Spatial domain prediction of optimal MIMO beam alignment pairs in D2D networks

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***Abstract*—The problem of resource-efficient beam alignment is a long standing one within massive MIMO (mMIMO) enabled wireless communication networks. In device-to-device enabled networks, the beam alignment problem is repeated for every new pair that appears and wishes to communicate, leading to a seemingly unbounded resource expenditure as the network grows dense. In this paper, we develop a new approach that uses the implicit geometric structure of such networks to break the spell. Instead of combatting it, our method exploits densification to facilitate alignment with minimal resources. Assuming a static (or slow varying) network, the intuition behind our approach is to utilize the beam alignment solutions at prior device pairs to predict optimal alignment in future pairs at independent new locations. We show the equivalence between this problem and a non-linear matrix completion (MC) problem under some sparsity condition. In order to solve it, we design a MC technique based on attention-based graph neural network (GNN) which proves effective to predict optimal beam pairs with little side-information.**

***Index Terms*—Beam alignment, D2D communication network, matrix completion, graph neural network, machine learning**

I. Introduction

Recent years have seen a rise in interest for 5G mmWave and in device-to-device (D2D) networks, sometimes combined together [1], [2]. In 5G mmWave communication systems, large-scale yet compact antenna arrays (mMIMO) may be equipped at both transmitter and receiver sides, such that high path losses can be compensated by generating narrow beams with strong beamforming gains. In this context, efficient communications hinges on the ability to identify beam pairs (one beam at TX, one beam at RX) which are well aligned with the channel propagation paths. Traditional beam alignment based on exhaustive scanning (beam sweeping) involves systematically steering a set of predefined beamforming vectors across the angular space to cover all potential beam directions. The receiver measures the received signal quality for each beam combination, such as the received signal strength or signal-to-noise ratio (SNR), and provides feedback to the transmitter. While exhaustive sweeping helps identify the directions with the strongest signal, the scanning leads to a long training process, particularly when the number of antennas and the number of device pairs is large. To address these issues, various beam alignment mechanisms have been proposed to reduce beam training overhead or to enhance beam alignment accuracy. The majority of existing works address beam alignment in the context of a single point-to-point link. For instance, methods based on iterative refinement can be found in [3]. Other lines of work includes exploiting the duality between beam pairing, which in Ricean channel environment is akin to predicting the orientation of the TX-RX axis, and the device localization problem [4]–[6]. Hence device localization can be utilized in order to supervise beam alignment.

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In scenarios where prior localization information is not accessible, the above methods can't be used and other approaches must be resorted to. In this work, we leverage instead the underlying spatial structure of beam pairing. Assuming a network with a number of devices communicating directly in D2D mode, we exploit past beam pairing solutions available for a subset of past device pairs at unknown locations in order to predict the optimal TX and RX beams at future new random device pairs at new independent locations. To the best of our knowledge, no prior attempt has been made to improve the optimal beam alignment prediction by utilizing secondary AoA information from other devices in the network.

The main contributions of this paper can be summarized as follows: First we note that in the particular case of Line of Sight (LoS) channels, the problem of predicting new beam directions is equivalent to predicting angle of arrival (AoA) and angle of departure (AoD) information. In this case, our first contribution demonstrates analytically the minimum number of prior beam pairing solution needed to reconstruct optimal beam direction for any new device pair in a N-device network. While the number of pairs grows with $\mathcal{O}(N^2)$, the number of pre-trained pairs in our solution only grows linear with $\mathcal{O}(N)$. In the second contribution, we extend our results to more general Ricean channels and develop a novel solution relying on GNN-based matrix completion (MC) to reconstruct optimal beams for any device pair. Interestingly, device localization itself has also previously been reformulated as a MC problem [7], [8]. In these papers the Euclidean distance matrix- and eventually the location map- is recovered from partially observed distance information. The Euclidean distance matrix in a k-dimensional Euclidean space is known to be at most of rank $k+2$ [8], whereas the rank of the AoA matrix is not known a priori in our case and a different MC form must be developed here.

Simulation results are provided to demonstrate the effectiveness of our proposed method and robustness with respect to multi-path outdoor model. We also shown how beam pairing performance improves with the density of devices distributed in the network as it allows a better leverage of the geometry structure, yielding a more resource-efficient approach compared with classical methods like beam sweeping.

Notations: We adopt x, $\mathbf{x}$, and $\mathbf{X}$ to denote a scalar, vector, and matrix, respectively. Superscripts $*, T$, and H stand for the conjugate, transpose, and conjugate transpose, respectively.

II. System Model & Problem Formulation

As shown in Fig. 1, we consider a static (or slow moving) communication network, where N devices each equipped with M-antenna MIMO arrays are distributed space. Each device can transmit (resp. receive) over a TX beam (resp. RX) beam selected from a codebook of M fixed beams. The network is D2D enabled, in the sense that device pairs can establish direct communication links, relying on the joint selection of a pair of the TX and RX beams to enhance the link SNR. Let the graph $G = \{\mathcal{N}, \mathcal{E}\}$ represent such a network, where the node set $\mathcal{N}$ includes all devices and the edges in $\mathcal{E}$ represent the beam pairs selected for communication between two nodes. In this work, we assume that a (small) random subset of initial device pairs have already performed beam pairing. The goal of the study is to analyze the identifiability of beam pairing solutions for all remaining device pairs, along with a suitable reconstruction/prediction algorithm for those beams.

A. Channel model and beam pairing performance

Let $\mathbf{H}_{i,j}$ denote a multi-path channel between device i and device j, classically based on the sum of the contributions of L scattering clusters [9]:

$$\mathbf{H}_{i,j} = \sum_{l=1}^{L} \beta_{i,j,l} \mathbf{a}(M, \vartheta_{i,j,l}) \mathbf{a}^H(M, \phi_{i,j,l}), \tag{1}$$

where $\beta_{i,j,l}$ is the complex gain of the l-th path between device i and device j, $\vartheta_{i,j,l} \triangleq \sin(\theta_{i,j,l})$ and $\phi_{i,j,l} \triangleq \sin(\varphi_{i,j,l})$, $\theta_{i,j,l}$ and $\varphi_{i,j,l}$ are AoA and AoD of the l-th path between device i and device j. The steering vector $\mathbf{a}(M, \vartheta)$ is defined as

$$\mathbf{a}(M, \vartheta) \triangleq \frac{1}{\sqrt{M}} \left[1, e^{j\frac{2\pi d}{\lambda}\vartheta}, \cdots, e^{j\frac{2\pi d}{\lambda}\vartheta(M-1)}\right]^T. \tag{2}$$

where λ denotes the carrier wavelength and d denotes the distance between two adjacent antennas.

The received signal at the RX j from TX i is given by

$$\mathbf{y}_{i,j} = \mathbf{H}_{i,j} \mathbf{w}_{i,j} x_{i,j} + \mathbf{n}_j, \tag{3}$$

where $\mathbf{w}_{i,j}$ and $x_{i,j}$ denote the beamforming vector and transmitted signal from the device i to device j, $\mathbf{n}_j \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$ models the additive noise with zero mean and variance σ^2. After the receiver beamforming $\mathbf{w}_{j,i}$ at device j, we have

$$\hat{x}_{i,j} = \mathbf{w}_{j,i}^H \mathbf{H}_{i,j} \mathbf{w}_{i,j} x_{i,j} + \mathbf{w}_{j,i}^H \mathbf{n}_j. \tag{4}$$

The beam pairs, i.e. $\mathbf{w}_{i,j}$ and $\mathbf{w}_{j,i}$, are selected from a predefined beamforming codebook, which consists of M equally spaced channel steering vectors pointing at M different directions:

$$\mathcal{W} \triangleq \{\mathbf{w}(1), \mathbf{w}(2), \cdots, \mathbf{w}(M)\}, \tag{5}$$

with

$$\mathbf{w}(m) = \mathbf{a}(M, -1 + (2m-1)/M). \tag{6}$$

The SNR between RX j and TX i is expressed as

$$\gamma_{i,j} = \frac{|\mathbf{w}_{j,i}^H \mathbf{H}_{i,j} \mathbf{w}_{i,j}|^2}{\sigma^2}. \tag{7}$$

Assuming negligible inter-device interference, the optimal beam pair is selected as the one maximizing the SNR:

$$\mathbf{w}_{j,i}^{\text{opt}}, \mathbf{w}_{i,j}^{\text{opt}} = \arg \max_{\mathbf{w}_{j,i}, \mathbf{w}_{i,j} \in \mathcal{W}^2} \gamma_{i,j}, \tag{8}$$

where $\mathbf{w}_{i,j}^{\text{opt}}$ and $\mathbf{w}_{j,i}^{\text{opt}}$ denote the SNR-optimal beamforming vectors for communication between node i and node j, selected from the common codebook. Note that the search complexity using full scanning is M^2 for each pair, hence up to $N(N-1)M^2/2$ for the whole network, with the $1/2$ saving obtained from symmetry arguments.

B. Beam alignment as AoA matrix completion

In order to build intuition and guide analytical work, we first consider a propagation environment with a strong enough LoS component, such that there a duality between optimal beam pairs and the AoA and AoD information (this assumption is then relaxed when designing the algorithm). Let θ_{ij} denote the AoA from TX j to RX i. Note that the AoD from TX j to RX i can be obtained by symmetry.

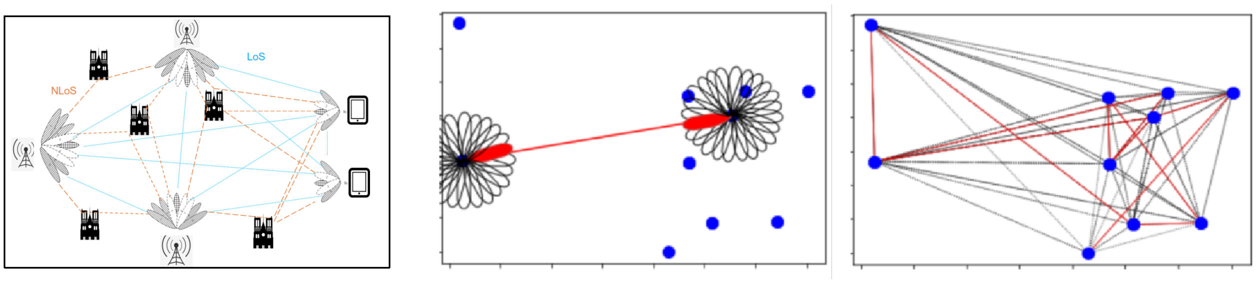
Let $\mathbf{A} \in \mathbb{R}^{N \times N}$ denote the entire AoA matrix, whose (i, j) element is θ_{ij} and the diagonal elements are left at 0. As mentioned above, we assume part of the AoA matrix has been observed through some pre-training on those device pairs that previously initiated communication. Let Ω and Ω' denote the set of observed and missing entries in matrix $\mathbf{A}$, respectively. To find the estimate of the full AoA matrix such that it matches all the known and observed values, any beam pairing algorithm can be interpreted as a MC problem on $\mathbf{A}$, with a reconstruction performance indicator chosen to be the root mean squared error (RMSE) between each of the estimated and observed values, which is:

$$\text{RMSE} = \sqrt{\frac{1}{|\Omega|} \sum_{\theta_{ij} \in \Omega} \left(\theta_{ij} - \hat{\theta}_{ij}\right)^2}, \tag{9}$$

where $\hat{\theta}_{ij}$ denote the estimated AoA from TX j to RX i.

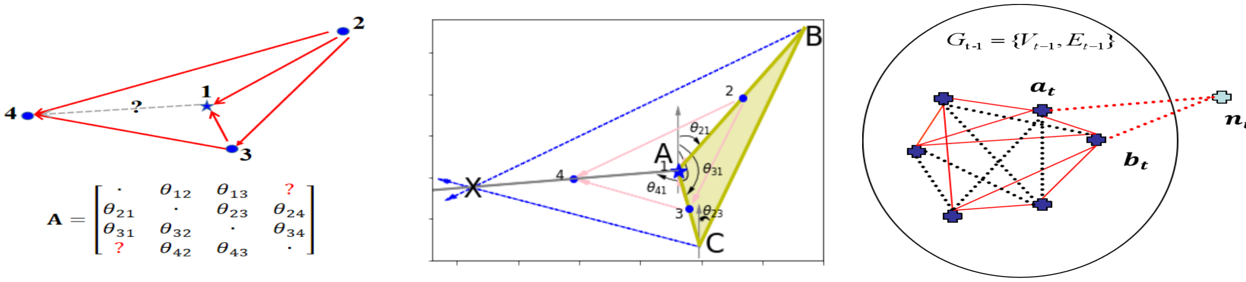
III. Analysis of AoA Identifiability Conditions

In this section, we analyze for the dominant LoS scenario the AoA identifiability conditions using geometric arguments allowing the completion of matrix $\mathbf{A}$ on the basis of a limited set of observed entries. In what followed we refer to all AoA elements of $\mathbf{A}$ which have been observed a priori as *trained* AoA. A *training pattern* refers to particular subset of trained AoA elements within $\mathbf{A}$.



(a) D2D Network (b) AoA-based beam alignment (c) AoA estimation

Fig. 1. Example of Beam alignment with mMIMO in an ultra-dense D2D communication networks. (a) An ultra-dense D2D network with N randomly located devices. (b) Example of AoA-based beam alignment where the beams are assumed to be infinitely narrow. The blue nodes denote the devices and the red links represent perfect-alignment of the transmitter and receiver beams between two devices, which signify successful communication between the devices. (c) Example of AoA estimation where the black dashed lines represent the AoA information that needs to be estimated.



(a) 4-device network (b) Scale-invariant triangle (c) N-device network

Fig. 2. Beam pairing as a AoA reconstruction problem. (a) AoA reconstruction for a 4 device networks: AoA for the (1,4) device pairing is feasible, based on previous pairing information for (1,2), (1,3), (2,3), (4,2), (4,3) (b) Geometric solution for 4 device network based on scale-invariant triangle. (c) Assuming full pairing among $N-1$ devices, the optimal beam pairs for the N-device network is feasible provided pairing is available for (a_t, n_t) and (b_t, n_t) device pairs.

Proposition 1. *For a D2D communication networks consisting of $N \geq 2$ static devices, hence with $N(N-1)/2$ ordered communication pairs, there exists a AoA training pattern with only $2N-3$ trained AoA elements allowing the reconstruction of all $N(N-1)$ AoA elements in* $\mathbf{A}$.

Proof: First note that in the LoS-dominant scenario, $\mathbf{A}$ exhibits some symmetry between upper and lower triangles. Without loss of generality, we reconstruct the $N(N-1)/2$ AoA elements in the upper triangle. The rest of the proof is by induction. It is trivial when $N = 2$ and $N = 3$. To prove the proposition in the general case, let us first examine $N = 4$, see Fig. 2(b). Specifically, we assume the AoA estimation has been solved for all D2D pairs within A, B, C network. Now consider device X: the AoA information for (X, A) can be found uniquely from AoA information for (X, B) , (X, C) and within A, B, C network as follows:

Consider Fig. 2(b), where without loss of generality device A is considered the reference point at location $x_A = (0, 0)$. Then, based on the AoA knowledge for $(A, C), (A, B)$ and (B, C) device pairs, the location of B and C, relative to X, is implicitly and uniquely determined. Based on location information for B and C and based on additional AoA for (B, X) and (C, X) pairs, the location of X is uniquely determined, hence AoA for (X, A) is uniquely determined. The AoA for the 4-device network is fully reconstructed with just 5 AoA information elements (instead of 6 in theory).

Now consider, as in Fig. 2(c), a network with $(t-1)$-devices (circled) for which AoA for all pairs have been determined, based on pre-training of $2(t-1)-3$ AoA elements. Assume a t-device network is created by injecting a new device n_t, where n_t seeks to connect to arbitrary devices within the existing $(t-1)$-device network. Assume two additional AoA elements are made available, between n_t and two arbitrary previous devices a_t and b_t. Assume n_t seek to align with another random device c_t. Then, based on the 4-device argument above, the AoA for (n_t, c_t) is uniquely determined from prior AoA in (n_t, b_t), (n_t, a_t) and the 3-device network (a_t, b_t, c_t). Hence AoA for the t-device network is full determined. Note that at each iteration, a new device is added and we assume two new links (between this new device and two arbitrary previous devices) are AoA trained. By induction, after N iterations and assuming the above AoA training pattern of $2N-3$ elements, the AoA for the entire matrix can be reconstructed.

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IV. ML-based Matrix Completion Approach

The above mathematical Proposition 1 provides the intuition that the underlying geometric structure can be leveraged to save a significant amount of resources when scanning across beam pairs for a dense network (i.e. N large). The proof is also a constructive one in that it also suggests a way to recover all missing AoA elements in N-device network when a subset of $2N-3$ device pairs have been beam-trained. This approach is referred to as Structured-Training Matrix Completion (ST-MC) because the choice of device pairs selected for initial beam training is done according to the above proof. This algorithm is summarized in the Algorithm 1.

Algorithm 1 Structured Training Matrix Completion (ST-MC) Algorithm

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Require: Node set with N devices ($\mathcal{N}$)
Ensure: Estimated AoA matrix for all devices ($\hat{\mathbf{A}}$)

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Initialize a graph G_0 = {V_0, E_0}
Randomly select two nodes a_0, b_0 ∈ N to construct initial set V_0 = {a_0, b_0}, and perform the beam searching between a_0 and b_0
Add θ_{a_0,b_0} into the initial edge set E_0
for t = 1, 2, ..., N − 1 do
    Randomly select one node n_t ∈ N \ V_{t−1}
    Construct V_t ← V_{t−1} ∪ n_t
    Randomly select two different nodes a_t, b_t ∈ V_{t−1} and perform the beam searching for (a_t, n_t) and (b_t, n_t) pairs, as shown in Fig. 2(c)
    Add θ_{a_t,n_t} and θ_{b_t,n_t} into edge set E_{t−1}
    for c_t ∈ V_{t−1} \ {a_t, b_t} do
        θ_{c_t,n_t} ← ADNH (θ_{a_t,v_t}, θ_{b_t,v_t}, θ_{a_t,b_t}, θ_{a_t,c_t}, θ_{b_t,c_t})
        Add θ_{c_t,n_t} into edge set
    end for
end for
Function ADNH (AoA Discovery with Neighbor Help)
Initialize device 1 as the reference point A = (0, 0)
Construct scale-invariant triangle as shown in Fig. 2(b) with ∠θ_21, ∠θ_31 and and ∠θ_21, where B = (x_B, y_B) and C = (x_C, y_C)
Construct vectors b⃗ and c⃗ from the θ_24 and θ_34 and the gradient of vectors are m_b⃗ = tan θ_24 and m_c⃗ = tan θ_34, respectively.
Find the intersection point of b⃗ and c⃗ by computing
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$$X = \begin{bmatrix} \tan(\theta_{24}) & -1 \\ \tan(\theta_{34}) & -1 \end{bmatrix}^{-1} \begin{bmatrix} \tan(\theta_{24}) x_B - y_B \\ \tan(\theta_{34}) x_c - y_c \end{bmatrix} \tag{10}$$

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Computing as θ_14 = arctan(x_X / y_X)
return Â ∈ R^{N×N}
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In reality however, some of the idealized assumptions used in the analysis should be relaxed: (i) The propagation may experience significant multi-path and beams may not be infinitely narrow: i.e., AoA estimation and optimal beam selection are no longer equivalent, (ii) the sequence of pre-observed beam pairs may not adhere to the pattern used in Proposition 1 and may be random. In what follow, we design a ML-aided MC algorithm providing a near-optimum solution for such realistic cases. In such cases, the element θ_{ij} in $\mathbf{A}$ is the AoA of path with the maximum SNR among all L paths from TX j to RX i, and $\mathbf{A}$ is not necessarily symmetric.

A. Collaborative Filtering Matrix Factorization (CFMF)

Consider a partly observed AoA matrix, whose known elements constitute a small subset Ω of the matrix, Collaborative Filtering Matrix Factorization (CFMF) methods aim to learn two latent feature matrices to recover the whole matrix by product of corresponding feature map [11]. The solution to the previous problem is equivalent to the following program:

$$\arg\min_{\mathbf{U},\mathbf{V}} \left\|\Omega \circ (\mathbf{A} - \mathbf{U}\mathbf{V}^T)\right\|_F^2 + \lambda(\|\boldsymbol{U}\|_F^2 + \|\boldsymbol{V}\|_F^2), \tag{11}$$

where $\mathbf{U}, \mathbf{V} \in \mathbb{R}^{N\times q}$ and $q \ll N$ denotes the dimension of the latent feature vector, $\Omega \circ (\cdot)$ is the binary projection operator that only counts observed entries of the matrix which lie in the set Ω, and $\|\cdot\|_F$ denotes the Frobenius norm. The first term and the second term denote reconstruction error and regularization penalty, respectively. Actually, CFMF-based MC method can be considered as a process of generating the original data by linear combinations of the latent features.

The CFMF has to be performed with only partially observed entries Ω in $\mathbf{A}$ and not the complete entries of $\mathbf{A}$, which remain mostly unknown under sparsity condition. To obtain $\mathbf{U}$ and $\mathbf{V}$, the two matrices are first initialized the with random values, and the error between the product of $\mathbf{U}$ and $\mathbf{V}$, i.e. $\hat{\mathbf{A}}$ to $\mathbf{A}$ for the observed entries Ω, are measured. This error is then minimized iteratively until a local minimum is obtained.

Note the AoA relationship is not linear as discussed in Algorithm 1, however, the performance of matrix factorization is well known to be hindered by the simple choice of interaction between the latent vectors, i.e., the inner products. This may not be able to capture the complex structure of the interactions between the row and column features. Additionally, CFMF methods are based on low-rank assumption, whereas in our case the rank of the AoA matrix is not known a priori.

B. GNN-based Matrix Completion (GNN-MC)

To tackle this high-sparse and non-linear MC problem, we propose a novel graph neural network (GNN) with attention mechanism to learn meaningful dense feature representations from sparse input data by fully utilizing node feature and geometric structure. Specifically, we perform attention-based graph filtering on the original sparse AoA matrix to supercharge the prediction with non-linearities, where graph convolution network (GCN) acts as a feature extractor and attention block helps to solve the non-linear issue in the propagation layer.

1) Graph Convolution: Given a graph $G = \{\mathcal{N}, \mathcal{E}\}$, let $\mathcal{L} = \mathbf{I} - \mathbf{D}^{-1/2}\mathcal{W}\mathbf{D}^{-1/2}$ denote the normalized graph Laplacian matrix, $\mathbf{D}$ denote the degree matrix. Considering that $\mathcal{L}$ is symmetric, it can be eigen-decomposed as $\mathcal{L} = \Phi\Lambda\Phi^T$. Until the loss function is small enough, the input data is propagated and aggregated across edges using an iterative process known as graph convolution. Let $\mathcal{X} = \{\S_1, \S_2, ..., \S_N\} \in \mathbb{R}^{N\times K_{\mathrm{i}}}, \S_i \in \mathbb{R}^{K_{\mathrm{i}}}$ denote the K_{i}-dimension input data for graph convolution layer, and $\mathcal{Y} = \{\dagger_1, \dagger_2, ..., \dagger_N\} \in \mathbb{R}^{N\times F_{\mathrm{o}}}, \dagger_i \in \mathbb{R}^{K_{\mathrm{o}}}$ denote the K_o-dimension output feature. The learning procedure can be expressed as $\dagger_j = \arctan(\sum_{i=1}^{K_i} \mathbf{\Phi}_k \hat{\mathbf{G}}_{ij} \mathbf{\Phi}_k \S_i)$, where $\arctan$ is a nonlinear activation function, $\Phi_k = (\phi_1, ..., \phi_k)$ denotes $N \times k$ matrix of the first k eigenvectors and $\hat{\mathbf{G}}_{ij} = \text{diag}\,(\hat{g}_{ij,1}, .., \hat{g}_{ij,k})$ denotes $k \times k$ diagonal matrix of spectral multipliers representing a learnable filter in frequency domain. This operation is computationally expensive due to matrix multiplication. In order to get around this problem, a normalized polynomial filter with Chebyshev coefficients can be used to extract features. The learning procedure is expressed as

$$\dagger_j = \arctan(\sum_{i=1}^{K_i} \sum_{p=0}^{P-1} \zeta_{ij,p} \mathbf{T}_p(\hat{\mathcal{L}}) \S_i), \tag{12}$$

where $\zeta_{ij,p}$ denotes learnable coefficients of Chebyshev polynomial filter, $\hat{\mathcal{L}} = 2\mathcal{L}/\lambda_{\max} - \mathbf{I}$, $\lambda_{\max}$ is the maximum eigenvalue, $\mathbf{T}_p(\cdot)$ denotes recursively-generated Chebyshev polynomial, and p denotes the Chebyshev polynomial order. This indicates that Laplacian is a local operator with p-hop neighborhood. To estimate the entire AoA matrix, let $\mathcal{H}^{(0)} = \mathbf{A}$ and for the t-th hidden layer, we have

$$\mathcal{H}^{(t+1)} = \arctan(\mathcal{H}^{(t)} \mathcal{W}_{\zeta}^{(t)}(\mathcal{L})) \tag{13}$$

where $\mathcal{W}_{\zeta}^{(t)}$ denotes weight matrix at the t-th layer, whose elements is $w_{\zeta_{ij}}^{(t)} = \sum_{p=0}^{p-1} \zeta_{ij,p}^{(t)} \mathbf{T}_p(\hat{\mathbf{L}})$.

2) Attention Mechanism: The relevance of different node to a target node can be adjusted by an attention score. In our model, each GCN block is followed by an attention-based propagation layer. The output weighted feature of node i after the l-th GCN block is

$$\tilde{\mathcal{H}}_i^{(t)} = \sum_{j \in \mathcal{N}(i)} \kappa_{ij}^{(t)} \mathcal{H}_j^{(t)}, \tag{14}$$

where $\mathcal{N}_i$ denote all one-hop neighborhood of node i including itself, $\kappa_{ij}^{(t)}$ denote the attention score, which is calculated as follows

$$\kappa_{ij}^{(t)} = \frac{e^{(\beta(t) \cos(\mathcal{H}_i^{(t)}, \mathcal{H}_j^{(t)}))}}{\sum\limits_{j' \in \mathcal{N}(i)} e^{(\beta(t) \cos(\mathcal{H}_i^{(t)}, \mathcal{H}_{j'}^{(t)}))}}, \tag{15}$$

where $\cos(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x}^{\mathrm{T}} \mathbf{y}}{\|\mathbf{x}\| \|\mathbf{y}\|}$. Considering the training instability of highly sparse MC problems, this paper adopts a relatively simple attention formulation with only one parameter per layer. Both the weight sharing matrix $\mathcal{W}_{\zeta}^{(t)}(\mathcal{L})$ and the attention hyper-parameter $\beta(t)$ are trained by minimizing the loss function given in (11).

V. Experiment Results

A. Experiment Setup

In this section, we numerically evaluate the performance of the proposed algorithm in the ultra-dense D2D massive MIMO communication network. The cardinality of analog beamforming codebook is set to be $M = 64$. The number of resolvable multipath in mmWave channel is set to be $L = 5$ for each device, while the complex channel gain is set as $\beta_{i,j,1} \sim \mathcal{CN}(0, \sigma_{\text{LoS}}^2)$ for LoS path and $\beta_{i,j,l} \sim \mathcal{CN}(0, \sigma_{\text{NLoS}}^2), i \neq 1$ for the other scattered paths. Let P be the total power of all paths, and K_r be the Ricean factor, thus, we have

$$\sigma_{\text{LoS}}^2 = \frac{K_r}{1 + K_r} P, \quad \sigma_{\text{NLoS}}^2 = \frac{P}{(L-1)(1 + K_r)}. \tag{16}$$

1) Baselines: In this work, we adopt beam sweeping and vanilla matrix factorization algorithm [10] as comparison benchmarks. We set the maximum number of iterations to 1000 and then calculated the average of 50 trials. The CF-MC, GNN-MC and ST-MC models are implemented on Google TensorFlow. The Adam optimizer was employed for training with a learning rate of 0.05 and weight decay of $1e^{-5}$. Cross-validation is used to tune hyper-parameters such as dropout rate and regularization factor.

2) Performance Metric: Since the absolute reference point for performance is given by the exhaustive beam sweeping baseline, we seek to bound the average SNR loss with respect to beam sweeping to a small desired value, such as -1dB.

$$\mathcal{L}_{\gamma} = \frac{\sum_{i,j \in \Omega'} |\hat{\mathbf{w}}_{j,i}^H \mathbf{H}_{i,j} \hat{\mathbf{w}}_{i,j}|^2}{\sum_{i,j \in \Omega'} |\{\hat{\mathbf{w}}_{j,i}^{\text{opt}}\}^H \mathbf{H}_{i,j} \{\hat{\mathbf{w}}_{i,j}^{\text{opt}}\}|^2}, \tag{17}$$

where $\hat{\mathbf{w}}_{j,i}$ denote the beamforming vector based on AoA estimation.

Under the constraint on SNR loss, the negative performance metric is measured by how many beam pairs need to be trained (measured) in order to predict the optimum beam pairs at any arbitrary device pair.

3) Data Pre-processing: In order to facilitate the processing of training data by ML models, the AoA information are first normalized adjusted for its cyclic quantity for the training and testing purpose. We will set the training pattern as different sizes in order to compare the performance of various algorithms under different sparsity conditions.

B. Performance Comparison

In this work, we will conduct experiments to verify that in an ultra-dense network, once a certain proportion of the beam alignment is determined, the beam alignment between all other nodes can be estimated without the need for extra time and resources to perform beam scanning.

1) Impact of the Number of Devices: As shown in Fig. 3, beam sweeping, as an exhaustive method, requires $N(N-1)M^2/2$ beam pair scans. The ST-MC method requires the least number of pre-trained beam pair scans, that is, $(2N-3)M^2$, nevertheless, it requires a structured selection of pre-trained RX-TX pairs. For a training pattern with random selection of RX-TX pairs, both CFMF-based and GNN-based MC methods need perform $|\Omega|M^2$ beam pair scans in advance. Compared with CFMF-MC method, GNN-MC method requires less communication overhead to achieve a given SNR loss. In other words, the GNN-MC algorithm can perform well in beam alignment prediction by selecting a training patter with a smaller size. The CFMF-MC method do not predict AoA information well under high sparsity conditions due to its linear operation and low-rank assumption. In fact,

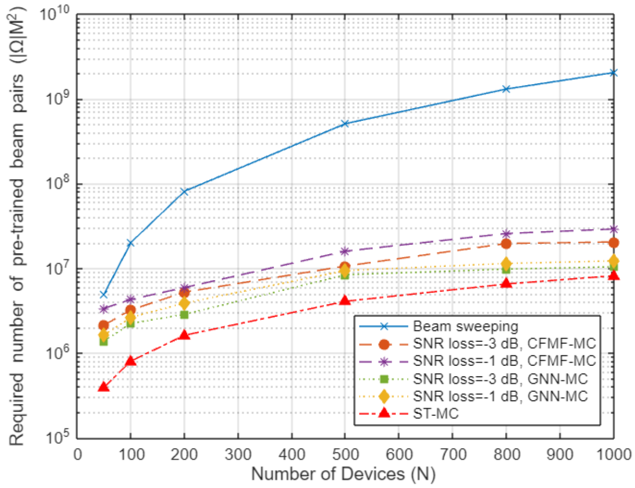

Fig. 3. The required number of pre-trained beam pair scans with different number of devices where the number of beams is 64. The proposed MC approaches save a factor of up to 70 times scanning resources over beam sweeping.

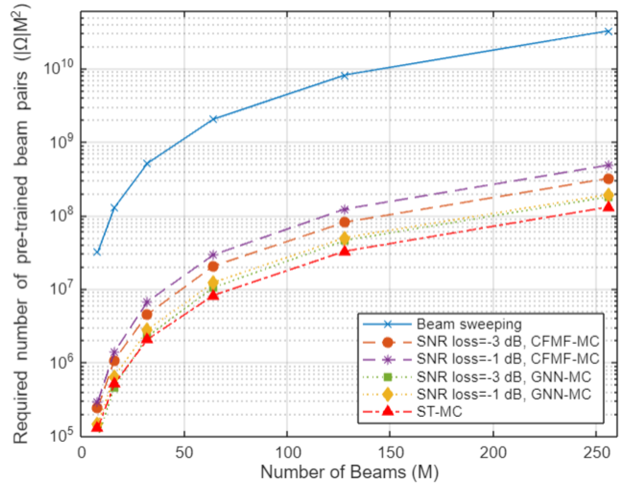

Fig. 4. The required number of pre-trained beam pair scans with different number of beams where the number of devices is 1000. The proposed MC approaches save a factor of up to 65 times scanning resources over beam sweeping.

in our scenario, the AoA matrix is full rank and the rank is equal to the number of devices. However, only a few of the eigenvalues of this matrix have relatively large magnitude, and the rest have relatively small magnitude (very close to 0), which indicates that this matrix can be approximated as a low-rank matrix to some extent. Therefore, the classical matrix factorization method could also accomplish this MC task, while GNN performs better because it does not rely on the low-rank assumption. Furthermore, the results demonstrate that, as the network becomes denser, the general trend is that the number of pre-training beam searching required by MC methods increases almost linearly with the number of devices.

2) Impact of the Number of Beams: As shown in Fig. 4, it can be seen that compared to beam sweeping, the MC-based methods can greatly reduce the required number of pre-trained beam pair scans regardless of the number of beams. We also set different Ricean factors, i.e., $K_r = 3, 5$ and 10, and found that the required number of pre-trained beam pairs for a given SNR loss is almost the same as long as the LoS transmission is dominant. This paper serves as a preliminary work for future research on this paradigm in order to analyze more realistic scenarios involving mobile nodes and more intricate networks with more reflective surfaces and large-size obstacles.

VI. CONCLUSION

This paper presents a new paradigm to perform fast beam alignment in large D2D network by scanning beams over a small subset of devices and leveraging the underlying geometric structure to recover all missing beam pairing information. For a N device network, the proposed method reduces the number of required beam scans from $\mathcal{O}(N^2)$ to $\mathcal{O}(N)$ to achieve an SNR performance close to that obtained under the classical beam sweeping method, as N grows large.

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