sin^2\theta_j + \tfrac{1}{4}\sin(2\theta_i)\sin(2\theta_j)\cos(\varphi_i - \varphi_j) \tag{35}$$

As a result,

$$\mathrm{E}(\cos\beta_{i,j}^2) = h_1h_2h_3 + h_4h_5 + \frac{1}{4}h_6h_7h_8 \tag{36}$$

where

$$h_1 = \mathrm{E}(\cos^2\theta_i) = \frac{1}{2} - \frac{1}{2}e^{-2\mu_i^2} + e^{-2\mu_i^2}\cos^2\theta_i\ ;\ h_2 = \mathrm{E}(\cos^2\theta_j) = \frac{1}{2} - \frac{1}{2}e^{-2\mu_j^2} + e^{-2\mu_j^2}\cos^2\theta_j\ ;$$

$$h_3 = \mathrm{E}(\cos^2(\varphi_i - \varphi_j)) = \frac{1}{2} - \frac{1}{2}e^{-2\left(\nu_i^2+\nu_j^2\right)} + e^{-2\left(\nu_i^2+\nu_j^2\right)}\cos^2(\varphi_i - \varphi_j)\ ;\ h_4 = \mathrm{E}(\sin^2\theta_i) = \frac{1}{2} - \frac{1}{2}e^{-2\mu_i^2} + e^{-2\mu_i^2}\sin^2\theta_i\ ;$$

$$h_5 = \mathrm{E}(\sin^2\theta_j) = \frac{1}{2} - \frac{1}{2}e^{-2\mu_j^2} + e^{-2\mu_j^2}\sin^2\theta_j\ ;\ h_6 = \mathrm{E}(\sin(2\theta_i)) = e^{-2\mu_i^2}\sin(2\theta_i)\ ;$$

$$h_7 = \mathrm{E}(\sin(2\theta_j)) = e^{-2\mu_j^2}\sin(2\theta_j)\ ;\ h_8 = \mathrm{E}(\cos(\varphi_i - \varphi_j)) = e^{-\left(\nu_i^2+\nu_j^2\right)/2}\cos(\varphi_i - \varphi_j)\ ;$$

The covariance of $\gamma_{i,j}$ and $\gamma_{i,l}$ is

$$s_{i,j,l}^2 = \mathrm{cov}(\gamma_{i,j}, \gamma_{i,l}) = \mathrm{E}(\gamma_{i,j}\gamma_{i,l}) - \mathrm{E}(\gamma_{i,j})\mathrm{E}(\gamma_{i,l}) \tag{37}$$

where $\mathrm{E}(\gamma_{i,j})$ is expressed in (33) and

$$\begin{aligned}&\gamma_{i,j}\gamma_{i,l} = (\cos\theta_i\cos\theta_j\cos(\varphi_i - \varphi_j) + \sin\theta_i\sin\theta_j)(\cos\theta_i\cos\theta_l\cos(\varphi_i - \varphi_l) + \sin\theta_i\sin\theta_l) = \\ &\tfrac{1}{2}\cos^2\theta_i\cos\theta_j\cos\theta_l(\cos(2\varphi_i - \varphi_j - \varphi_l) + \cos(\varphi_j - \varphi_l)) + \tfrac{1}{2}\sin(2\theta_i)\sin\theta_j\cos\theta_l\cos(\varphi_i - \varphi_l) + \tfrac{1}{2}\sin(2\theta_i)\cos\theta_j\sin\theta_l\cos(\varphi_i - \varphi_j) \\ &+ \sin^2\theta_i\sin\theta_j\sin\theta_l\end{aligned} \tag{38}$$

As a result,

$$\mathrm{E}(\gamma_{i,j}\gamma_{i,l}) = \frac{1}{2}m_1m_2m_3(m_4 + m_5) + \frac{1}{2}m_6m_7m_8m_9 + \frac{1}{2}m_{10}m_{11}m_{12}m_{13} + m_{14}m_{15}m_{16} \tag{39}$$

where

$$m_1 = \mathrm{E}(\cos^2\theta_i) = h_1\ ;\ m_2 = \mathrm{E}(\cos\theta_j) = e^{-\mu_j^2/2}\cos\theta_j\ ;\ m_3 = \mathrm{E}(\cos\theta_l) = e^{-\mu_l^2/2}\cos\theta_l\ ;$$

$$m_4 = \mathrm{E}(\cos(2\varphi_i - \varphi_j - \varphi_l)) = e^{-\left(4\nu_i^2+\nu_j^2+\nu_l^2\right)/2}\cos(2\varphi_i - \varphi_j - \varphi_l)\ ;\ m_5 = \mathrm{E}(\cos(\varphi_j - \varphi_l)) = e^{-\left(\nu_i^2+\nu_j^2\right)/2}\cos(\varphi_j - \varphi_l)\ ;$$

$$m_6 = \mathrm{E}(\sin(2\theta_i)) = h_6\ ;\ m_7 = \mathrm{E}(\cos\theta_l) = e^{-\mu_l^2/2}\cos\theta_l;\ m_8 = \mathrm{E}(\cos\theta_l) = m_3\ ;\ m_9 = \mathrm{E}(\cos(\varphi_i - \varphi_l)) = e^{-\left(\nu_i^2+\nu_l^2\right)/2}\cos(\varphi_i - \varphi_l)\ ;$$

$$m_{10} = \mathrm{E}(\sin(2\theta_j)) = h_7\ ;\ m_{11} = \mathrm{E}(\cos\theta_j) = m_2\ ;\ m_{12} = \mathrm{E}(\sin\theta_l) = e^{-\mu_l^2/2}\sin\theta_l\ ;\ m_{13} = \mathrm{E}(\cos(\varphi_i - \varphi_j)) = h_8;$$

$$m_{14} = \mathrm{E}(\sin^2\theta_i) = h_4\ ;\ m_{15} = \mathrm{E}(\sin\theta_j) = e^{-\mu_j^2/2}\sin\theta_j;\ m_{16} = \mathrm{E}(\sin\theta_l) = m_{12}$$

B. Covariance matrix of $\boldsymbol{b}$

$$\mathrm{E}\left(\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^T\right) = \begin{bmatrix} \sigma_1^2 d_2^2\gamma_{1,2}^2 + \sigma_2^2 d_1^2\gamma_{1,2}^2 + s_{1,2}^2 d_1^2 d_2^2 & \sigma_1^2 d_2 d_3\gamma_{1,2}\gamma_{1,3} + s_{1,2,3}^2 d_1^2 d_2 d_3 & \dots & \sigma_1^2 d_2 d_N\gamma_{1,2}\gamma_{1,N} + s_{1,2,N}^2 d_1^2 d_2 d_N \\ \sigma_1^2 d_2 d_3\gamma_{1,2}\gamma_{1,3} + s_{1,2,3}^2 d_1^2 d_2 d_3 & \sigma_1^2 d_3^2\gamma_{1,3}^2 + \sigma_3^2 d_1^2\gamma_{1,3}^2 + s_{1,3}^2 d_1^2 d_3^2 & \dots & \sigma_1^2 d_3 d_N\gamma_{1,3}\gamma_{1,N} + s_{1,3,N}^2 d_1^2 d_3 d_N \\ \dots & \dots & \dots & \dots \\ \sigma_1^2 d_2 d_N\gamma_{1,2}\gamma_{1,N} + s_{1,2,N}^2 d_1^2 d_2 d_N & \sigma_1^2 d_3 d_N\gamma_{1,3}\gamma_{1,N} + s_{1,3,N}^2 d_1^2 d_3 d_N & \dots & \sigma_1^2 d_N^2\gamma_{1,N}^2 + \sigma_N^2 d_1^2\gamma_{1,N}^2 + s_{1,N}^2 d_1^2 d_N^2 \end{bmatrix} \tag{40}$$