Interference Mitigation in Dynamic TDD MIMO Interference Channels

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***Abstract*—Dynamic Time Division Duplexing (DynTDD) is one of the key features that enable the dynamic or flexible uplink and downlink transmission and reception for a specific sub-frame in 5G mobile networks. However, the advantages of the DynTDD system are difficult to fully utilize due to the cross-link interference (CLI) arising from neighboring cells using different transmission directions on the same or partially- overlapping time-frequency resources. There are two types of cross-link interference; between the Base Stations (BS), which is known as BS-to-BS or DL-to-UL interference, and between User Equipment (UE) which is known as UE-to-UE or UL- to-DL interference. Rank deficiency of channel matrices is an important aspect of Multi-Input Multi-Output (MIMO) wireless systems. Poor scattering and the presence of single or very few direct paths are some reasons for rank deficiency in wireless channels. While the implications of rank deficient channels are well understood for the single user (SU) point-to-point setting, less is known for interference networks. In this paper, we give a sufficient condition for a full rank MIMO interfering channel, that outperforms the current state of the art, we provide also tighter necessary conditions for Interference Alignment (IA) feasibility, that are very close to the necessary and sufficient condition, which gives all the feasible cases. Then we extend a MIMO Interference Alignment (IA) feasibility framework to rank deficient channels, by investigating our published theorem for the necessary and sufficient condition in this case.**

***Index Terms*—Dynamic TDD, MIMO, rank deficient, interference alignment, Degree of Freedom**

I. Introduction

Beside the potential to significantly improve the overall resource utilization [1] and considerately reduce the latency [2], Dynamic Time Division Duplexing (DynTDD), brings some new challenges because of the introduction of cross-link interference (CLI), including DL-to-UL and UL-to-DL interference.

Studies focusing on resolving the BS-to-BS interference problem rather than the UE-to-UE interference have been much more prevalent. The reason for this is that during an UL transmission, DL to UL interference may cause substantial performance degradation, contrarily to the DL transmission where the DynTDD is only used in favor of it [3]. But according to [4], the UE-to-UE interference power level is low for the UEs in the center of the cell region but very high for the cell-edge UEs. Moreover, for network stability, it is also very important to handle the UE-to-UE interference of edge UEs. Thus, to further improve network capacity significantly, we have to resort to concurrent transmissions. Multiple concurrent transmission techniques (e.g., Zero Forcing (ZF), Interference Alignment (IA), and distributed MIMO) are proposed in which multiple senders jointly encode signals to multiple receivers so that interference is aligned or canceled and each receiver is able to decode its desired information.

The feasibility conditions of IA have been analyzed in [5]–[11]. [12] also mathematically characterizes the achievable Degrees-of-Freedom (DoF) of their proposed DIA technique for a given number of antennas at BS/MS. In [5] the authors show that for the MIMO Interfering Broadcast Channel (IBC) where each user has one desired data stream, a proper system is feasible. For the symmetric (which we call here uniform) MIMO Interfering Broadcast Channel (IBC), they provide a proper but infeasible region of antenna configurations by analyzing the difference between the necessary conditions and the sufficient conditions of linear IA feasibility. [11] established a necessary and sufficient condition on IA feasibility for the (full rank) MIMO Interfering Broadcast Multiple Access Channel (IBMAC), which characterizes the optimal sum DoF for various practical network configurations.

The main contributions of this paper are summarized as follows: the work reported in this paper comes to go beyond our study in [13] and [14]. We start by reporting the proper conditions considering centralized design for IA feasibility. Then we give a sufficient condition in a full rank MIMO IC, which is written in terms of the problem dimension (number of UEs, number of antennas, and the number of streams). After that, we propose tighter necessary conditions that show motivating numerical results since it is very close to the sufficient and necessary condition for IA feasibility. In the end, we give the numerical result that shows the gap between our sufficient condition, the sufficient and necessary condition, and the existing sufficient condition in the current state of the art. We highlight also the gap between our different proposed conjectures and the sufficient and necessary condition. In addition, for the reduced Rank MIMO IBMAC-IC we establish for a given system a comparative table between our previous IA methods in [13] and the necessary and sufficient condition in [14], to confirm the conclusion about the choice of the IA method, based on the rank of the interfering channel, that increase the feasible DoF.

II. Dynamic TDD System Model

We consider a MIMO system with two cells, one operating in DL and the other one in UL. Each cell has one BS of M antennas, with K_{ul} and K_{dl} interfering/interfered users in the UL and DL cell respectively. The kth DL UE and the lth UL UE have $N_{dl,k}$ and $N_{ul,l}$ antennas respectively. This scenario brings the two types of interference, the BS-to-BS interference, and the UE-to-UE interference between the UEs that are particularly on the edge of the two cells as shown in Fig 1. The channel between the lth user in the UL cell and the kth user in the DL cell is denoted as $\boldsymbol{H}_{k,l} \in \mathbb{C}^{N_{dl,k} \times N_{ul,l}}$ with $k \in [1, ..., K_{dl}]$ and $l \in [1, ..., K_{ul}]$. Denote $d_{dl,k}$ and $d_{ul,l}$ as the number of data streams from the DL BS to the kth DL UE and from the lth UL UE to the UL BS respectively. We denote the rank of the UE-to-UE interference channel (IC) as $r_{k,l}$. We have $r_{k,l}$ distinguishable significant paths contribute to $\boldsymbol{H}_{k,l}$, where distinguishable means with linearly independent antenna array responses from other paths, at both the Tx side and the Rx side. Then we can factorize $\boldsymbol{H}_{k,l}$ as:

$$\boldsymbol{H}_{k,l} = \boldsymbol{B}_{k,l} \boldsymbol{A}_{k,l}^{H} \quad (1)$$

with $\boldsymbol{B}_{k,l} \in \mathbb{C}^{N_{dl,k} \times r_{k,l}}$ and $\boldsymbol{A}_{k,l} \in \mathbb{C}^{N_{ul,l} \times r_{k,l}}$ are full rank matrices.

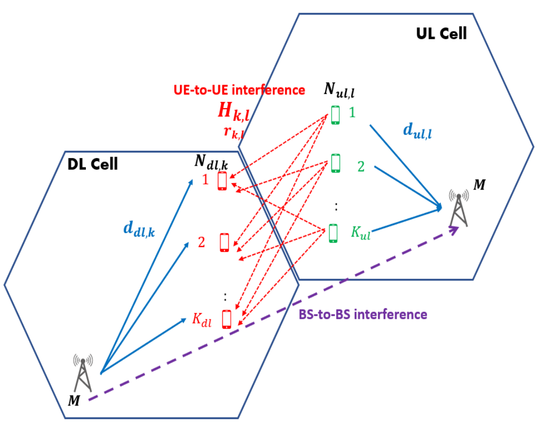

Fig. 1: DynTDD system model.

To analyze the UE-to-UE interference, we have both the DL and UL UEs will contribute to cancel each link of interference between them. We consider $\boldsymbol{F}_k \in \mathbb{C}^{N_{dl,k} \times d_{dl,k}}$ and $\boldsymbol{G}_l \in \mathbb{C}^{N_{ul,l} \times d_{ul,l}}$ as the Rx/Tx beamforming (BF) matrices at the kth DL and the lth UL users respectively. ZF from UL UE l to the DL UE k requires:

$$\boldsymbol{F}_k^H \boldsymbol{H}_{k,l} \boldsymbol{G}_l = 0, \forall k \in \{1, ..., K_{dl}\}, \forall l \in \{1, ..., K_{ul}\}\,. \quad (2)$$

Our system of Fig.1 is also called IBMAC (Interfering Broadcast–Multiple Access Channel) in [11] which corresponds to a two cells system with one cell being in DL (BC) and another in UL (MAC) and with interference between the two cells.

In our study, we suppose that the number of base station antennas is large enough so that all UL or DL UE streams can be supported, and that the BS-to-BS interference can be mitigated by exploiting a limited rank BS-to-BS channel [12]. Hence the IBMAC problem is then limited to the interference from UL users to DL users, which we may call IBMAC-IC (IBMAC Interference Channel). We assume:

$$d_{dl,k} \geq 1 \quad \text{and} \quad d_{ul,l} \geq 1. \quad (3)$$

III. IA Feasibility Conditions for DynTDD UE-to-UE generic Rank MIMO IBMAC

In this section we analyze the overall UL UE to DL UE interference, considering generic rank MIMO channels.

A. Proper Conditions

In [13] we have established the proper conditions, where the global proper conditions are given by [13, eq.(6)].

Note that this condition subsumes the SU MIMO conditions $d_{ul,l} \leq N_{ul,l}$, $d_{dl,k} \leq N_{dl,k}$ so that the number of variables on the LHS is non-negative. Apart from this proper condition for the overall system, we get an overall set of proper conditions by considering all subsystems also.

Theorem 1. ***Overall Proper Conditions for IA Feasibility in generic rank MIMO IBMAC-IC***
The conditions in [13, eq.(6)] should be satisfied also by any subsystem, i.e. the IBMAC-IC formed by any subset of the UL users and any subset of the DL users, such that:

$$\begin{aligned} &\sum_{l \in U_{ul}} d_{ul,l}(N_{ul,l} - d_{ul,l}) + \sum_{k \in U_{dl}} d_{dl,k}(N_{dl,k} - d_{dl,k}) \\ &\geq \sum_{k \in U_{dl}} \sum_{k \in U_{dl}} \min(r_{k,l} d_{dl,k}, r_{k,l} d_{ul,l}, d_{ul,l} d_{dl,k})\,. \quad (4) \\ &\forall\, U_{ul} \subseteq [1, ..., K_{ul}] \ \ \textit{and} \ \ \forall\, U_{dl} \subseteq [1, ..., K_{dl}] \end{aligned}$$

B. Necessary and sufficient Conditions

In our previous work [14] we have revisited the feasibility analysis framework of [7], [6] and [5]. Thus we have provided a detailed analysis of the UE-to-UE interference by shedding light on the channel matrices and the beamformers at Tx and Rx, which was of huge use to provide the necessary and sufficient condition for IA feasibility in a Reduced Rank MIMO IBMAC-IC. To better understand the following theorems and conjectures, we report here the representation of Jacobian matrices $\boldsymbol{J}$ and $\boldsymbol{J}_H$ that include all the UE-to-UE interference channels of our system model, where the details to obtain these matrices are given in [14]:

$$J = \begin{bmatrix} I_{d_{ul,1}} \otimes H_{11}^{(2)} & 0 & (H_{11}^{(3)})^T \otimes I_{d_{dl,1}} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & I_{d_{ul,K_{ul}}} \otimes H_{1K_{ul}}^{(2)} & (H_{1K_{ul}}^{(3)})^T \otimes I_{d_{dl,1}} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ I_{d_{ul,1}} \otimes H_{K_{dl}1}^{(2)} & 0 & 0 & (H_{K_{dl}1}^{(3)})^T \otimes I_{d_{dl,K_{dl}}} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & I_{d_{ul,K_{ul}}} \otimes H_{K_{dl}K_{ul}}^{(2)} & 0 & (H_{K_{dl}K_{ul}}^{(3)})^T \otimes I_{d_{dl,K_{dl}}} \end{bmatrix} \quad (5)$$

where the first two block columns form J_G and the last two form J_F.

The block $I_{d_{ul,l}} \otimes H_{kl}^{(2)}$ in J_G has dimensions $d_{ul,l}d_{dl,k} \times d_{ul,l}(N_{ul,l} - d_{ul,l})$, the block $(H_{kl}^{(3)})^T \otimes I_{d_{dl,k}}$ in J_F has dimensions $d_{ul,l}d_{dl,k} \times (N_{dl,k} - d_{dl,k})d_{dl,k}$.
The dimensions are for matrix J_G:
$\sum_{l=1}^{K_{ul}} \sum_{k=1}^{K_{dl}} d_{ul,l}d_{dl,k} \times \sum_{l=1}^{K_{ul}} (N_{ul,l} - d_{ul,l})\, d_{ul,l}$,
for matrix J_F :
$\sum_{l=1}^{K_{ul}} \sum_{k=1}^{K_{dl}} d_{ul,l}d_{dl,k} \times \sum_{k=1}^{K_{dl}} (N_{dl,k} - d_{dl,k})\, d_{dl,k}$.

$$A_{kl}^H = \begin{bmatrix} A_{kl}^{(1)} & A_{kl}^{(2)} \end{bmatrix},\ B_{kl}^H = \begin{bmatrix} B_{kl}^{(1)} & B_{kl}^{(2)} \end{bmatrix}. \quad (6)$$

The matrix blocks $A_{kl}^{(1)}$ and $B_{kl}^{(1)}$ have dimensions $r_{kl} \times d_{ul,l}$ and $r_{kl} \times d_{dl,k}$ respectively.

$$J_H = \begin{bmatrix} (I_{d_{ul,1}} \otimes B_{11}^{(1)H}) & 0 & (A_{11}^{(1)T} \otimes I_{d_{dl,1}}) & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & (I_{d_{ul,K_{ul}}} \otimes B_{K_{dl}K_{ul}}^{(1)H}) & 0 & (A_{K_{dl}K_{ul}}^{(1)T} \otimes I_{d_{dl,K_{dl}}}) \end{bmatrix} \quad (7)$$

where the first two block columns form J_B and the last two form J_A.

The necessary and sufficient condition for IA feasibility in a regular MIMO IBMAC-IC is given in the following Theorem:

Theorem 2. ***Necessary and Sufficient Condition for IA Feasibility in a Regular MIMO IBMAC-IC***
For a full rank MIMO IBMAC-IC, the DoF tuple $(d_{ul,1}, ..., d_{ul,K_{ul}}, d_{dl,1}, ..., \ d_{ul,K_{dl}})$ *is feasible almost surely if and only if* J *has full row rank.*

And the necessary and sufficient condition for IA feasibility in a reduced rank MIMO IBMAC-IC is provided and proved in [14], this condition is given by the following Theorem:

Theorem 3. ***Necessary and Sufficient Condition for IA Feasibility in Reduced Rank MIMO IBMAC-IC***
For a deficient rank MIMO IBMAC-IC, the DoF $(d_{ul,1}, ..., d_{ul,K_{ul}}, d_{dl,1}, ..., \ d_{ul,K_{dl}})$ *are feasible almost surely if and only if:*

$$\mathit{rank}(J) = \mathit{rank}(J_J) = \mathit{rank}([J \ \ J_H]) \quad (8)$$

i.e., the column space of J_H *in* (7) *should be contained in the column space of* J *in* (5).

IV. IA Feasibility Conditions for DynTDD UE-to-UE Full Rank MIMO IBMAC

In this section, we focus on the sufficient condition of IA feasibility in a full rank MIMO IBMAC-IC.The aim here is to give an easy formulation of a sufficient condition, in terms of the problem dimensions: the number of antennas at UL and DL UEs, the number of data stream and the number of users that are included in the UE-to-UE interference, rather than the sufficient and necessary condition in Theorem 2 given by a Jacobian matrix row rank. [11] finds sufficiency in the limited scenario in which all DL UEs and UL UEs have the same number of data streams $d_{dl,k} = d_{dl}, d_{ul,l} = d_{ul}$, for this assumption the number of antennas at DL $N_{dl,k}$ and at UL $N_{ul,l}$ must satisfy $mod(N_{dl,k} - d_{dl}, d_{ul}) = mod(N_{ul,l} - d_{ul}, d_{dl}) = 0$ so the IA is feasible. Within this scope, we establish a sufficient condition for IA feasibility given by our following Theorem 4, which gives a much greater DoF than the recent work in [11].

Theorem 4. ***Sufficient Condition for IA Feasibility in a Regular MIMO IBMAC-IC***
For a full rank MIMO IBMAC-IC, respecting the proper condition of Theorem 1, and if:

$$\forall k, l : (N_{ul,l} - d_{ul,l}) \geq d_{dl,k} \ \ \mathit{and} \ \ (N_{dl,k} - d_{dl,k}) \geq d_{ul,l} \quad (9)$$

then $(d_{ul,1}, ..., d_{ul,K_{ul}}, d_{dl,1}, ..., \ d_{ul,K_{dl}})$ *is feasible.*

The equation in (9) means that both the block matrix $I_{d_{ul,l}} \otimes H_{kl}^{(2)}$ in J_G and the block matrix $(H_{kl}^{(3)})^T \otimes I_{d_{dl,k}}$ in J_F should be full row rank.

For the proof, see the Appendix.

In the following, we introduce three conjectures that represents another sufficient condition, and two tighter necessary conditions for IA feasibility in regular MIMO IBMAC-IC.

Conjecture 1. ***Sufficient Condition for IA Feasibility in a Regular MIMO IBMAC-IC***
For a full rank MIMO IBMAC-IC, respecting the proper condition of Theorem 1, and if:

$$\forall k, l : (N_{ul,l} - d_{ul,l}) \geq d_{dl,k} \ \ \mathit{or} \ \ (N_{dl,k} - d_{dl,k}) \geq d_{ul,l} \quad (10)$$

and:

$$\sum_{k=1}^{K_{dl}} d_{dl,k} \min(N_{dl,k} - d_{dl,k}, \sum_{l} d_{ul,l} - \max_{i}(d_{ul,i})) + \sum_{l=1}^{K_{ul}} d_{ul,l} \min(N_{ul,l} - d_{u,l}, \sum_{k} d_{dl,k} - \max_{i}(d_{dl,i})) \geq \sum_{k=1}^{K_{dl}} \sum_{l=1}^{K_{ul}} d_{dl,k}d_{ul,l} \quad (11)$$

then $(d_{ul,1}, ..., d_{ul,K_{ul}}, d_{dl,1}, ..., \ d_{ul,K_{dl}})$ *is feasible.*

The equation in (10) means that either the block matrix $I_{d_{ul,l}} \otimes H_{kl}^{(2)}$ in J_G or the block matrix $(H_{kl}^{(3)})^T \otimes I_{d_{dl,k}}$ in J_F should be full row rank. The equation in (11) represents the tighter necessary version of the proper condition in (4).

Conjecture 2. ***Tighter Necessary Condition for IA Feasibility in a Regular MIMO IBMAC-IC***

For full rank MIMO channels, if the tuple of DoF $(d_{ul,1},...,d_{ul,K_{ul}},d_{dl,1},...,d_{dl,K_{dl}})$ *respects the proper condition of Theorem 1 and is feasible, then it may satisfy the following necessary condition:*

$$\sum_{k=1}^{K_{dl}} d_{dl,k} \min(N_{dl,k}-d_{dl,k}, \max_{l}(d_{ul,l}))+ \sum_{l=1}^{K_{ul}} d_{ul,l} \min(N_{ul,l}-d_{ul,l}, \max_{k}(d_{dl,k})) \geq \sum_{l=1}^{K_{ul}} \sum_{k=1}^{K_{dl}} d_{ul,l} d_{dl,k} \tag{12}$$

The LHS of equation (12) represents the number of non zero columns of the Jacobian matrix $\boldsymbol{J}$ in equation (5), and the RHS is the number of rows of $\boldsymbol{J}$.

Conjecture 3. ***Tighter Necessary Condition for IA Feasibility in a Regular MIMO IBMAC-IC***
For full rank MIMO channels, if the tuple of DoF $(d_{ul,1},...,d_{ul,K_{ul}},d_{dl,1},...,d_{dl,K_{dl}})$ *respects the proper condition of Theorem 1 and is feasible, then it may satisfy the following necessary condition:*

$$\begin{gathered} \forall k,l : (N_{ul,l}-d_{ul,l}) \geq d_{dl,k} \\ or \\ \forall k,l : (N_{dl,k}-d_{dl,k}) \geq d_{ul,l} \end{gathered} \tag{13}$$

The conjecture 3 means that either $\boldsymbol{J}_G$ or $\boldsymbol{J}_F$ is full row rank.

V. Results and Discussion

For the evaluation of the different given conditions for the IA feasibility, we give the numerical results in Table I. Thus we compare the number of combinations (a combination is a given number of data streams at each UL and DL UE) for different sum DoF, when considering the proper condition in Theorem 1, the necessary and sufficient condition in Theorem 2, our sufficient condition in Theorem 4, and the sufficient condition in [11, Theorem 3]. We choose as an example $K_{ul} = 2$ and $K_{dl} = 3$, for the following three systems:

- System 1: $N_{ul,1} = 3, N_{ul,2} = 7, N_{dl,1} = 2, N_{dl,2} = 3$ and $N_{dl,3} = 8$, which is the system that has been chosen in [11],
- System 2: $N_{ul,1} = 4, N_{ul,2} = 7, N_{dl,1} = 4, N_{dl,2} = 5$ and $N_{dl,3} = 6$,
- System 3: $N_{ul,1} = 7, N_{ul,2} = 7, N_{dl,1} = 6, N_{dl,2} = 5$ and $N_{dl,3} = 6$.

We get the following numerical results by doing an exhaustive search for all the possible combinations that satisfies each given theorem in Table I, and this process is repeated for different sum DoF. We give here a example to better understand the meaning of a combination, for System 1 when $SumDoF = 6$, the different possible combinations that respect the proper condition in Theorem 1 are:

$d_{ul,1} = 2, d_{ul,2} = 1, d_{dl,1} = 1, d_{dl,2} = 1$ and $d_{dl,3} = 1$
$d_{ul,1} = 1, d_{ul,2} = 2, d_{dl,1} = 1, d_{dl,2} = 1$ and $d_{dl,3} = 1$
$d_{ul,1} = 1, d_{ul,2} = 1, d_{dl,1} = 2, d_{dl,2} = 1$ and $d_{dl,3} = 1$
$d_{ul,1} = 1, d_{ul,2} = 1, d_{dl,1} = 1, d_{dl,2} = 2$ and $d_{dl,3} = 1$
$d_{ul,1} = 1, d_{ul,2} = 1, d_{dl,1} = 1, d_{dl,2} = 1$ and $d_{dl,3} = 2$

From these results, we can conclude that:

- The gap in term of the number of combinations between the proper (Theorem 1) and the necessary and sufficient condition (Theorem 2) is not negligible, and it is proportional to the number of antennas. Thus a feasible Sum DoF needs to be associated to feasible combinations (distribution of the DoF at UL and DL UE), so the IA is feasible,
- All the feasible cases are given by the necessary and sufficient condition (Theorem 2), our sufficient condition (Theorem 4) comes to cover a subset of these feasible cases, the size of this subset is quite interesting, since Theorem 4 is written in term of the problem dimension, and does not need the full row rank test on $\boldsymbol{J}$.
- When considering our sufficient condition (Theorem 4) with the sufficient condition mentioned before in the state of the art [11, Theorem 3], we notice how much our sufficient condition outperforms and improves the available state of the art.

Sum_{DoF}	5	6	7	8	9	10	11	12	13	14	15
Proper Theorem 1_{SYS1}	1	5	10	15	20	21	19	5	0	0	0
Theorem 2_{SYS1}	1	5	10	15	20	21	16	3	0	0	0
Theorem 4 $(9)_{SYS1}$	1	2	1	0	0	0	0	0	0	0	0
[11, Theorem $3]_{SYS1}$	1	0	0	1	0	0	0	0	0	0	0
Proper Theorem 1_{SYS2}	1	5	15	33	58	83	80	26	4	0	0
Theorem 2_{SYS2}	1	5	15	31	50	67	60	21	4	0	0
Theorem 4 $(9)_{SYS2}$	1	5	15	22	20	9	2	0	0	0	0
[11, Theorem $3]_{SYS2}$	1	0	0	0	0	0	1	0	0	0	0
Proper Theorem 1_{SYS3}	1	5	15	35	70	125	189	241	187	51	8
Theorem 2_{SYS3}	1	5	15	35	70	125	173	197	167	51	8
Theorem 4 $(9)_{SYS3}$	1	5	15	35	61	76	72	52	28	12	3
[11, Theorem $3]_{SYS3}$	1	0	0	1	0	0	1	0	0	0	0

TABLE I: Number of combinations for different Sum DoF in a full rank interference channel, $K_{ul} = 2$ and $K_{dl} = 3$

In Table II we give some numerical results for the three systems mentioned before, to evaluate the gap between the given conjectures and the necessary and sufficient condition in Theorem 2.

Sum_{DoF}	5	6	7	8	9	10	11	12	13	14	15
Proper Theorem 1_{SYS1}	1	5	10	15	20	21	19	5	0	0	0
Theorem 2_{SYS1}	1	5	10	15	20	21	16	3	0	0	0
Theorem 4 $(9)_{SYS1}$	1	2	1	0	0	0	0	0	0	0	0
Conjecture 1_{SYS1}	1	3	1	0	0	0	0	0	0	0	0
Conjecture 2_{SYS1}	1	4	6	8	12	14	8	2*	0	0	0
Conjecture 3_{SYS1}	1	5	9	11	11	7	2	0	0	0	0
Proper Theorem 1_{SYS2}	1	5	15	33	58	83	80	26	4	0	0
Theorem 2_{SYS2}	1	5	15	31	50	67	60	21	4	0	0
Theorem 4 $(9)_{SYS2}$	1	5	15	22	20	9	2	0	0	0	0
Conjecture 1_{SYS2}	1	5	4	11	11	4	1	0	0	0	0
Conjecture 2_{SYS2}	1	5	15	31	50	66	57	11	1	0	0
Conjecture 3_{SYS2}	1	5	15	31	49	61	46*	14*	3	0	0
Proper Theorem 1_{SYS3}	1	5	15	35	70	125	189	241	187	51	8
Theorem 2_{SYS3}	1	5	15	35	70	125	173	197	167	51	8
Theorem 4 $(9)_{SYS3}$	1	5	15	35	61	76	72	52	28	12	3
Conjecture 1_{SYS3}	1	5	4	12	14	30	24	38	28	10	0
Conjecture 2_{SYS3}	1	5	15	35	69	119	160	161	85	12	0
Conjecture 3_{SYS3}	1	5	15	35	70	125	173	197	167	51	8

TABLE II: Number of combinations for different Sum DoF in a full rank interference channel, $K_{ul} = 2$ and $K_{dl} = 3$

(*) : the given condition gives some combinations that are proper but not feasible. (Feasible = Theorem 2 is satisfied).
From these results, we can notice that:

- Conjecture 1 is another sufficient condition for IA feasibility that gives more feasible combinations comparing to Theorem 4, from our observation, Conjecture 1 can find some combinations which are not found by

Theorem 4, so Conjecture 1 and Theorem 4 can be complementary,
- Conjecture 2 and Conjecture 3 are very close to the necessary and sufficient conditions in Theorem 2, but at some DoF these conditions gives some combinations (at most two combinations) that are proper but not feasible, it is for this reason that these conditions are mentioned as a tighter necessary condition.

In Table III we move to a reduced rank MIMO IBMAC-IC, and we evaluate the DoF of uniform system ($N_{ul,l} = N_{ul}$, $N_{dl,k} = N_{dl}$, $d_{ul,l} = d_{ul}$, $d_{dl,k} = d_{dl}$, $r_{kl} = r$) with $N_{dl} = 4$, $N_{ul} = 5$, $K_{dl} = 4$ and $K_{ul} = 2$, for the different conditions established in [13] and the proper and sufficient conditions given by Theorem 3. In Table III we give different example than [14] in order to show that the previous conclusion mentioned in [14] are also true at different number of antennas. In the following we give the description of each element in Table III, where a generic tuple $(d_{dl}, d_{ul}, d_{tot})$ denotes the uniform DoF of a DL UE, an UL UE, and the overall UL and DL sum DoF:

- $(d_{p,dl}, d_{p,ul}, d_{p,tot})$ considering Theorem 2 in the centralized case,
- $(d_{d,dl}, d_{d,ul}, d_{d,tot})$ considering the distributed method, with DL UE DoF as in [13, eq. (31a)], UL UE DoF as in [13, eq. (31b)] (with n_F, n_G in Table III optimized as n_{F_d}, n_{G_d}),
- $(d_{c,dl}, d_{c,ul}, d_{c,tot})$ considering the combined method, with DL UE DoF as in [13, eq. (26)], the UL UE as in [13, eq. (27)] (with n_F, n_G in Table III optimized as n_{F_c}, n_{G_c}),
- $(d_{r,dl}, d_{r,ul}, d_{r,tot})$ considering Rx side ZF only as in [13, eq. (26)] with $n_F = K_{ul}$,
- $(d_{t,dl}, d_{t,ul}, d_{t,tot})$ considering Tx side ZF only as in [13, eq. (27)] with $n_G = K_{dl}$,
- $(d_{T3,dl}, d_{T3,ul}, d_{T3,tot})$ considering Theorem 3.

r	0	1	2	3	4
$(d_{p,dl}, d_{p,ul}, d_{p,tot})$	(5,4,28)	(4,2,20) or (3,4,20)	(4,1,18)	(3,1,14)	(3,1,14)
$(d_{d,dl}, d_{d,ul}, d_{d,tot})$	(5,4,28)	(4,2,20)	(1,4,12)	(2,0,8)$^{(**)}$	(0,4,8)$^{(**)}$
$(n_{F,d}, n_{G,d})$	(1,2)	(1,2)	(2,0)	(1,2)	(2,0)
$(d_{c,dl}, d_{c,ul}, d_{c,tot})$	(5,4,28)	(4,2,20)	(3,1,14)	(3,1,14)	(3,1,14)
$(n_{F,c}, n_{G,c})$	(1,2)	(1,2)	(2,0)	(2,0)	(2,0)
$(d_{r,dl}, d_{r,ul}, d_{r,tot})$	(5,4,28)	(3,4,20)	(3,1,14)	(3,1,14)	(3,1,14)
$(d_{t,dl}, d_{t,ul}, d_{t,tot})$	(5,4,28)	(5,0,20)$^{(**)}$	(5,0,20)$^{(**)}$	(5,0,20)$^{(**)}$	(5,0,20)$^{(**)}$
$(d_{T3,dl}, d_{T3,ul}, d_{T3,tot})$	(5,4,28)	(4,2,20)	(3,2,16)	(3,1,14)	(3,1,14)

TABLE III: DoF per user as a function of the rank of any cross link channel with $N_{ul} = 4$, $N_{dl} = 5$, $K_{ul} = 2$ and $K_{dl} = 4$.

$^{(**)}$: the given DoF does not satisfy the conditions in (3). If negative DoF results from a formula, this DoF will be set to zero logically.

The observations in [14, Table I] are always true in this case.

VI. Conclusions

In this paper we address the IA feasibility in MIMO IBMAC-IC, thus by considering a full rank interference channel, we establish a sufficient condition for IA feasibility, that is written in terms of the problem dimension (number of UEs, number of antennas, and number of streams). This condition outperform the existing sufficient condition in [11] in term of the feasible DoF region (combinations), to highlight this we provide a comparative table between our and the existing sufficient condition in the state of the art for IA feasibility in full rank MIMO IC. We propose also two conjectures, written in terms of the problem dimension, as a tighter necessary conditions for IA feasibility, and by numerical results, we show how much these conjectures are interesting since their results are very close to the necessary and sufficient condition given in Theorem 2. For the rank deficient MIMO IC we evaluate the DoF for a number of dimensions and compare the results between the proper conditions mentioned before in our previous work [13] including the centralized and distributed design, the zero-forcing in shared fashion or only considering Tx or Rx, and our sufficient condition given by Theorem 3, here we take another number of dimension than the one taken in [14] to confirm the observations given before in [14] regarding the rank of the MIMO IC for which the distributed method is interesting to be applied for the IA. The byproduct of our results is that the IA feasibility conditions that we provide are applicable to any MIMO interference network, for our system model as an example we consider the DynTDD interference network.

Appendix

Proof of Theorem 4: ***Diagonal Shift method***
For the ease of understanding of the following proof, we can divide $\boldsymbol{J}_F$ and $\boldsymbol{J}_G$ into sub-matrices $\boldsymbol{J}_{F_k}$ and $\boldsymbol{J}_{G_k}$ respectively, regarding the k^{th} receiver, such as:

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}_{G_1} & \boldsymbol{J}_{F_1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{J}_{G_2} & \boldsymbol{0} & \boldsymbol{J}_{F_2} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & & \ddots & \vdots \\ \boldsymbol{J}_{G_{K_{dl}}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_{F_{K_{dl}}} \end{bmatrix} \tag{14}$$

For each receiver $k \in [1, \ldots, K_{dl}]$, the matrices $\boldsymbol{J}_{F_k}$ and $\boldsymbol{J}_{G_k}$ are given by (15) and (16) respectively:

$$\boldsymbol{J}_{F_k} = \begin{bmatrix} \boldsymbol{J}_{F_{k1}}^T & \boldsymbol{J}_{F_{k2}}^T & \ldots & \boldsymbol{J}_{F_{kK_{ul}}}^T \end{bmatrix}^T \tag{15}$$

$$\boldsymbol{J}_{G_k} = \begin{bmatrix} \boldsymbol{J}_{G_{k1}} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}_{G_{k2}} & \ldots & \boldsymbol{0} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{J}_{G_{kK_{ul}}} \end{bmatrix} \tag{16}$$

With $l \in [1, \ldots, K_{ul}]$, we have:

$$\boldsymbol{J}_{F_{kl}} = \boldsymbol{H}_{kl}^{(3)\,T} \otimes \boldsymbol{I}_{d_{dl,k}} \tag{17a}$$

$$\boldsymbol{J}_{G_{kl}} = \boldsymbol{I}_{d_{ul,l}} \otimes \boldsymbol{H}_{kl}^{(2)} \tag{17b}$$

We prove here that for any system $(N_{dl,k}, N_{ul,l}, d_{dl,k}, d_{ul,l})$ satisfying Theorem 4, the associated matrix $\boldsymbol{J}$ to this system can be transformed to a permutation matrix with a rank equal to the number of rows of $\boldsymbol{J}$, i.e. $\boldsymbol{J}$ is a full row matrix, thus the IA is feasible. This transformation can be done following the coming steps acting on $\boldsymbol{J}_F$ then on $\boldsymbol{J}_G$ side, we call this proof as *Diagonal Shift method*:

- **Building Diagonals on $\boldsymbol{J}_F$**:
  - *First diagonal:* On the given matrix $\boldsymbol{J}$ at $\boldsymbol{J}_F$, we choose the longest diagonal*** from the 1^{st} element of $\boldsymbol{J}_{F_1}$ and we put to zero the other elements in the rows including this diagonal, we note the number of elements of this diagonal as n_1. If n_1 is equal or smaller than the number of rows of $\boldsymbol{J}_{F_1}$, we set the variable sh to 0 or 1 respectively,
  - *Second diagonal:* We choose the longest diagonal from the element at the 1^{st} column and the $(sh \times d_{dl,2} + 1)^{th}$ row of $\boldsymbol{J}_{F_2}$, i.e. the diagonal is shift down by $sh \times d_{dl,2}$ elements. We put to zero the other elements in the rows including this diagonal. We note the number of elements of this diagonal as n_2. If n_2 is equal or smaller than the number of rows of $\boldsymbol{J}_{F_2}$, we don't increment sh or we increment by 1 respectively,

  $\vdots$

  - K_{dl}^{th} *diagonal:* We choose the longest diagonal from the element at the 1^{st} column and the $(sh \times d_{dl,K_{dl}} + 1)^{th}$ row of $\boldsymbol{J}_{F_{K_{dl}}}$, i.e. the diagonal is shift down by $sh\, d_{dl,K_{dl}}$ elements.We put to zero the other elements in the rows including this diagonal.
- **Choosing elements from $\boldsymbol{J}_G$**:
  The following process is done for each $k \in [1, \ldots, K_{dl}]$:
  Whenever n_k, with $k \in [1, \ldots, K_{dl}]$, is smaller than the number of rows of $\boldsymbol{J}_{F_k}$ noted as m_k, we work on the $m_k - n_k$ remaining rows of $\boldsymbol{J}$ that don't include element from the previous chosen diagonals on $\boldsymbol{J}_{F_k}$. For those rows, we choose $m_k - n_k$ elements from $\boldsymbol{J}_{G_k}$. The column and row of each chosen element should be different from each other and also different from the previously selected element in $\boldsymbol{J}_{G_1}, \ldots, \boldsymbol{J}_{G_{k-1}}$.

***: Our longest diagonal should take n elements for a matrix $\boldsymbol{A} \in (m \times n)$ with $m \geq n$, it begins always at the 1^{st} column of $\boldsymbol{A}$:

- If the diagonal begins at the i^{th} row with $i \leq (m - n + 1)$, the diagonal will end at the n^{th} column and the $(i + n - 1)^{th}$ row;
- If the diagonal begins at i^{th} row with $i > (m-n+1)$; the diagonal will be stopped at the $(m-i+1)^{th}$ column and the m^{th} row and goes forward from $(m-i+2)^{th}$ column and the 1^{st} row.

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