Semantics-Aware Source Coding in Status Update Systems

Pouya Agheli [1], Nikolaos Pappas [2], Marios Kountouris [1]

[1]Communication Systems Department, EURECOM, France

[2]Department of Science and Technology, Linköping University, Sweden

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Overview

Introduction

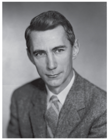
Shannon-Weaver's Model

- Shannon and Weaver's joint model identifies that problems in communication are at three levels [1]:

Level A– The technical problems are concerned with the accuracy of transference of information from sender to receiver.

Level B– The semantic problems are concerned with the interpretation pf meaning by the receiver, as compared with the intended meaning of the sender.

Level C– The problems of influence or effectiveness are concerned with the success with which the meaning conveyed to the receiver leads to the desired conduct on his part.



Claude Shannon (1916–2001)

Warren Weaver (1894–1978)

Shannon-Weaver's Model

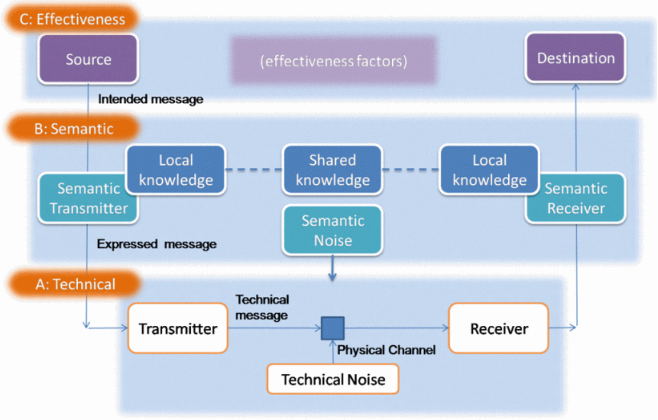


Figure 1: A 3-Level Communication Model [2].

Semantics of Information

Semantics of information (SoI)

The SoI is a metric which measures the importance and usefulness of messages with respect to the goal of data exchange in communication networks.

General SoI formulation

The SoI is generally a composite function, such as

$$\mathcal{S}(t) = \nu(\psi(\mathcal{I})) \tag{1}$$

where $\psi : \mathbb{R}^m \to \mathbb{R}^z$, $m \geq z$ is a function of $m \in \mathbb{Z}^+$ information attributes $\mathcal{I} \in \mathbb{R}^m$, and $\nu : \mathbb{R}^z \to \mathbb{R}$ is a context-dependent, cost-aware function [4, 5].

Goal-Oriented Semantic Communications

- This new paradigm
  1. redefined importance, timing, and effectiveness [3, 4],
  2. needs joint data acquisition, transmission, and reception,
  3. alters processes among all network layers, including
     - source and channel coding
     - multiple access techniques
     - network routing and switching
     - cross-layer resource allocations

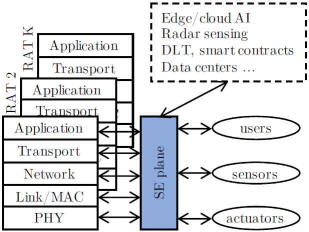


Figure 2: Interaction of different nodes with the protocol stack in 5G [3].

Single-User Status Update Systems

System Model

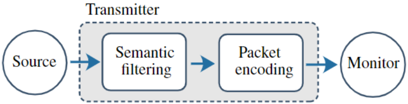


Figure 3: System model of semantics-aware transmission.

- The source X generates discrete symbols as *status updates* from a finite set $\mathcal{X}$.
- The set $\mathcal{X} = \{x_i \mid i \in \mathcal{I}_n\}$ for $\mathcal{I}_n = \{1, 2, ..., n\}$ where each realization has a probability of $\tilde{p}_i = P_X(x_i)$.
- Here, $P_X(\cdot)$ is a known pmf, and $\tilde{p}_i \geq \tilde{p}_j, \forall i \leq j$.
- The sequence of observations is i.i.d, and packets generations are based on a *Poisson* distribution with rate λ.

Semantic Filtering

- The process of admitting k important packets of n and discarding the remaining $n-k$ packets.
- The arrival's importance is proportional to the reverse of it's probability. I.e., the more *infrequent* an event is, the more *significant* is for the remote monitor.
- The realization set becomes $\mathcal{X} = \{x_i \mid i \in \mathcal{I}_k\}$ where $\mathcal{I}_k \subseteq \mathcal{I}_n$.
- If $q_k = \sum_{i \in \mathcal{I}_k} \tilde{p}_i$, the probabilities are refined as

$$p_i = \begin{cases} \tilde{p}_i / q_k, \forall i \in \mathcal{I}_k \\ 0, \text{otherwise.} \end{cases} \tag{2}$$

Key Metric of Interest

- We assume *timeliness* as a primary contextual attribute on the form of a non-increasing utility function $f:\mathbb{R}_0^+ \to \mathbb{R}$ of information freshness.

The SoI's formula

The definition in (1) becomes

$$\mathcal{S}(t) = f\left(\Delta(t)\right) \tag{3}$$

where $\Delta(t) = t - u(t)$ is the age of information (AoI) at the receiver.

Average SoI

By the use of (3), the average SoI is derived as

$$\bar{\mathcal{S}} = \lim_{T\to\infty} \frac{1}{T}\int_0^T f\left(\Delta(t)\right)\mathrm{d}t. \tag{4}$$

Problem Statement

1. **Objective:** Find the optimal codeword ℓ_i for the *i*-th realization
2. **Optimization:** Maximize the average SoI in (4) & minimize a cost function $\phi(\ell) : \mathbb{R}_0^+ \to \mathbb{R}_0^+$, i.e., $\sum_{i \in \mathcal{I}_k} p_i \phi(\ell_i)$
3. **Constraints:** The Kraft-McMillan inequality, i.e., $\sum_{i \in \mathcal{I}_k} 2^{-\ell_i} \leq 1$, and the positive integer value of the codeword length, i.e., $\ell_i \in \mathbb{Z}^+$

Note: Maximizing the average SoI in (4) is equivalent to *minimizing* the average penalty of lateness given as follows.

Average penalty of lateness

$$L(\Delta) = \lim_{T \to \infty} \frac{1}{T} \int_0^T g\left(\Delta(t)\right) \mathrm{d}t \tag{5}$$

where $g : \mathbb{R}_0^+ \to \mathbb{R}$ is a non-decreasing function [6].

Optimization problem

$$\mathcal{P}_1: \min_{\{\ell_i\}} \; L(\Delta) + w \sum_{i \in \mathcal{I}_k} p_i \phi(\ell_i)$$
$$\text{s.t.} \; \sum_{i \in \mathcal{I}_k} 2^{-\ell_i} \leq 1, \tag{6}$$
$$\ell_i \in \mathbb{Z}^+$$

where $w > 0$ denotes a weight parameter.

- We can relax the integer value constraint in (6) by letting real values for ℓ_i. Then, the final integer value of the codeword length can be derived via applying a round-off operation.
- We also assume a quadratic cost function under binary alphabetic, as $\phi(x) = \alpha x + \beta x^2$, $\alpha, \beta \geq 0$ [7].

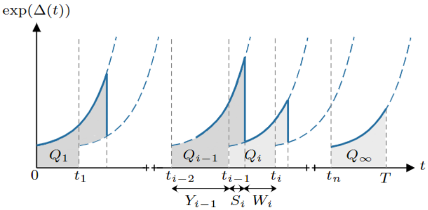


Figure 4: Sample evolution for $g\left(\Delta(t)\right) = \exp(\Delta(t))$.

- $L(\Delta)$ is derived by summing the area below the curve of $g\left(\Delta(t)\right)$ over the span of $[0, T]$.
- As a sample path for $g\left(\Delta(t)\right) = \exp(\Delta(t))$, (5) becomes

$$L(\Delta) = \lim_{T\to\infty} \frac{1}{T} \left\{ \sum_{i=1}^{\mathcal{N}(T)} Q_i + Q_\infty \right\} = \eta \mathbb{E}[Q] \tag{7}$$

where $\eta = \lim_{T\to\infty} \frac{\mathcal{N}(T)-1}{T}$, and $\mathcal{N}(T)$ is the number of admitted packet by time T.

New optimization problem

From (6)–(7), and by merging η with w, we have

$$\begin{aligned}\mathcal{P}_2 : \min_{\{\ell_i\}} \quad & \underbrace{\mathbb{E}[Q] + w \sum_{i \in \mathcal{I}_k} p_i(\alpha \ell_i + \beta \ell_i^2)}_{\triangleq \mathcal{J}_{\mathrm{SoI}}} \\ \text{s.t.} \quad & \sum_{i \in \mathcal{I}_k} 2^{-\ell_i} \leq 1, \\ & \ell_i \in \mathbb{R}^+.\end{aligned} \tag{8}$$

Semantics-Aware Optimal Codeword Design

- Three different instances of $g\left(\Delta(t)\right)$ are assumed to compute the optimal codeword lengths according to (8), as

$$g\left(\Delta(t)\right) = \begin{cases} \exp(\rho\Delta(t)) & \text{EDT case} \\ \ln(\rho\Delta(t)) & \text{LDT case} \\ \rho(\Delta(t))^{\kappa} & \text{PDT case} \end{cases} \tag{9}$$

where $\rho \geq 0$, and $\kappa \in \mathbb{Z}^{+}$.

- The above cases respectively correspond to the scenarios of
  1. *exponentially* (E-),
  2. *logarithmically* (L-),
  3. *polynomially* decreasing timeliness (PDT).

a. EDT Case

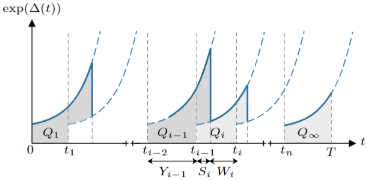


Figure 5: Sample evolution for $g\left(\Delta(t)\right) = \exp(\Delta(t))$.

- According to Figure 5, we obtain

$$\mathbb{E}[Q] \approx \frac{\rho}{2}\mathbb{E}[Y^2] + \rho\mathbb{E}[S]\mathbb{E}[Y] + \mathbb{E}[Y]$$
$$\overset{(b)}{=} \frac{\rho}{2}\mathbb{E}[L^2] + \rho(\mathbb{E}[L])^2 + (1+2\rho\gamma)\mathbb{E}[L] + \rho\gamma^2 + \gamma. \quad (10)$$

To achieve (b), we have $\gamma = \frac{1}{\lambda q_k}$. Also, since $S_i = \ell_i$ for a channel transferring one bit at a unit time, $\mathbb{E}[S] = \mathbb{E}[L]$, $\mathbb{E}[Y] = \mathbb{E}[L] + \gamma$, and $\mathbb{E}[Y^2] = \mathbb{E}[L^2] + 2\gamma\mathbb{E}[L] + 2\gamma^2$ [8].

- To find optimal ℓ_i, $\forall i \in \mathcal{I}_k$, we have the following steps.
  1. Importing the computed $\mathbb{E}[Q]$ into $\mathcal{P}_2$
  2. Defining a Lagrangian function $\mathcal{L}(\ell_i; .)$
  3. Writing the Karush-Kuhn-Tucker (KKT) condition
  4. Applying some algebraic manipulations

Optimal codeword length for the EDT case

$$\ell_i = -\ln_2\left(\frac{\mathcal{C}_1 p_i}{\mu(\ln(2))^2} W_0\left(\frac{\mu(\ln(2))^2}{\mathcal{C}_1 p_i} 2^{\frac{\mathcal{C}_2}{\mathcal{C}_1}}\right)\right) \tag{11}$$

where $W_0(.)$ is the principal branch of Lambert W function, $\mu \geq 0$ is the Lagrange multiplier, $\mathcal{C}_1 = \rho + 2w\beta$, and

$$\mathcal{C}_2 = \frac{2\rho\mu\ln(2) + \mathcal{C}_1(1+2\rho\gamma+w\alpha)}{\mathcal{C}_1 + 2\rho}.$$

b. LDT Case

- After applying the same steps as for the EDT case, we obtain

Optimal codeword length for the LDT case

$$\ell_i = -\ln_2\left(\frac{\mathcal{C}_3 p_i}{\mu'(\ln(2))^2} W_0\left(\frac{\mu'(\ln(2))^2}{\mathcal{C}_3 p_i} 2^{\frac{\mathcal{C}_4}{\mathcal{C}_3}}\right)\right) \tag{12}$$

where $\mu' > 0$, $\mathcal{C}_3 = 2\rho + 2w\beta$, and

$$\mathcal{C}_4 = \frac{4\rho\mu' \ln(2) + 2\mathcal{C}_3(2\rho\gamma - 1 + \frac{w\alpha}{2})}{\mathcal{C}_3 + 4\rho}.$$

c. PDT Case

- After applying the same steps as for the EDT and LDT cases, we have the following formula for ℓ_i, $\forall i \in \mathcal{I}_k$.

Optimal codeword length for the PDT case

$$\ell_i = -\ln_2\left(\frac{\mathcal{C}_1 p_i}{\mu''(\ln(2))^2} W_0\left(\frac{\mu''(\ln(2))^2}{\mathcal{C}_1 p_i} 2^{\frac{\mathcal{C}_5}{\mathcal{C}_1}}\right)\right) \tag{13}$$

where $\mu'' > 0$, and

$$\mathcal{C}_5 = \frac{2\rho\mu'' \ln(2) + \mathcal{C}_1(2\rho\gamma + w\alpha)}{\mathcal{C}_1 + 2\rho}.$$

Numerical Results

- For simulations and figures, we assume the following setup.
  - We utilize a Zipf(n, s) distribution with pmf

$$P_X(x) = \frac{1/x^s}{\sum_{j=1}^{n} 1/j^s}, \tag{14}$$

    with $n = |\mathcal{X}| = 100$ and $s = 0.4$.
  - We set $\rho = 0.5$, $\alpha = \beta = 1$ and $T = 10\,[\text{sec}]$.
  - The weight w is set in a way that the value range of average SoI and coding cost penalty terms becomes comparable.

a. Finding Optimal k

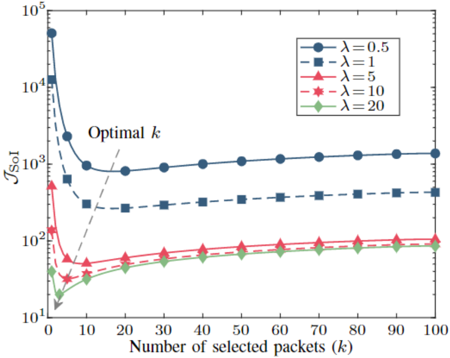


Figure 6: The objective function versus k for the EDT case and Zipf(100,0.4) distribution.

- Increasing the arrival rate *reduces* $\mathcal{J}_{\mathrm{SoI}}$ as well as the optimal k.
- Comparing with a linear age scenario, e.g., $g(\Delta(t)) = \rho\Delta(t)$, an exponential penalty (nonlinear age) results in lower values for optimal k.
- From $\mathcal{J}_{\mathrm{SoI}}$ perspective: PDT $<$ EDT $<$ LDT

b. Effect of Event Distribution

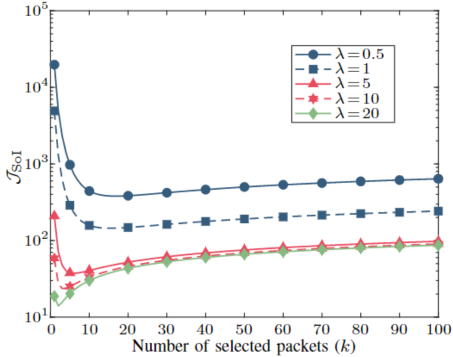


Figure 7: The objective function versus k for the EDT case and uniform probability distribution.

- Increasing the arrival rate *reduces* $\mathcal{J}_{\text{SoI}}$ as well as the optimal k.
- The optimal k for the uniform distribution is slightly smaller than that for the Zipf pmf.
- For instance, for $\lambda = 10$, the Zipf and the uniform distribution results in optimal $k = 5$ and $k = 3$, respectively.

c. Finding Optimal λ

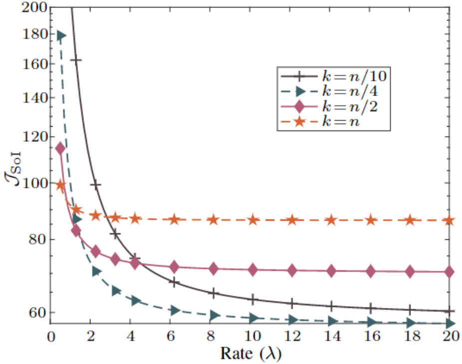


Figure 8: The objective function versus λ for the EDT case and Zipf(100,0.4) distribution.

- Increasing the input rate decreases $\mathcal{J}_{\mathrm{SoI}}$; however, this decrease diminishes and saturates at higher rate values.
- By increasing the number of selected packets, lower input rates are required to reduce the penalty terms.
- From $\mathcal{J}_{\mathrm{SoI}}$ perspective: PDT $<$ EDT $<$ LDT

d. Finding Optimal k, α And β

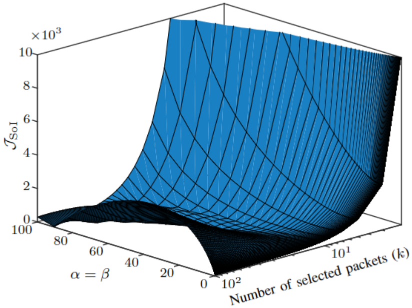


Figure 9: The objective function versus k, α, and β for the EDT case and $\lambda = 1$.

- Increasing the input rate, hence decreasing the optimal k, the optimal values of cost variables increase.

Table 1: Optimal parameters under the EDT scenario.

λ	k	$\alpha = \beta$	λ	k	$\alpha = \beta$
0.5	20	1.26	10	5	2.5
1	18	1.58	20	2	12.59
5	10	1.99			

Publication I

Semantics-Aware Source Coding in Status Update Systems

Pouya Agheli*, Nikolaos Pappas†, and Marios Kountouris*
*EURECOM, Communication Systems Department, Sophia Antipolis, France
†Dept. of Science and Technology, Linköping University, Norrköping Campus, Sweden
Email: pouya.agheli@eurecom.fr, nikolaos.pappas@liu.se, marios.kountouris@eurecom.fr

***Abstract*—We consider a communication system in which the destination receives status updates from an information source that observes a physical process. The transmitter performs semantics-empowered filtering as a means to send only the most "important" samples to the receiver in a timely manner. As a first step, we explore a simple policy where the transmitter selects to encode only a fraction of the least frequent realizations of the observed random phenomenon, treating the remaining ones as not informative. For this timely source coding problem, we derive the optimal codeword lengths in the sense of maximizing a semantics-aware utility function and minimizing a quadratic average length cost. Our numerical results show the optimal number of updates to transmit for different arrival rates and encoding costs and corroborate that semantic filtering results in higher performance in terms of timely delivery of important updates.**

I. Introduction

The evolution of the latest generations of mobile communication systems has been mainly driven by setting highly ambitious, often hard to achieve, goals. Although this maximalistic approach may trigger technological advances, it often comes with inflated requirements in terms of resources to meaningfully scale. Wireless networks are currently evolving in enabling effective communication of concise information that is both timely and valuable for achieving end users' requirements. Age of information (AoI) performance metrics [9], [10], which describe information freshness in networks, and value of information (VoI) [11], [12], which quantifies the information utility or gain in decision making, can be viewed as simple, quantitative surrogate for information semantics.

In this paper, we consider a communication system in which a transmitter receives status updates generated from a known discrete distribution with finite support and seeks to communicate them to a remote receiver. The updates generated by the information source may correspond to observations or measurements of a random phenomenon. The transmitter performs semantics-aware filtering and sends to the receiver only the most relevant randomly arriving source symbols in a timely fashion over an error-free channel. We consider a simple coding scheme focusing on less frequent events, i.e., the transmitter encodes only a fraction of the least frequent realizations, treating the remaining ones as not informative or irrelevant, thus providing more information about events that happen less often. Additionally, the semantics of information is captured through a timeliness metric for the received updates,

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Double-User Status Update Systems

System Extensions

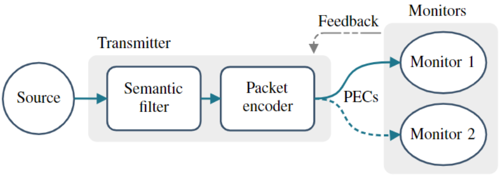


Figure 10: Semantics-aware transmission over a double-user network.

- The same input, packet arrival, ... as the single-user model.
- Transmission occurs over a noisy network modeled by two identical packet erasure channels (PECs) with erasure probability δ.
- A simple ARQ protocol is used for fixing potential transmission errors.

Semantic Filtering

- The process of admitting k important packets of n and discarding the remaining $n-k$ packets.
- Two value assessment policies are considered:

Policy 1: An arrival's importance is proportional to the reverse of it's probability $\rightarrow$ Monitor 1

Policy 2: An arrival's importance is proportional to it's probability $\rightarrow$ Monitor 2

- The realization set becomes $\mathcal{X} = \{x_i \mid i \in \mathcal{I}_k\}$ where $\mathcal{I}_k \subseteq \mathcal{I}_n$.
- $\mathcal{I}_k \subseteq \mathcal{I}_n$, is obtained from $\mathcal{I}_k = \mathcal{I}_{k_1} \cup \mathcal{I}_{k_2}$, where
  - $\mathcal{I}_{k_1}$ is the set of the k_1 least frequent arrivals,
  - $\mathcal{I}_{k_2}$ is the set of the k_2 most frequent arrivals.
- Some observations can be important for both monitors.

Problem Reformulation

1. **Objective:** Find the optimal codeword ℓ_i for the i-th realization
2. **Optimization:** Maximize the weighted sum of the monitors' average SoI, i.e., $\bar{\mathcal{S}}_1$ and $\bar{\mathcal{S}}_2$
3. **Constraints:** The Kraft-McMillan inequality, i.e., $\sum_{i\in\mathcal{I}_k} 2^{-\ell_i} \leq 1$, and the positive integer value of the codeword length, i.e., $\ell_i \in \mathbb{Z}^+$

Note: For the convenience of analytical derivation and to ensure positiveness, we *minimize* the average penalty of lateness.

Average penalty of lateness

$$L_r(\Delta_r) = \lim_{T\to\infty} \frac{1}{T}\int_0^T g\left(\Delta_r(t)\right)\mathrm{d}t \qquad (15)$$

where $r=1$ and $r=2$ show monitors 1 and 2, respectively. Also, $g:\mathbb{R}_0^+ \to \mathbb{R}$ is a non-decreasing function.

Optimization problem

$$\mathcal{P}_3: \min_{\{\ell_i\}} \quad w_1 L_1(\Delta_1) + w_2 L_2(\Delta_2)$$
$$\text{s.t.} \quad \sum_{i \in \mathcal{I}_k} 2^{-\ell_i} \leq 1, \tag{16}$$
$$\ell_i \in \mathbb{Z}^+$$

where $w_1, w_2 > 0$ are weight parameters.

- We can relax the integer value constraint in (16) by letting real values for ℓ_i.
- The final integer value of the codeword length can be derived via applying a round-off operation.

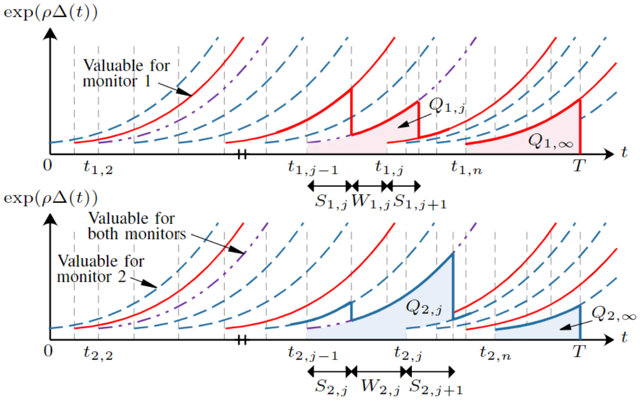


Figure 11: Sample evolution for the EDT case over time for $\rho = 0.2$.

- The same as the single-user model, EDT, LDT, and PDT scenarios are considered to model $g\left(\Delta_r(t)\right)$.

- $L(\Delta)$ is derived by summing the area below the curve of $g\left(\Delta(t)\right)$ over the span of $[0, T]$, as

$$L_r(\Delta_r) = \lim_{T\to\infty} \frac{1}{T}\left\{ \sum_{j=1}^{\mathcal{N}_r(T)} Q_{r,j} + Q_{r,\infty} \right\} = \eta_r \mathbb{E}[Q_r] \qquad (17)$$

where $\eta_r = \lim_{T\to\infty} \frac{\mathcal{N}_r(T)-1}{T}$.

- $\mathcal{N}_r(T) \leq \mathcal{N}(T)$ is the number of admitted packets for monitor 1 ($r = 1$) or monitor 2 ($r = 2$) by time T, where $\mathcal{N}(T)$ is the number of all admitted packets.

New optimization problem

From (16)–(17), and by merging η_r with w_r, we have

$$\begin{aligned} \mathcal{P}_4 : \min_{\{\ell_i\}} \quad & \underbrace{w_1\mathbb{E}[Q_1] + w_2\mathbb{E}[Q_2]}_{\triangleq\, \mathcal{J}_{\mathrm{SoI}}} \\ \text{s.t.} \quad & \sum_{i\in\mathcal{I}_k} 2^{-\ell_i} \leq 1, \\ & \ell_i \in \mathbb{R}^+. \end{aligned} \tag{18}$$

Codeword Design (a. EDT Case)

- To find $\ell_i, \forall i \in \mathcal{I}_k$, we have the following steps.
  1. Importing the computed $\mathbb{E}[Q_1]$ and $\mathbb{E}[Q_2]$ into $\mathcal{P}_4$
  2. Defining a Lagrangian function $\mathcal{L}(\ell_i;.)$
  3. Writing the Karush-Kuhn-Tucker (KKT) condition
  4. Applying some algebraic manipulations

Optimal codeword length for the EDT case

$$\ell_i = -\ln_2\left(\frac{\mathcal{C}_1 p_i}{\mu(\ln(2))^2} W_0\left(\frac{\mu(\ln(2))^2}{\mathcal{C}_1 p_i} 2^{\frac{\mathcal{C}_2}{\mathcal{C}_1}}\right)\right) \tag{19}$$

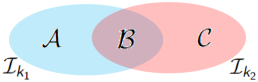
where $\mu \geq 0$ is the Lagrange multiplier, $\mathcal{C}_1 = \rho\pi_2(\varpi_1+\varpi_2)$,

$$\mathcal{C}_2 = \frac{2\mu\rho\ln(2)\pi_1^2(\varpi_1\chi_1+\varpi_2\chi_2) + \mathcal{C}_1\mathcal{C}_3\pi_1}{\mathcal{C}_1 + 2\rho\pi_1^2(\varpi_1\chi_1+\varpi_2\chi_2)},$$

and $\mathcal{C}_3 = \varpi_1(1+2\rho\gamma_1) + \varpi_2(1+2\rho\gamma_2)$.

Definitions and recalling:

- $\gamma_r = \frac{1}{\lambda q_{k_r}}$, where λ is the rate of Poisson distributed arrivals, and $q_{k_r} = \sum_{i \in \mathcal{I}_{k_r}} \tilde{p}_i$.
- The service time of realization x_i being important for monitor 1 and/or 2 during the j-th arrival is $S_{r,j} = \psi_j \ell_i$ time units.
- ψ_j is the number of transmissions for the j-th packet and is geometrically distributed with success prob. $1 - \epsilon_0$, and first and second moments $\pi_1 = \frac{1}{1-\epsilon_0}$ and $\pi_2 = \frac{1+\epsilon_0}{(1-\epsilon_0)^2}$, respectively.
- $\varpi_1 := w_1 \varrho_1$ and $\varpi_2 := w_2 \varrho_2$, with $\varrho_1, \varrho_2 \in \{0, 1\}$ being indicator parameters, initialized as follows:
  - $\varrho_1 = 1, \varrho_2 = 0 \Longleftarrow x_i$ belongs to set $\mathcal{A} = \mathcal{I}_{k_1} - \mathcal{B}$
  - $\varrho_1 = \varrho_2 = 1 \Longleftarrow x_i$ belongs to set $\mathcal{B} = \mathcal{I}_{k_1} \cap \mathcal{I}_{k_2}$
  - $\varrho_1 = 0, \varrho_2 = 1 \Longleftarrow x_i$ belongs to set $\mathcal{C} = \mathcal{I}_{k_2} - \mathcal{B}$



Definitions and recalling:

- The values of χ_1 and χ_2 are calculated via **Algorithm 1**.

1. We assume uniform p_i, and assign $\chi_1^{(0)} = k_1/k$ and $\chi_2^{(0)} = k_2/k$.
2. We find ℓ_i and compute new χ_1 and χ_2. Then, the new values for ℓ_i are found.
3. This process continues until the convergence criterion ε is satisfied.

Algorithm 1: Solution for deriving χ_1 and χ_2

Input: Fixed parameters $\mathcal{I}_k$, $\mathcal{I}_{k_1}$, $\mathcal{I}_{k_2}$, and p_i, $\forall i \in \mathcal{I}_k$. Stopping accuracy ε. Initial parameters $\mu^{(0)}$, $\chi_1^{(0)}$, $\chi_2^{(0)}$, $\beta^{(0)}$, and $\ell_i^{(0)}$, $\forall i$.

Output: Final-form parameters $\chi_1 = \chi_1^{(n)}$, $\chi_2 = \chi_2^{(n)}$, $\beta = \beta^{(n)}$, $\ell_i = \ell_i^{(n)}$, $\forall i$, and $\mu = \mu^{(m)}$.

1 *Iteration* m:
2 *Iteration* n:
3 Update $\beta^{(n)}$ and $\ell_i^{(n)}$ using (10) and (9), respectively.
4 Compute $\mathbb{E}[L] = \sum_{i\in\mathcal{I}_k} p_i \ell_i^{(n)}$, $\mathbb{E}[L]_1 = \sum_{i\in\mathcal{I}_{k_1}} p_i \ell_i^{(n)}$, and $\mathbb{E}[L]_2 = \sum_{i\in\mathcal{I}_{k_2}} p_i \ell_i^{(n)}$.
5 Update $\chi_1^{(n)}$ and $\chi_2^{(n)}$ based on 4.
6 **if** *Criterion* $\left|\chi_1^{(n)} - \chi_1^{(n-1)}\right| > \varepsilon$ *or* $\left|\chi_2^{(n)} - \chi_2^{(n-1)}\right| > \varepsilon$ **then** set $n = n+1$, and **goto** 2.
7 Compute $\beta^{(n)}$ from (10), and derive $\ell_i^{(n)}$ from (9).
8 **if** $\sum_{i\in\mathcal{I}_k} p_i \ell_i^{(n)} = 1$ **then** stop the process, and **goto** 11.
9 **else if** $\sum_{i\in\mathcal{I}_k} p_i \ell_i^{(n)} < 1$ **then** decrease $\mu^{(m)}$, set $m = m+1$, and **goto** 1.
10 **else** increase $\mu^{(m)}$, set $m = m+1$, and **goto** 1.
11 Save $\chi_1^{(n)}$, $\chi_2^{(n)}$, $\beta^{(n)}$, $\ell_i^{(n)}$, $\forall i$, and $\mu^{(m)}$.

b. LDT Case

- Similar to the EDT case, we can derive codeword length ℓ_i, $\forall i \in \mathcal{I}_k$, for the LDT case, as follows.

Optimal codeword length for the LDT case

$$\ell_i = -\ln_2\left(\frac{\mathcal{C}_4 p_i}{\mu'(\ln(2))^2} W_0\left(\frac{\mu'(\ln(2))^2}{\mathcal{C}_4 p_i} 2^{\frac{\mathcal{C}_5}{\mathcal{C}_4}}\right)\right) \tag{20}$$

where $\mu' \geq 0$ is the Lagrange multiplier, $\mathcal{C}_4 = 2\rho\pi_2(\varpi_1+\varpi_2)$,

$$\mathcal{C}_5 = \frac{4\mu'\rho\ln(2)\pi_1^2(\varpi_1\chi_1+\varpi_2\chi_2) + 2\mathcal{C}_4\mathcal{C}_6\pi_1}{\mathcal{C}_4 + 4\rho\pi_1^2(\varpi_1\chi_1+\varpi_2\chi_2)},$$

and $\mathcal{C}_6 = \varpi_1(2\rho\gamma_1-1) + \varpi_2(2\rho\gamma_2-1)$.

c. PDT Case

- After applying the same steps as for the EDT and LDT cases, we have

Optimal codeword length for the PDT case

$$\ell_i = -\ln_2\left(\frac{\mathcal{C}_1 p_i}{\mu''(\ln(2))^2} W_0\left(\frac{\mu''(\ln(2))^2}{\mathcal{C}_1 p_i} 2^{\frac{\mathcal{C}_7}{\mathcal{C}_1}}\right)\right) \tag{21}$$

where $\mu'' \geq 0$ denotes the Lagrange multiplier,

$$\mathcal{C}_7 = \frac{2\mu\rho \ln(2)\pi_1^2(\varpi_1\chi_1+\varpi_2\chi_2) + \mathcal{C}_1\mathcal{C}_8\pi_1}{\mathcal{C}_1 + 2\rho\pi_1^2(\varpi_1\chi_1+\varpi_2\chi_2)},$$

and $\mathcal{C}_8 = \varpi_1(1+2\rho\gamma_1) + \varpi_2(1+2\rho\gamma_2)$.

Numerical Results

- The following setup is considered for plotting the figures.
  - We utilize a Zipf(n, s) distribution with $n = |\mathcal{X}| = 100$ and $s = 0.4$.
  - We set $\rho = 0.2$, $\delta = 0.5$, $w_1 = w_2 = 1$, and $T = 10\,[\text{sec}]$.
  - The packet error rate ϵ_0 is initialized according to its upper bound $\Phi\left(\frac{\delta}{\sqrt{\delta(1-\delta)}}\right)$, where δ and $\Phi(.)$ denote the erasure probability and the cumulative distribution function of the standard normal Gaussian distribution, respectively.

a. Finding Optimal k_1 and k_2 (Different λ)

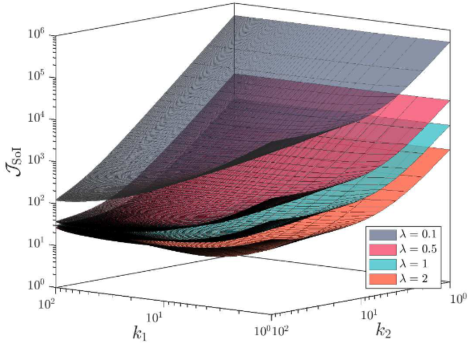


Figure 12: The interplay between the objective function and k_1 and k_2 for the EDT case and Zipf(100,0.4) distribution.

- Increasing the arrival rate *reduces* $\mathcal{J}_{\mathrm{SoI}}$ as well as the optimal k_1 and k_2.
- The same results hold for the other cases.
- From $\mathcal{J}_{\mathrm{SoI}}$ perspective: PDT $<$ EDT $<$ LDT

- The derived optimal values of k_1 and k_2 for all cases with different arrivals rates:

Table 2: Optimal number of selected packets for $n = 100$.

	EDT case		LDT case		PDT case	
Arrival rate	k_1	k_2	k_1	k_2	k_1	k_2
$\lambda = 0.1$	100	94	100	91	100	89
$\lambda = 0.5$	100	45	100	42	100	41
$\lambda = 1$	45	14	46	13	46	12
$\lambda = 2$	45	10	46	8	46	8

b. Finding Optimal k_1 and k_2 (Different w_1 and w_2)

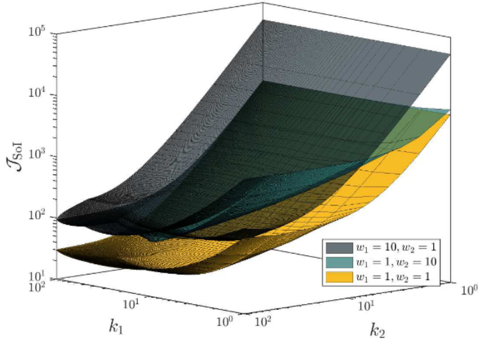


Figure 13: The interplay between the objective function and k_1 and k_2 for the EDT case and Zipf(100,0.4) distribution.

- Giving 10 times more weight to the arrivals of monitor 2 compared to those of monitor 1 equalizes the optimal k_1 and k_2.
- In this case, the transmitter equally filters around 33% of frequent and infrequent arrivals.

- The obtained information from Figure 13 and from its extension for $\delta = 0.25$ is given in the following table.

Table 3: Optimal number of selected packets for $n = 100$ and $\lambda = 1$.

	Weight parameters (w_1, w_2)					
	$(1,1)$		$(10,1)$		$(1,10)$	
Erasure probability	k_1	k_2	k_1	k_2	k_1	k_2
$\delta = 0.25$	47	17	100	60	33	34
$\delta = 0.5$	45	14	50	5	33	33

c. Finding Optimal λ

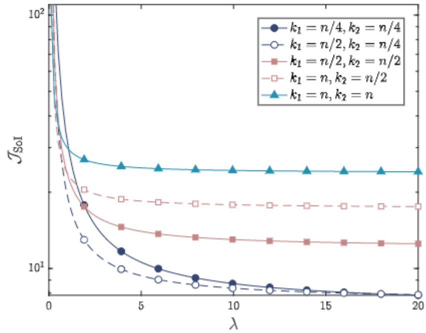


Figure 14: The objective function versus λ for the EDT case and Zipf(100,0.4) distribution.

- Increasing the input rate decreases $\mathcal{J}_{\text{SoI}}$; however, this decrease diminishes and saturates at higher rate values.
- By increasing the number of selected packets, lower input rates are required to reduce the penalty terms.
- From $\mathcal{J}_{\text{SoI}}$ perspective: PDT $<$ EDT $<$ LDT

Publication II

Semantic Source Coding for Two Users with Heterogeneous Goals

Pouya Agheli*, Nikolaos Pappas†, and Marios Kountouris*
*Communication Systems Department, EURECOM, Sophia Antipolis, France
†Dept. of Science and Technology, Linköping University, Norrköping Campus, Sweden
Email: pouya.agheli@eurecom.fr, nikolaos.pappas@liu.se, marios.kountouris@eurecom.fr

***Abstract*—We study a multiuser system in which an information source provides status updates to two monitors with heterogeneous goals. Semantic filtering is first performed to select the most useful realizations for each monitor. Packets are then encoded and sent so that each monitor can timely fulfill its goal. In this regard, some realizations are important for both monitors, while every other realization is informative for only one monitor. We determine the optimal real codeword lengths assigned to the selected packet arrivals in the sense of maximizing a weighted sum of semantics-aware utility functions for the monitors. Our analytical and numerical results provide the optimal design parameters for different arrival rates and highlight the improvement in timely status update delivery using semantic filtering and source coding.**

I. Introduction

Goal-oriented semantic communication is recently considered as a promising and timely research avenue towards realizing the long-standing vision of Shannon and Weaver [1] and incorporating the significance and the importance of information into the existing theoretic edifice. Despite various past endeavors [2]–[6], which remained at a conceptual level, leading to hardly any or no practically relevant application, the quest for such theory has recently gained new impetus [7]–[9], fueled by the emergence of networked intelligent systems and

timely source coding scheme for two users with heterogeneous goals. Specifically, we consider that only a fraction of the "least" ("most") frequent source realizations is important for the first (second) monitor. This setting model for instance the case in which one user is interested in regular/standard information for monitoring purposes or typical actuation (normal mode), whereas the other monitor tracks the outliers that could potentially represent some kind of threat to the system or a possibly dangerous situation (alarm mode). The notion of semantics (importance) is captured here through a metric of timeliness, which is a nonlinear function of AoI, for the received updates at both monitors

This work falls within the realm of source coding problem for status update systems in which the goal is to minimize the average age of information, such as in [14]–[17]. In [18], we proposed a semantics-aware encoding scheme for a single user in an error-free point-to-point status update link. Our paper extends prior work into multiuser systems with heterogeneous goals, which could also be competing or diverging for certain realizations. Specifically, we derive the optimal real codeword lengths that maximize a weighted sum of the semantics-aware utility functions for two heterogeneous monitors. Our analytical and numerical results characterize the promising performance

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Conclusion & Future Works

Conclusion

1. Studying the problem of timely source coding in single- and double-user status update systems.
2. Deriving the optimal codeword lengths to optimize
   - a weighted sum of timeliness and quadratic coding cost penalty in the single-user model,
   - a weighted sum of timeliness utility functions assigned to two users with heterogeneous goals in the double-user model.
3. Finding that semantic filtering and source coding can significantly reduce the number of required update packets in both models.

Future Works

- Some tasks for the near future:
  1. Defining a multi-dimensional formula for the value of arrival samples
  2. Developing an adaptive semantic filtering scheme
  3. Considering the impacts of error-prone channels and error control techniques
  4. Proceeding to PHY and MAC layers
  5. ...

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Thank You!