Transceiver Design in Dynamic TDD with Reduced-Rank MIMO Interference Channels

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***Abstract*—In Dynamic Time Division Duplexing (DynTDD), the time slot allocation can be changed dynamically based on the changing communication requirements. This provides greater flexibility compared to traditional Time Division Duplexing (TDD) systems, where the time slot allocation is fixed. However, the advantages of the DynTDD system are difficult to fully utilize due to the cross-link interference (CLI) arising from neighboring cells using different transmission directions on the same or partially- overlapping time-frequency resources. There are two types of cross-link interference; between the Base Stations (BS), which is known as BS-to-BS or DL-to-UL interference, and between User Equipment (UE) which is known as UE-to-UE or UL-to-DL interference. Coordinated beamforming is an important signal-processing technique for interference mitigation in a cellular communication system. This paper studies zero-forcing (ZF) transmit beamforming design at initialization with and without water-filling, and the iterative weighted minimum mean-square error (WMMSE) algorithm to maximize the sum rate in Multiple Input Multiple Output UE-to-UE Interference Channel (MIMO-IC). This paper also investigates the non-uniform Degrees-of-Freedom (DoF) at UL UEs and/or at DL UEs which could increase the sum of DoF that is represented by the slope of the sum rate and leads to a higher sum rate at high SNR.**

***Index Terms*—Dynamic TDD, MIMO, rank deficient, interference alignment, Degree of Freedom, sum rate, WMMSE, zero-forcing, water-filling**

I. Introduction

MIMO systems have great potential to achieve high throughput in wireless systems [1]. In Downlink (DL) communications, when a certain knowledge of the Channel State Information (CSI) at the transmitter is available, the system throughput can be maximized. In our study we consider Dynamic Time Division Duplexing (DynTDD) systems, that have the potential to significantly improve the overall resource utilization [2] and considerately reduce the latency [3]. DynTDD brings some new challenges because of the introduction of cross-link interference (CLI), including DL-to-UL and UL-to-DL interference. Studies focusing on resolving the BS-to-BS interference problem rather than the UE-to-UE interference has been much more prevalent. The reason for this is that during the UL transmission, DL to UL interference may cause substantial performance degradation, contrarily to the DL transmission where the DynTDD is only used in favor of it [4]. But according to [5], the UE-to-UE interference power level is low for the UEs in the center of the cell region but very high for the cell-edge UEs. Moreover, for network stability, it is also very important to handle the UE-to-UE interference of edge UEs. Thus, to further improve network capacity significantly, we have to resort to concurrent transmissions. Multiple concurrent transmission techniques (e.g., Zero Forcing (ZF), Interference Alignment (IA), and distributed MIMO) are proposed in which multiple senders jointly encode signals to multiple receivers so that interference is aligned or canceled and each receiver is able to decode its desired information. The feasibility conditions of IA have been analyzed in [6]–[12]. [13] also mathematically characterizes the achievable DoF of their proposed Distributed Interference Alignment (DIA) technique for a given number of antennas at BS/ Mobile Station (MS).

The main contributions of this paper go beyond the results of our previous works on [14] and [15], we exploit the non-uniformity of the DoF at DL UEs and/or at the UL UEs, to increase the sum of DoF so the rate at high SNR. We validate this by numerical results and sum rate simulations, in which we consider a complete DynTDD system that uses ZF transmit filters at the DL BS to handle the intracell interference. For the sum rate maximization, we adopt an algorithm that uses at the initialization, the ZF beamformers at DL and UL UEs to cancel the UE-to-UE interference, and the ZF transmitter at DL BS, so the intracell interference between DL UEs could be canceled, and then in the iterative process, we use the WMMSE filters. We also consider the water-filling method to improve the performance at low SNR.

II. System Model and Problem Formulation

Consider a MIMO system with two cells (each having one BS), one operating in DL and the other one in UL. The UL and DL cells have M_{ul} and M_{dl} antennas respectively, with K_{ul} and K_{dl} interfering/interfered users in the UL and DL cell respectively. The k^{th} DL UE and the l^{th} UL UE are equipped with $N_{dl,k}$ and $N_{ul,l}$ antennas respectively. The different configuration in DynTDD between neighbors cells gives rise to two types of interference the UE-to-UE interference between the UEs that are particularly on the edge of the two cells as shown in Fig 1, and the BS-to-BS interference. Our system in Fig.1 is also called IBMAC (Interfering Broadcast–Multiple Access Channel) in [12] which corresponds to a two-cell system with one cell being in DL (BC) and another in UL (MAC) and with interference between the two cells.

In our study, we suppose that the number of base station antennas is large enough so that all UL or DL UE streams can be supported, and that the BS-to-BS interference can be mitigated by exploiting a limited rank BS-to-BS channel [13]. Hence the IBMAC problem is then limited to the interference from UL users to DL users, which we may call IBMAC-IC (IBMAC Interference Channel).

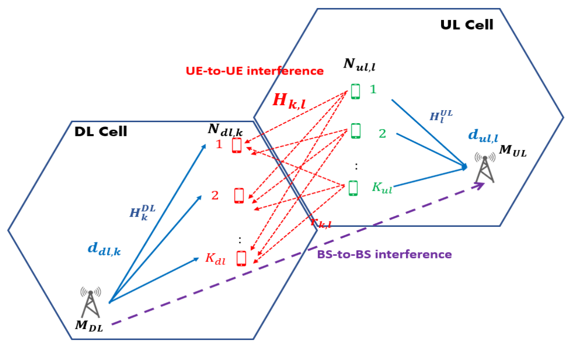

Fig. 1: DynTDD system Model

The l^{th} UL user sends $d_{ul,l}$ independent streams to the UL BS, $p_{ul,l}$ is non-negative UL power at user l, at the same time the k^{th} DL user receive $d_{dl,k}$ independent streams from the DL BS with the non-negative DL power allocation $p_{dl,k}$. Let $V_{dl,k} \in \mathbb{C}^{M_{dl} \times d_{dl,k}}$ denotes the beamformer that the DL BS uses to transmit the signal $s_{dl,k} \in \mathbb{C}^{d_{dl,k} \times 1}$ to the k^{th} DL UE. And $\boldsymbol{V}_{ul,l} \in \mathbb{C}^{N_{ul,l} \times d_{ul,l}}$ denotes the beamformer that the l^{th} UL UE uses to transmit the signal $s_{ul,l} \in \mathbb{C}^{d_{ul,l} \times 1}$ to the UL BS. We assume $E[\boldsymbol{s}_{dl,k}\boldsymbol{s}_{dl,k}^H] = \boldsymbol{I}$ and $E[\boldsymbol{s}_{ul,l}\boldsymbol{s}_{ul,l}^H] = \boldsymbol{I}$. We consider $\boldsymbol{U}_{dl,k} \in \mathbb{C}^{N_{dl,k} \times d_{dl,k}}$ and $\boldsymbol{U}_{ul,l} \in \mathbb{C}^{M_{ul} \times d_{ul,l}}$ as the Rx beamforming (BF) matrices at the k^{th} DL UE and the UL BS (from the l^{th} UL UE) respectively. Then the received signal at the k^{th} DL UE is given by $\boldsymbol{y}_{dl,k}$:

$$\boldsymbol{y}_{dl,k} = \underbrace{\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}\boldsymbol{s}_{dl,k}}_{\text{desired signal}} + \underbrace{\sum_{j=1,j\neq k}^{K_{dl}} \boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,j}\boldsymbol{s}_{dl,j}}_{\text{intracell interference}} + \underbrace{\sum_{l=1}^{K_{ul}} \boldsymbol{H}_{k,l}\boldsymbol{V}_{ul,l}s_{ul,l}}_{\text{UL To DL interference}} + \underbrace{\boldsymbol{n}_{dl,k}}_{\text{noise}}, \tag{1}$$

where the matrix $\boldsymbol{H}_k^{DL} \in \mathbb{C}^{N_{dl,k} \times M_{dl}}$ represents the channel from the DL BS to the k^{th} DL UE, and $H_l^{UL} \in \mathbb{C}^{M_{ul} \times N_{ul,l}}$ is the matrix of the channel from the l^{th} UL UE to the UL BS. We call $\boldsymbol{H}_k^{DL}$ and $\boldsymbol{H}_l^{UL}$ the direct channels. The interference channel between the l^{th}UL and DL UEs is denoted as $H_{k,l} \in \mathbb{C}^{N_{dl,k} \times N_{ul,l}}$. $n_{dl,k} \in \mathbb{C}^{N_{dl,k} \times 1}$ denotes the additive white Gaussian noise with distribution $\mathcal{CN} \in (0, \sigma_{dl,k}^2\boldsymbol{I})$ at the k^{th} DL UE. ZF from UL UE l to the DL UE k requires:

$$\boldsymbol{U}_{dl,k}^H\boldsymbol{H}_{k,l}\boldsymbol{V}_{ul,l} = \boldsymbol{0}, \forall k \in \{1, ..., K_{dl}\}, \forall l \in \{1, ..., K_{ul}\}. \tag{2}$$

For this system the achievable rate for the UL user l is given as:

$$\boldsymbol{R}_{ul,l} = \log\det\Bigg(\boldsymbol{I}_{M_{ul}} + \boldsymbol{H}_l^{UL}\boldsymbol{V}_{ul,l}\boldsymbol{V}_{ul,l}^H(\boldsymbol{H}_l^{UL})^H \Big(\sum_{i=1,i\neq l}^{K_{ul}} \boldsymbol{H}_i^{UL}\boldsymbol{V}_{ul,i}\boldsymbol{V}_{ul,i}^H(\boldsymbol{H}_i^{UL})^H + \sigma_{ul,l}^2\boldsymbol{I}_{M_{ul}}\Big)^{-1}\Bigg). \tag{3}$$

In our study we consider a ZF precoders $\boldsymbol{V}_{dl,l}$ at each UL UE given as:

$$\boldsymbol{V}_{ul,l} = \sqrt{\frac{p_{ul,l}}{\mathrm{Tr}(\boldsymbol{G}_{z,l}\boldsymbol{G}_{z,l}^H)}}\boldsymbol{G}_{z,l}. \tag{4}$$

such that $\boldsymbol{G}_{z,l}$ is the beamformer at l^{th} UL UE, the outcome from the ZF process satisfying (2), which is iterative in general, but can be in closed-form for some special cases. The detailed process to obtain $\boldsymbol{G}_{z,l}$ for a special system is mentioned in section V-A.

The achievable rate for the DL user k is given as:

$$\boldsymbol{R}_{dl,k} = \log\det\Bigg(\boldsymbol{I}_{N_{dl,k}} + \boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}\boldsymbol{V}_{dl,k}^H(\boldsymbol{H}_k^{DL})^H \Big(\sum_{j=1,j\neq k}^{K_{dl}} \boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,j}\boldsymbol{V}_{dl,j}^H(\boldsymbol{H}_k^{DL})^H + \sum_{l=1}^{K_{ul}} \boldsymbol{H}_{k,l}\boldsymbol{V}_{ul,l}\boldsymbol{V}_{ul,l}^H\boldsymbol{H}_{k,l}^H + \sigma_{dl,k}^2\boldsymbol{I}_{N_{dl,k}}\Big)^{-1}\Bigg). \tag{5}$$

In our study we choose $\boldsymbol{V}_{dl,k}$ as ZF transmit filter at the DL BS for the k^{th} DL UE, which is computed as:

$$\boldsymbol{V}_{dl} = b\bar{\boldsymbol{V}} = [\boldsymbol{V}_{dl,1}, \quad \boldsymbol{V}_{dl,2}, \quad \ldots, \quad \boldsymbol{V}_{dl,K_{dl}}], \tag{6a}$$

$$\bar{\boldsymbol{V}}_{dl} = \boldsymbol{H}^H\boldsymbol{F}\Big(\boldsymbol{F}^H\boldsymbol{H}\boldsymbol{H}^H\boldsymbol{F}\Big)^{-1}, \tag{6b}$$

$$b = \sqrt{\frac{\sum_{k=1}^{K_{dl}} p_{dl,k}}{\mathrm{Tr}(\bar{\boldsymbol{V}}_{dl}\bar{\boldsymbol{V}}_{dl}^H)}}. \tag{6c}$$

where $\boldsymbol{H} \in \mathbb{C}^{K_{dl}N_{dl,k} \times M_{dl}}$ contains the different DL channel matrices stacked row-wise and $\boldsymbol{F} \in \mathbb{C}^{K_{dl}N_{dl,k} \times K_{dl}d_{dl,k}}$ is blocked diagonal matrix, and are given such that:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1^{DL} \\ \vdots \\ \boldsymbol{H}_{K_{dl}}^{DL} \end{bmatrix} \tag{7}$$

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F}_{z,1} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F}_{z,2} & \ldots & \boldsymbol{0} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{F}_{z,K_{dl}} \end{bmatrix} \tag{8}$$

$\boldsymbol{F}z,k$ is the beamformer at k^{th} DL UE, the outcome from the ZF process satisfying (2) which is iterative in general but can be in closed-form for some special cases. The detailed process to obtain $\boldsymbol{F}_{z,l}$ for a special case is mentioned in sectionV-A. In the case of the WMMSE study, we use sometimes $\boldsymbol{U}_{dl,k} = \boldsymbol{F}_{z,k}$ to find the beams of initialization at DL-BS.

In the following table, we summarize the meanings of the notations used in this paper to provide a clear reference:

notation	references
$d_{dl,k}$, $d_{ul,l}$	number of data streams at the k^{th} DL UE, at the l^{th} UL UE respectively
$N_{dl,k}$, $N_{ul,l}$	number of antennas at the k^{th} DL UE, at the l^{th} UL UE respectively
K_{dl}, K_{ul}	number of DL UEs, of UL UEs respectively
M_{dl}, M_{ul}	number of antennas at the DL BS, at the UL BS respectively
$p_{dl,k}$, $p_{ul,l}$	the power at DL BS for the k^{th} DL UE, at the l^{th} UL UE respectively
$s_{dl,k}$, $s_{ul,l}$	Tx signal from DL BS to the k^{th} DL UE, from the l^{th} UL UE respectively
$\boldsymbol{H}_k^{DL}$,$\boldsymbol{H}_l^{UL}$	direct channel from the DL BS to the k^{th} DL UE, from the l^{th} UL UE to the UL BS respectively
$\boldsymbol{H}_{k,l}$	interference channel between the l^{th} UL UE and the k^{th} DL UE
$\boldsymbol{V}_{dl,k}$, $\boldsymbol{V}_{ul,l}$	Tx beamforming at the DL BS for the k^{th} DL UE, at the l^{th} UL UE respectively
$\boldsymbol{U}_{dl,k}, \boldsymbol{U}_{ul,l}$	Rx beamforming at the k^{th} DL UE, at the UL BS

TABLE I: Notation.

III. IA Feasibility Conditions for DynTDD UE-to-UE generic Rank MIMO IBMAC

In [14] we have established the proper conditions, where the global proper conditions are given by [14, eq.(6)], we have also given different conditions for IA feasibility derived from centralized and distributed designs. And in [15] we have revisited the feasibility analysis framework of [8], [7] and [6]. Thus we have provided a detailed analysis of the UE-to-UE interference by shedding light on the channel matrices and the beamformers at Tx and Rx, which was of huge use to provide the necessary and sufficient condition for IA feasibility in a Reduced Rank MIMO IBMAC-IC, that is represented in [15, Theorem 4].

In this section, we analyze the feasibility of the combined method that is given in [14, eq.(26), eq.(27)]. For this, we compare the DoF given by the combined method in [14, eq.(26), eq.(27)] to the DoF given by the sufficient and necessary condition for a generic rank interference channel in [15, Theorem 4], which is a precise characterization of the feasible DoF. And we make our observation in the following conjecture:

Conjecture 1.
For a DynTDD system, if the DoF tuple $(d_{ul,1},...,d_{ul,K_{ul}},d_{dl,1},...,d_{dl,K_{dl}})$ satisfies the condition for the combined method in [14, eq.(26), eq.(27)], then this DoF is almost surely feasible.

Then we exploit the non-uniform DoF between DL UEs and between UL UEs, i.e. when the number of the data stream at each DL UE, $d_{dl,k}$, or at each UL UE, $d_{ul,l}$, could be different from each other. As a result, we give the following conjecture:

Conjecture 2.
In DynTDD systems, if the DoF tuple $(d_{ul,1},...,d_{ul,K_{ul}},d_{dl,1},...,d_{dl,K_{dl}})$ is feasible for IA (i.e. satisfy [15, Theorem 4]), and present a non-uniform DoF at Rx (DL UEs) and/or at Tx (UL UEs), so the resulting sum of DoF would be surely equal or greater than the sum of DoF when assuming the feasible uniform DoF.

IV. Numerical Results

To analyze the observations given in Conjecture 1 and Conjecture 2, we give Table II, in which we consider a MIMO IBMAC-IC, and we evaluate the DoF of the system $N_{ul}=3$, $N_{dl}=6$, $K_{ul}=2$ and $K_{dl}=4$. In this table, we evaluate the different conditions established in [14] and the proper and sufficient conditions given by [15, Theorem 4]. Where a generic tuple (d_{dl},d_{ul},d_{tot}) denotes the DoF at DL UEs, at UL UEs, and the overall UL and DL sum of DoF:

- $(d_{p,dl},d_{p,ul},d_{p,tot})$ considering [15, Theorem 2] in the centralized case,
- $(d_{d,dl},d_{d,ul},d_{d,tot})$ considering the distributed method, with DL UE DoF as in [14, eq. (31a)], UL UE DoF as in [14, eq. (31b)] (with n_F, n_G in Table II optimized as n_{F_d}, n_{G_d}),
- $(d_{c,dl},d_{c,ul},d_{c,tot})$ considering the combined method, with DL UE DoF as in [14, eq. (26)], the UL UE as in [14, eq. (27)] (with n_F, n_G in Table II optimized as n_{F_c}, n_{G_c}),
- $(d_{r,dl},d_{r,ul},d_{r,tot})$ considering Rx side ZF only as in [14, eq. (26)] with $n_F=K_{ul}$,
- $(d_{t,dl},d_{t,ul},d_{t,tot})$ considering Tx side ZF only as in [14, eq. (27)] with $n_G=K_{dl}$,
- $(d_{T4,dl},d_{T4,ul},d_{T4,tot})$ considering [15, Theorem 4].

r	0	1	2	3
$(d_{p,dl}$,$d_{p,ul}$,$d_{p,tot})$	(6,3,30)	(5,2,25)	(5,1,22)	(5,1,22)
$(d_{d,dl}$,$d_{d,ul}$,$d_{d,tot})$	(6,3,30)	(5,1,22)	(2,3,14)or (4,0,16)*	(3,0,12)*
$(n_{F,d}$,$n_{G,d})$	(1,2)	(1,2)	(1,2) or (2,0)	(1,2)
$(d_{c,dl}$,$d_{c,ul}$,$d_{c,tot})$	(6,3,30)	(5,1,22)	(4,1,18)	(4,1,18)
$(n_{F,c}$,$n_{G,c})$	(1,2)	(1,2)	(2,0)	(2,0)
$(d_{r,dl},d_{r,ul},d_{r,tot})$	(6,3,30)	(4,3,18)	(2,3,14)	(0,3,6)*
$(d_{t,dl},d_{t,ul},d_{t,tot})$	(6,3,30)	(6,0,24)*	(6,0,24)*	(6,0,24)*
$(d_{T4,dl},d_{T4,ul},d_{T4,tot})$	(6,3,30)	(5,1,22)	((5,5,4,4),1,20)**	(4,1,18)

TABLE II: DoF per user as a function of the rank of any cross-link channel with $N_{ul}=3$, $N_{dl}=6$, $K_{ul}=2$ and $K_{dl}=4$.

(*): the given DoF does not satisfy the condition in [15, eq.(3)],
(**): the given DoF represents a non-uniform DoF at DL UEs, of the form $((d_{dl,1},d_{dl,2},d_{dl,3},d_{dl,4}),d_{ul},d_{tot})$

In Table II we can conclude that all the given DoF by the combined method [14, eq. (26), eq.(27)] is feasible as long as this DoF satisfies the necessary and sufficient condition in [15, Theorem 4].

For Conjecture 2, we can observe, in Table II for $r=2$ and when considering the condition in [15, Theorem 4], that the non uniform tuple DoF $d_{ul,1}=d_{ul,2}=1, d_{dl,1}=d_{dl,2}=5, d_{dl,3}=d_{dl,4}=4$, which gives a sum of DoF equal to 20, is feasible. Otherwise, if we assume a uniform DoF (i.e. $d_{ul,1}=d_{ul,2}$ and $d_{dl,1}=d_{dl,2}=d_{dl,3}=d_{dl,4}$) we are limited to a feasible sum of DoF equal to 18. So considering a different number of the data streams at Rx and Tx users could be interesting to increase the sum of DoF, so the rate at high SNR.

V. Simulations

In this section, we evaluate the sum rate at DL and UL UEs for different scenarios regarding the rank of the MIMO IBMAC-IC and the used beamformers. We start by giving an example of how to obtain the ZF precoders at UL UEs and the ZF decoders at DL UEs, for a closed-form case, and describe the algorithm that we followed to perform the water-filling. Then we show the result of the sum rate simulation.

A. The ZF precoders at UL UEs and the ZF decoders at DL UEs

In this subsection, we give some details about how we obtain the ZF precoders $\boldsymbol{G}_{z,l}$ and the ZF decoders $\boldsymbol{F}_{z,k}$ in closed form case, which allow us to satisfy the condition to cancel all the interference links from the UL UEs to the DL UEs given in equation (2). We take the system $N_{ul} = 3$, $N_{dl} = 6$, $K_{ul} = 2$ and $K_{dl} = 4$, with interference channel matrix of rank $r = 2$ and the data stream $d_{ul,1} = d_{ul,2} = 1, d_{dl,1} = d_{dl,2} = 5$ and $d_{dl,3} = d_{dl,4} = 4$. We give the following steps that show how we obtain $\boldsymbol{G}_{z,l}$ and $\boldsymbol{F}_{z,k}$:

Step 0: We generate all the interference channels matrices $\boldsymbol{H}_{11}, \boldsymbol{H}_{12}, \boldsymbol{H}_{21}, \boldsymbol{H}_{22}, \boldsymbol{H}_{31}, \boldsymbol{H}_{32}, \boldsymbol{H}_{41}$ and $\boldsymbol{H}_{42}$ of rank $r = 2$.

Step 1: The UL UE 1 cancels the stream from UL UE 1 to DL UE 1. So the singular value decomposition (SVD) of the interference channel matrix $\boldsymbol{H}_{11}$ gives:

$$[\boldsymbol{U}_{t1}\boldsymbol{S}_{t1}\boldsymbol{V}_{t1}] = SVD(\boldsymbol{H}_{11}). \tag{9}$$

$\boldsymbol{S}_{t1}$ [1] is given such that:

$$\boldsymbol{S}_{t1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta_{1,1} & 0 \\ 0 & 0 & \beta_{1,2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

with $\beta_{1,1}$ and $\beta_{1,2}$ are the non-zero singular values of $\boldsymbol{H}_{11}$. Then we take $\boldsymbol{V}_{N1} = \boldsymbol{V}_{t1}$ and apply it to Tx 1(UL UE 1), so the new interference channel matrices become:

$$\boldsymbol{H}_{N1,k1} = \boldsymbol{H}_{k1}\boldsymbol{V}_{N1}, \forall k \in [1, ..., K_{dl}] \tag{11}$$

The resulting $\boldsymbol{H}_{N1,11}$ has zeros at the first column, thus the interference from the UL UE 1 to the DL UE 1 is canceled by the UL UE 1.

Step 2: The UL UE 2 cancels the stream from UL UE 2 to DL UE 2. So the singular value decomposition (SVD) of the interference channel matrix $\boldsymbol{H}_{22}$ gives:

$$[\boldsymbol{U}_{t2}\boldsymbol{S}_{t2}\boldsymbol{V}_{t2}] = SVD(\boldsymbol{H}_{22}). \tag{12}$$

where the positions of the two non-zero singular values of $\boldsymbol{S}_{t2}$ are as those of $\boldsymbol{S}_{t1}$.
Then we take $\boldsymbol{V}_{N2} = \boldsymbol{V}_{t2}$ and apply it to Tx 2 (UL UE 2), so the new interference channel matrices become:

$$\boldsymbol{H}_{N2,k2} = \boldsymbol{H}_{k2}\boldsymbol{V}_{N2}, \forall k \in [1, ..., K_{dl}] \tag{13}$$

The resulting $\boldsymbol{H}_{N2,22}$ has zeros at the first column, thus the interference from the UL UE 2 to the DL UE 2 is canceled by UL UE 2.

Step 3: The DL UE 1 cancels the stream coming from UL UE 2. So we take the new channel from UL UE 2 to DL UE 1 after step 2 $\boldsymbol{H}_{N2,12}$, and we consider the SVD of $\boldsymbol{H}_{N2p,12}$ which is the first column of $\boldsymbol{H}_{N2,12}$ (the stream from UL UE 2 to DL UE 1):

$$[\boldsymbol{U}_1\boldsymbol{S}_1\boldsymbol{V}_1] = SVD(\boldsymbol{H}_{N2p,12}). \tag{14}$$

Then we take $\boldsymbol{U}_1^H$ and apply it to Rx 1 (DL UE 1), so the new interference channel matrices become:

$$\boldsymbol{H}_{n1,1l} = \boldsymbol{U}_1^H\boldsymbol{H}_{Nl,1l}, \forall l \in [1, ..., K_{ul}] \tag{15}$$

$\boldsymbol{S}_1$ [1] is given such that:

$$\boldsymbol{S}_1 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \gamma_1]^T \tag{16}$$

with γ_1 is the non-zero singular value of $\boldsymbol{H}_{N2p,12}$.
The resulting $\boldsymbol{H}_{n1,12}$ has $d_{dl,2}$ zeros at the first column, thus the interference from the UL UE 2 to the DL UE 1 is canceled at the DL UE 1.

Step 4: The DL UE 2 cancels the stream coming from UL UE 1. So we take the new channel from UL UE 1 to DL UE 2 after step 1 $\boldsymbol{H}_{N1,21}$, and we consider the SVD of $\boldsymbol{H}_{N1p,21}$ which is the first column of $\boldsymbol{H}_{N1,21}$ (the stream from UL UE 1 to DL UE 2):

$$[\boldsymbol{U}_2\boldsymbol{S}_2\boldsymbol{V}_2] = SVD(\boldsymbol{H}_{N1p,21}). \tag{17}$$

where the positions of the non-zero singular value of $\boldsymbol{S}_2$ is as that of $\boldsymbol{S}_1$.
Then we take $\boldsymbol{U}_2^H$ and apply it to Rx 2 (DL UE 2), so the new interference channel matrices become:

$$\boldsymbol{H}_{n2,2l} = \boldsymbol{U}_2^H\boldsymbol{H}_{Nl,2l}, \forall l \in [1, ..., K_{ul}] \tag{18}$$

The resulting $\boldsymbol{H}_{n2,21}$ has $d_{dl,1}$ zeros at the first column, thus the interference from the UL UE 1 to the DL UE 2 is canceled at the DL UE 2.

Step 5: The DL UE 3 cancels the stream coming from UL UE 1 and the stream coming from UL UE 2. So we consider the SVD of the matrix $\boldsymbol{H}_{c,3}$ given by:

$$\boldsymbol{H}_{c,3} = \begin{bmatrix} h_{N1,31}^{11} & h_{N1,31}^{21} & h_{N1,31}^{31} & h_{N1,31}^{41} & h_{N1,31}^{51} \\ h_{N2,32}^{11} & h_{N2,32}^{21} & h_{N2,32}^{31} & h_{N2,32}^{41} & h_{N2,32}^{51} \end{bmatrix}^T \tag{19}$$

such that $h_{N1,31}^{ji}$ represents the element of $\boldsymbol{H}_{N1,31}$ at the i^{th} column and the j^{th} line:

$$[\boldsymbol{U}_3\boldsymbol{S}_3\boldsymbol{V}_3] = SVD(\boldsymbol{H}_{c,3}) \tag{20}$$

$\boldsymbol{S}_3$ [1] is given such that:

$$\boldsymbol{S}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & \gamma_{3,1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{3,2} \end{bmatrix}^T \tag{21}$$

with $\gamma_{3,1}$ and $\gamma_{3,2}$ are the non-zero singular values of $\boldsymbol{H}_{c,3}$. Then we take $\boldsymbol{U}_3^H$ and apply it to Rx 3 (DL UE 3), so the new interference channel matrices become:

$$\boldsymbol{H}_{n3,3l} = \boldsymbol{U}_3^H\boldsymbol{H}_{Nl,3l}, \forall l \in [1, ..., K_{ul}] \tag{22}$$

The resulting $\boldsymbol{H}_{n3,31}$ and $\boldsymbol{H}_{n3,32}$ have $d_{dl,3}$ zeros at the first column, thus the interference from the UL UE 1 and from UL UE 2 to the DL UE 3 are canceled at the DL UE 3.

Step 6: The DL UE 4 cancels the stream coming from UL UE 1 and the stream coming from UL UE 2. So we consider the SVD of the matrix $\boldsymbol{H}_{c,4}$ which is similar to $\boldsymbol{H}_{c,3}$ with considering $\boldsymbol{H}_{N1,41}$ and $\boldsymbol{H}_{N2,42}$:

$$[\boldsymbol{U}_4\boldsymbol{S}_4\boldsymbol{V}_4] = SVD(\boldsymbol{H}_{c,4}). \tag{23}$$

where the positions of the two non-zero singular values of $\boldsymbol{S}_4$ are as those of $\boldsymbol{S}_3$.
Then we take $\boldsymbol{U}_4^H$ and apply it to Rx 4 (DL UE 4), so the new interference channel matrices become:

$$\boldsymbol{H}_{n4,4l} = \boldsymbol{U}_4^H\boldsymbol{H}_{Nl,4l}, \forall l \in [1, ..., K_{ul}] \tag{24}$$

The resulting $\boldsymbol{H}_{n4,41}$ and $\boldsymbol{H}_{n4,42}$ have $d_{dl,4}$ zeros at the first column, thus the interference from the UL UE 1 and from UL UE 2 to the DL UE 4 are canceled at the DL UE 4.

Finally, $\boldsymbol{F}_{z,1} = \boldsymbol{U}_1[:, 1 : d_{dl,1}]$, $\boldsymbol{F}_{z,2} = \boldsymbol{U}_2[:, 1 : d_{dl,2}]$, $\boldsymbol{F}_{z,3} = \boldsymbol{U}_3[:, 1 : d_{dl,3}]$ and $\boldsymbol{F}_{z,4} = \boldsymbol{U}_4[:, 1 : d_{dl,4}]$; $\boldsymbol{G}_{z,1} = \boldsymbol{V}_{N1}[:, 1 : d_{ul,1}]$ and $\boldsymbol{G}_{z,2} = \boldsymbol{V}_{N2}[:, 1 : d_{ul,2}]$.

[1]This distribution of singular values is used to dedicate the first effective antennas to the reception/transmission of the useful signal

B. Waterfilling algorithm

In the following, a method for the application of the MIMO water-filling algorithm for broadband channels is presented. The sum rate at DL for the initialization (when considering the ZF between UL and DL UEs and also the ZF between DL BS and DL UEs) could be written such that:

$$\begin{aligned}\boldsymbol{R}_{dl} &= \sum_{k=1}^{K_{dl}} \log\det\Big(\boldsymbol{I}_{N_{dl,k}} + \frac{1}{\sigma_n^2}(\boldsymbol{F}_{z,k}^H\boldsymbol{F}_{z,k})^{-1} \\ &\Big(\boldsymbol{F}_{z,k}^H\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}\boldsymbol{Q}_{dl,k}\boldsymbol{V}_{dl,k}^H(\boldsymbol{H}_k^{DL})^H\boldsymbol{F}_{z,k}\Big)\Big) \\ &= \sum_{k=1}^{K_{dl}} \log\det\Big(\boldsymbol{I}_{N_{dl,k}} + \frac{1}{\sigma_n^2}\Big(\boldsymbol{V}_{dl,k}^H(\boldsymbol{H}_k^{DL})^H\boldsymbol{F}_{z,k} \\ &(\boldsymbol{F}_{z,k}^H\boldsymbol{F}_{z,k})^{-1}\boldsymbol{F}_{z,k}^H\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}\boldsymbol{Q}_{dl,k}\Big)\Big),\end{aligned} \tag{25}$$

with $\boldsymbol{Q}_{dl,k} = \boldsymbol{I}_{d_{dl,k}}$, and the DL transmit power constraint is $\sum_{k=1}^{K_{dl}} \mathrm{Tr}(\boldsymbol{Q}_{dl,k}\boldsymbol{V}_{dl,k}^H\boldsymbol{V}_{dl,k}) = P$, P is the power budget available at the DL BS.

Now, we consider the eigendecomposition of $\boldsymbol{V}_{dl,k}^H\boldsymbol{V}_{dl,k}$ given by:

$$\boldsymbol{V}_{dl,k}^H\boldsymbol{V}_{dl,k} = \tilde{\boldsymbol{X}}_{dl,k}\tilde{\boldsymbol{\Sigma}}_{dl,k}\tilde{\boldsymbol{X}}_{dl,k}^H. \tag{26}$$

where $\tilde{\boldsymbol{X}}_{dl,k}\tilde{\boldsymbol{X}}_{dl,k}^H = \tilde{\boldsymbol{X}}_{dl,k}^H\tilde{\boldsymbol{X}}_{dl,k} = \boldsymbol{I}$, and $\tilde{\boldsymbol{\Sigma}}_{dl,k} = \tilde{\boldsymbol{\Sigma}}_{dl,k}^{1/2}\tilde{\boldsymbol{\Sigma}}_{dl,k}^{1/2}$ is a positive diagonal matrix. Let $\boldsymbol{Q}_{dl,k}^{'} = \tilde{\boldsymbol{\Sigma}}_{dl,k}^{1/2}\tilde{\boldsymbol{X}}_{dl,k}^H\boldsymbol{Q}_{dl,k}\tilde{\boldsymbol{X}}_{dl,k}\tilde{\boldsymbol{\Sigma}}_{dl,k}^{1/2}$ and $\boldsymbol{V}_{dl,k}^{'} = \boldsymbol{V}_{dl,k}\tilde{\boldsymbol{X}}_{dl,k}\tilde{\boldsymbol{\Sigma}}_{dl,k}^{-1/2}$. So with $\boldsymbol{Q}_{dl,k}^{'}$ and $\boldsymbol{V}_{dl,k}^{'}$ (25) could be written such that:

$$\begin{aligned}\boldsymbol{R}_{dl} &= \sum_{k=1}^{K_{dl}} \log\det\Big(\boldsymbol{I}_{N_{dl,k}} + \frac{1}{\sigma_n^2}(\boldsymbol{V}_{dl,k}^{'H}(\boldsymbol{H}_k^{DL})^H \\ &\boldsymbol{F}_{z,k}(\boldsymbol{F}_{z,k}^H\boldsymbol{F}_{z,k})^{-1}\boldsymbol{F}_{z,k}^H\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}^{'}\boldsymbol{Q}_{dl,k}^{'})\Big),\end{aligned} \tag{27}$$

with the DL transmit power constraint$\sum_{k=1}^{K_{dl}} \mathrm{Tr}(\boldsymbol{Q}_{dl,k}^{'}) = P$.

Then, we consider the following eigendecomposition:

$$\begin{aligned}&\frac{1}{\sigma_n^2}\boldsymbol{V}_{dl,k}^{'H}(\boldsymbol{H}_k^{DL})^H\boldsymbol{F}_{z,k}(\boldsymbol{F}_{z,k}^H\boldsymbol{F}_{z,k})^{-1}\boldsymbol{F}_{z,k}^H\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}^{'} \\ &= \boldsymbol{X}_{dl,k}\boldsymbol{\Sigma}_{dl,k}\boldsymbol{X}_{dl,k}^H.\end{aligned} \tag{28}$$

where $\boldsymbol{X}_{dl,k}\boldsymbol{X}_{dl,k}^H = \boldsymbol{X}_{dl,k}^H\boldsymbol{X}_{dl,k} = \boldsymbol{I}$, and $\boldsymbol{\Sigma}_{dl,k} = \boldsymbol{\Sigma}_{dl,k}^{1/2}\boldsymbol{\Sigma}_{dl,k}^{1/2}$ is a positive diagonal matrix. We note $\boldsymbol{V}_{dl,k}^{''} = \boldsymbol{V}_{dl,k}^{'}\boldsymbol{X}_{dl,k}$ and $\boldsymbol{Q}_{dl,k}^{''} = \boldsymbol{X}_{dl,k}^H\boldsymbol{Q}_{dl,k}^{'}\boldsymbol{X}_{dl,k}$, then $\boldsymbol{V}_{dl,k}^{'}\boldsymbol{Q}_{dl,k}^{'}\boldsymbol{V}_{dl,k}^{'H} = \boldsymbol{V}_{dl,k}^{''}\boldsymbol{Q}_{dl,k}^{''}\boldsymbol{V}_{dl,k}^{''H}$.

So the sum rate at DL in (27) becomes:

$$\begin{aligned}\boldsymbol{R}_{dl} &= \sum_{k=1}^{K_{dl}} \log\det\Big(\boldsymbol{I}_{N_{dl,k}} + \frac{1}{\sigma_n^2}(\boldsymbol{V}_{dl,k}^{"H}(\boldsymbol{H}_k^{DL})^H \\ &\boldsymbol{F}_{z,k}(\boldsymbol{F}_{z,k}^H\boldsymbol{F}_{z,k})^{-1}\boldsymbol{F}_{z,k}^H\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}^{"}\boldsymbol{Q}_{dl,k}^{"})\Big) \\ &= \sum_{k=1}^{K_{dl}} \log\det\left(\boldsymbol{I}_{N_{dl,k}} + \boldsymbol{\Sigma}_{dl,k}\boldsymbol{Q}_{dl,k}^{"}\right),\end{aligned} \tag{29}$$

The DL transmit power constraint becomes $\sum_{k=1}^{K_{dl}} \mathrm{Tr}(\boldsymbol{Q}_{dl,k}^{''}) = \sum_{k=1}^{K_{dl}} \mathrm{Tr}(\boldsymbol{Q}_{dl,k}^{'}\boldsymbol{X}_{dl,k}\boldsymbol{X}_{dl,k}^H) = \sum_{k=1}^{K_{dl}} \mathrm{Tr}(\boldsymbol{Q}_{dl,k}^{'}) = P$.

We have $\boldsymbol{Q}_{dl,k}^{''} = diag\{p_{k,1},\ldots,p_{k,d_{dl,k}}\}$ and $\boldsymbol{\Sigma}_{dl,k} = diag\{\sigma_{k,1},\ldots,\sigma_{k,d_{dl,k}}\}$, $p_{k,i}$ represents the power given to the k^{th} DL UE at the antennas with i^{th} data stream. Hence (29) becomes:

$$\boldsymbol{R}_{dl} = \sum_{k=1}^{K_{dl}}\sum_{i=1}^{d_{dl,k}} \log(1 + \sigma_{k,i}p_{k,i}). \tag{30}$$

with the power constraint $\sum_{k=1}^{K_{dl}}\sum_{i=1}^{d_{dl,k}} p_{k,i} = P$. We use the Kuhn–Tucker conditions to verify that the solution $\sum_{k=1}^{K_{dl}}\sum_{i=1}^{d_{dl,k}} p_{k,i} = \sum_{k=1}^{K_{dl}}\sum_{i=1}^{d_{dl,k}} \left[\lambda - \frac{1}{\sigma_{k,i}}\right]_+ = P$ is the assignment that maximizes the sum rate, where the optimal λ can be solved using bisection method. In the subsection V-C, the P of here will be denoted as P_{DL-BS}.

C. sum rate Performance

We evaluate the sum rate for the system $N_{ul} = 3$, $N_{dl} = 6$, $K_{ul} = 2, K_{dl} = 4, M_{dl} = 20$ and $M_{ul} = 4$. For this, we consider several cases of initialization of the beamformers and repeat the WMMSE algorithm in an iterative process to maximize the sum rate. In the following we describe the meaning of each notation associated with a given simulation:

- **init (UE2UE ZF + BS2UE ZF)**: the simulation gives the sum rate at the initialization with UE-to-UE ZF, by considering the precoders at UL UEs $\boldsymbol{G}_{z,l}$ and the decoders at DL UEs $\boldsymbol{F}_{z,k}$, and also the ZF between the DL UEs by considering the ZF precoders at the DL BS given in (6),
- **init (UE EigR + BS2UE ZF)**: the simulation gives the sum rate at the initialization without the UE-to-UE ZF, by considering the precoders at UL UEs and the decoders at DL UEs as the reception vectors from the SVD of the channel matrices at UL and DL sides, and also the ZF between the DL UEs by considering the ZF precoders at the DL BS given in (6),
- **init (UE2UE ZF + BS2UE ZF+ WF)**: the simulation is similar to the simulation in **init (UE2UE ZF + BS2UE ZF)** with the addition to the water-filling algorithm discussed in the subsection V-B,
- **init (UE2UE ZF + BS2UE ZF)+ WMMSE, iter=n**: this simulation consider the initialization as explained in **init (UE2UE ZF + BS2UE ZF)** simulation, then running the WMMSE algorithm as given in [16] or [17], and gives the sum rate at the n^{th} iteration of the WMMSE algorithm,
- **init (UE EigR + BS2UE ZF)+ WMMSE, iter=n**: this simulation consider the initialization as explained in **init (UE EigR + BS2UE ZF)** simulation, then running the WMMSE algorithm as given in [16] or [17], and gives the sum rate at the n^{th} iteration of the WMMSE algorithm,
- **init (UE2UE ZF + BS2UE ZF+ WF)+ WMMSE, iter=n**: this simulation consider the initialization as explained in **init (UE2UE ZF + BS2UE ZF+ WF)** simulation, then running the WMMSE algorithm as given in [16] or [17], and gives the sum rate at the n^{th} iteration of the WMMSE algorithm.

We evaluate the sum rate at DL with $\boldsymbol{R}_{dl,k}$ of (5) and at UL with $\boldsymbol{R}_{ul,l}$ of (3) by Monte Carlo averaging over 100 channel realizations. The elements of the direct channel matrices are generated as i.i.d. Gaussian random variables $\mathcal{CN}(0,\,1)$ and the receive noise covariance is normalized, i.e. $\boldsymbol{R}_{n_k n_k} = \boldsymbol{I}_{N_{dl,k}}$. Since both noise and beamformer $\boldsymbol{V}_k$ at the DL BS are normalized, for the simulations without the water-filling we assume the same power at each UL UE i.e. $p_{ul,1} = p_{ul,2} = P$, and a total power of $K_{dl}P$ at DL BS $\sum_{k=1}^{K_{dl}} p_{dl,k} = K_{dl}P = P_{DL-BS}$ we define $P = 10^{\frac{SNR}{10}}$.

We evaluate in Fig.2 the sum rate at the DL and at the UL UEs, for the system $N_{ul} = 3$, $N_{dl} = 6$, $K_{ul} = 2, K_{dl} = 4, M_{dl} = 20$ and $M_{ul} = 4$. We consider for this system two cases regarding the rank of the interference channel between the UL UEs and the DL UEs $rank(\boldsymbol{H}_{k,l}) = r$:

- Reduced rank MIMO IBMAC-IC: $r = 2$ such that the DoF at each UL and DL UE is: $d_{ul,1} = d_{ul,2} = 1$ and $d_{dl,1} = d_{dl,2} = 5, d_{dl,3} = d_{dl,4} = 4$,
- Full rank MIMO IBMAC-IC: $r = 3$ such that the DoF at each UL and DL UE is $d_{ul,1} = d_{ul,2} = 1$ and $d_{dl,1} = d_{dl,2} = d_{dl,3} = d_{dl,4} = 4$.

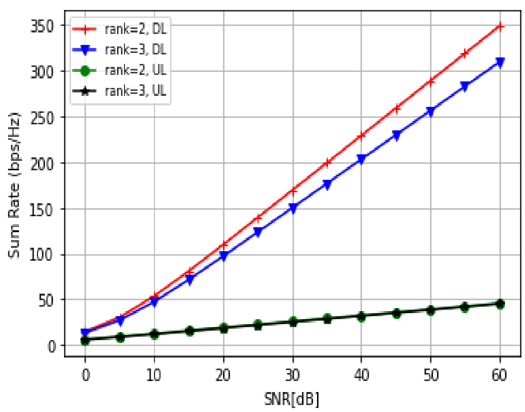

Fig. 2: sum rate performance with $N_{ul} = 3$, $N_{dl} = 6$, $K_{ul} = 2$ and $K_{dl} = 4$.

In this simulation Fig.2, we investigate the sum rate performance at UL and DL when considering two different ranks of the MIMO IBMAC-IC, $r = 2$ and $r = 3$. We observe in Fig.2 that at UL the sum rate is approximately the same in the two cases, this is because in this example, and based on our IA feasibility condition in [15, Theorem 4], we already know that for this system dimension, it isn't possible to increase the DoF (so the rate at high SNR) at UL UEs (Table II), otherwise the IA wouldn't be feasible. Then if we consider the DL side, with the ZF beamformer $\boldsymbol{V}_k$ we can see in Fig.2 at high SNR that for $r = 2$ the sum rate is greater than the sum rate for $r = 3$, which is confirmed before in the numerical results (Table II). For the latter the consideration of a non-uniform DoF at DL UEs (Conjecture 2) allows us to increase the sum rate at high SNR.

In Fig.3, we want to highlight the impact of the UE-to-UE interference on the performance of the DynTDD system, so we consider the simulations with the two initialization: init (UE2UE ZF + BS2UE ZF) and init (UE EigR + BS2UE ZF). From the simulation results in Fig.3, it is observed that a significant improvement in the sum rate could be achieved when we consider the ZF of the UE-to-UE interference. So the proposed ZF decoders $\boldsymbol{F}_{z,k}$ and precoders $\boldsymbol{G}_{z,l}$ mitigate the UE-to-UE interference, which improves the performance of the system remarkably.

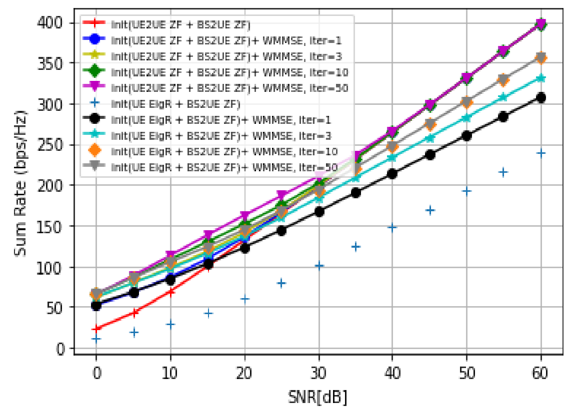

Fig. 3: sum rate performance with $N_{ul} = 3$, $N_{dl} = 6$, $K_{ul} = 2$, $K_{dl} = 4$ and $r = 2$

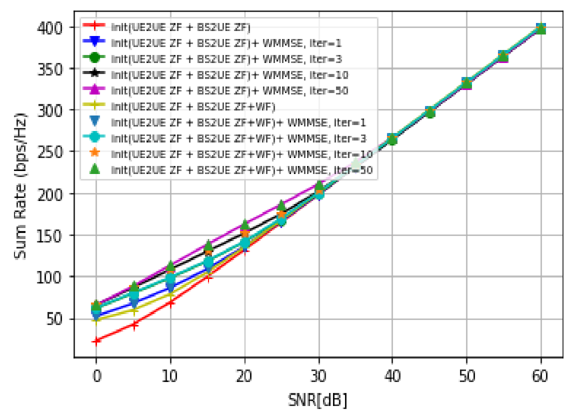

Fig. 4: sum rate performance with $N_{ul} = 3$, $N_{dl} = 6$, $K_{ul} = 2$, $K_{dl} = 4$ and $r = 2$

To evaluate the water-filling algorithm, we give in Fig.4 a comparison between the average sum rate versus SNR for the four simulations: init (UE2UE ZF + BS2UE ZF), init (UE2UE ZF + BS2UE ZF+ WF), init (UE2UE ZF + BS2UE ZF)+ WMMSE, iter=n and init (UE2UE ZF + BS2UE ZF+ WF)+ WMMSE, iter=n. In Fig.4, we compare the sum rate at initialization (init (UE2UE ZF + BS2UE ZF)) and the sum rate at the 1^{th},3^{rd}, 10^{th} and the 50^{th} iteration of the WMMSE algorithm, the results illustrate the convergence behavior of the WMMSE algorithm. At low SNR the WMMSE algorithm improves the sum rate and outperforms the ZF solution. The simulation results show that the water-filling algorithm with the ZF can approach closer to the performance of the WMMSE algorithm at low SNR.

VI. Conclusions

In this paper we highlight some new observations about the IA feasibility, thus the benefit of a non-uniform DoF at DL UEs and/or UL UEs regarding the sum of DoF maximization, so the rate at high SNR. This paper studies beamforming design for MIMO IBMAC-IC in DynTDD systems to maximize the weighted sum rate. In this scope we give for a closed-form case, detailed steps to construct the ZF beamformers at DL and UL UEs, so all the UL-to-DL interference links are canceled, and we consider a ZF transmitter at the DL BS so the intracell interference is mitigated. In our simulation we use these ZF filters at the initialization, then for the sum rate maximization, we consider the WMMSE iterative algorithm which is a potential candidate for practical low-complexity transmit beamforming implementations. We study the impact of the water-filling algorithm at initialization, and how this could improve the performance at low SNR. Numerical results studying the sum rate show that the UE-to-UE interference in the DynTDD system could be harmful to the system's performance.

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