Simulation of Separable Quantum Measurements on Bipartite States via Likelihood POVMs

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***Abstract*—By developing a new framework of likelihood POVMs, analysis techniques and a new proof of the quantum covering lemma, we address the simulation of separable quantum measurement over bipartite states. In addition to a new one shot inner bound that naturally generalizes to the asymptotic case, we demonstrate the power, generality and universality of the developed techniques in the most general distributed measurement scenario by recovering all current known inner bounds. In addition to the above results, this framework is appealing in being the most natural and simple POVM simulation protocol.**

I. Introduction

The measurement compression problem (MCP) [1]–[4] of quantifying the information content in a quantum measurement's outcome is of fundamental interest. Here, we develop a new framework of measurements and tools to solve the MCP in diverse network scenarios and thereby derive a new inner bound for a distributed network MCP in the one-shot regime.

An elegant formulation coupled with his profound insight into the measurement process, Winter [4] solved the MCP in the single Tx-Rx scenario. At the heart of the MCP is Quantum Classical covering [5], a notion central to several fundamental problems [6]–[10]. This has motivated MCP studies in diverse network scenarios [11], [12] and fostered connections to several problems. Going beyond conventional asymptotic setting [4], [13], recent studies [14] focus on the one-shot regime. One-shot studies [15], [16], unable to utilize ideas like time sharing and measure concentration tools, are more general and challenging. These studies have led to powerful tools such as convex-split[17]–[19], position-based decoding [20], among others [21].

In this article, we consider the distributed MCP scenario in its full generality (Fig. 1), wherein the outcome of a separable POVM on a bipartite state distributed among two Txs has to be simulated at a central Rx. First, we consider(Sec. IV) simulation via POVMs based on IID random codebooks. Developing the likelihood POVM and new techniques (Sec. III), we derive a new inner bound to the MCP in the one-shot regime (Thm. 1) that naturally generalizes to the current known best inner bound achievable (Thm. 2) via IID random code based POVMs in the asymptotic regime.

In contrast to conventional/widely-held belief, simulation via POVMs based on IID random codebooks can be strictly sub-optimal for distributed MCP. Specifically, if the separable POVM to be simulated has a 'certain coupled' structure, then simulation via *structured POVMs*, i.e POVMs based on *jointly designed codes possessing algebraic (closure) properties* can yield strictly smaller inner bounds in the asymptotic regime. This can be traced back to the ingenious work of Körner and Marton [22] in the context of mod-2 sum recovery - an instance of distributed classical covering - and subsequent works [23]–[25]. Recently, the use of *structured POVMs* for MCP has been studied in [26] where it has been proven [26, Rem. 1, Ex. 1,2] that structured POVMs can outperform IID unstructured POVMs.

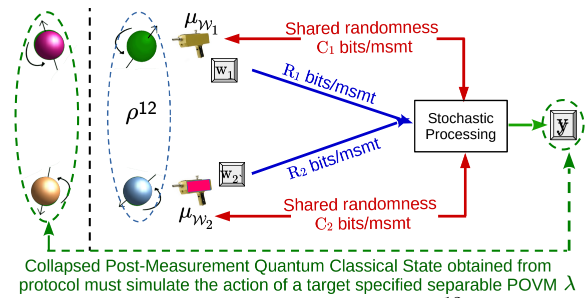

Fig. 1. Txs j, holding system j of bipartite state ρ^{12} shares (i) common randomness at rate C_j bits/measurement (msmt) and (ii) noiseless bit pipe of rate R_j bits/msmt, both with Rx. The goal is to simulate separable POVM λ.

While random structured codes - random jointly designed codes with algebraic properties - can yield strictly better inner bounds in multiple network scenarios [24], it is more involved. Indeed, the distribution of jointly designed random codes with inter and intra- algebraic properties do not possess simplifying properties such as mutual independence etc., that IID random codes enjoy, lending the former's analysis more involved. For instance, the operator Chernoff bound (OCB) of Ahlswede Winter [27] which is often used to analyze performance in MCP, relies on mutual independence and cannot be employed to analyze structured POVMs. Moreover, while we are able to achieve better inner bounds using structured codes in the asymptotic regime, there have been no studies aimed at achieving corresponding inner bounds in the one-shot regime. These point to the challenges of deriving inner bounds via structured codes.

Beyond the distributed MCP scenario (Fig. 1) we mentioned earlier, the larger contribution of this work is a unified framework of likelihood POVMs, ideas and analytical tools (Sec. III) that address all the above problems in a unified framework. The latter enable us treat both unstructured (IID random code) POVMs and structured POVMs. This includes a simplified of a general measure-transformed quantum covering lemma (QCL) (Lemma 1) that does not rely on, and whose proof is different from that of the OCB [27]. For instance, our QCL (Lemma 1) does not require mutual independence, and it suffices that the random codewords are pairwise independent - a property critical for proving Thm. 3. While the inner bounds in Thms. 1 and 2 are known [11], [26], our framework is much simplified, result in shorter proofs and is applicable even in centralized joint POVM MCP with distributed Rxs [12]. These demonstrate power & universality of the proposed framework.

In Sec. III, we introduce our likelihood POVM framework, indicate the challenges and our approaches and our tools. We build on this in Sec. IV to derive a new result - a one shot inner bound for distributed MCP.

II. Preliminaries and Problem Statement

We supplement notation in [5] with the following. For $n \in \mathbb{N}$, we let $[n] \triangleq \{1, \cdots, n\}$. An underline denotes an appropriate aggregation of related objects. For ex. if $\mathcal{H}_{X_1}, \mathcal{H}_{X_2}$ are Hilbert spaces, $\mathcal{H}_{\underline{X}}$ denotes $\mathcal{H}_{X_1} \otimes \mathcal{H}_{X_2}$, whereas when $k_j \in [K_j]$ for $j \in [2]$ are elements in index sets, $\underline{k} = (k_1, k_2)$. $\mathcal{L}(\mathcal{H})$, $\mathcal{P}(\mathcal{H})$ and $\mathcal{D}(\mathcal{H})$ denote linear, positive and density operators acting on $\mathcal{H}$. For state $\rho \in \mathcal{D}(\mathcal{H})$ with spectral decomposition $\rho = \sum_x \alpha_x |e_x\rangle\langle e_x|$, $|\phi_\rho\rangle \in \mathcal{H}_R \otimes \mathcal{H}$ denotes a purification, $|\varphi_\rho\rangle = \sum_x \sqrt{\alpha_x}\, |e_x\rangle \otimes |e_x\rangle \in \mathcal{H} \otimes \mathcal{H}$ denotes the *canonical purification* and we let $\phi_\rho \triangleq |\phi_\rho\rangle\langle\phi_\rho|, \varphi_\rho \triangleq |\varphi_\rho\rangle\langle\varphi_\rho|$. Associated with POVM $\lambda_{\mathcal{Y}} = \{\lambda_y \in \mathcal{P}(\mathcal{H}) : y \in \mathcal{Y}\}$ is a Hilbert space $\mathcal{H}_{\mathcal{Y}} \triangleq \text{span}\{|y\rangle : y \in \mathcal{Y}\}$ with $\langle\hat{y}|y\rangle = \delta_{\hat{y}y}$ and CPTP maps $\overline{\mathscr{E}}^{\lambda}(\cdot), \mathscr{E}^{\lambda}(\cdot)$ defined as $\overline{\mathscr{E}}^{\lambda}(s) = \sum_{y \in \mathcal{Y}} \sqrt{\lambda_y} s \sqrt{\lambda_y} \otimes |y\rangle\langle y|$ and $\mathscr{E}^{\lambda}(s) = \sum_y \text{tr}(\lambda_y s)\, |y\rangle\langle y|$. For a stochastic matrix $(p_{Y|W}(y|w) : (w, y) \in \mathcal{W} \times \mathcal{Y})$, we let $\overline{\mathscr{E}}_p^{Y|W}(\cdot), \mathscr{E}_p^{Y|W}(\cdot)$ denote the CPTP maps $\overline{\mathscr{E}}_p^{Y|W}(a) \triangleq \sum_{(w,y) \in \mathcal{W} \times \mathcal{Y}} p_{Y|W}(y|w)\, |w\rangle \langle w|a|w\rangle \langle w| \otimes |y\rangle\langle y|$ and $\mathscr{E}_p^{Y|W}(a) \triangleq \sum_{(w,y) \in \mathcal{W} \times \mathcal{Y}} p_{Y|W}(y|w) \langle w|a|w\rangle\, |y\rangle\langle y|$.

Defn 1. *A POVM $\lambda_{\mathcal{Y}} = \{\lambda_y \in \mathcal{P}(\mathcal{H}_{\underline{X}}) : y \in \mathcal{Y}\}$ acting on bipartite states is separable if there exists POVMs $\mu_{\mathcal{W}_j} = \{\mu_{w_j} \in \mathcal{P}(\mathcal{H}_{X_j}) : w_j \in \mathcal{W}_j\}$ for $j \in [2]$ and a stochastic matrix $p_{Y|\underline{W}} = p_{Y|W_1W_2}$ such that*

$$\sum_{w_1, w_2} p_{Y|\underline{W}}(y|w_1, w_2) \mu_{w_1} \otimes \mu_{w_2} = \lambda_y \text{ for all } y \in \mathcal{Y}. \quad (1)$$

We now describe the scenario of interest (Fig. 1). Tx j holds system j of the bipartite state $\rho_{\underline{X}} = \rho_{X_1X_2} \in \mathcal{D}(\mathcal{H}_{\underline{X}})$ where $\mathcal{H}_{\underline{X}} \triangleq \mathcal{H}_{X_1} \otimes \mathcal{H}_{X_2}$. The goal is to simulate the action of the separable POVM $\lambda_{\mathcal{Y}} = \{\lambda_y \in \mathcal{P}(\mathcal{H}_{\underline{X}}) : y \in \mathcal{Y}\}$ with the classical outcome available at the Rx. To achieve this, Tx j shares (i) C_j bits/msmt of common (independent) randomness and (ii) a noiseless bit pipe of R_j bits/msmt, both with Rx.

These two resources are used (in the one-shot setting) as follows. For $j \in [2]$, a bank of $K_j = 2^{C_j}$ POVMs are designed, each with atmost $M_j = 2^{R_j}$ outcomes. The shared common random bits $k_j \in [K_j]$ is used by Tx j to choose a POVM in its bank. The chosen POVM is performed and its outcome is communicated to Rx via the noiseless bit pipe. The Rx has all common random bits $\underline{k} = (k_1, k_2)$ and the observed outcomes of the $\underline{k}$-indexed POVMs. It employs a decoder CPTP $\Delta : \mathcal{L}(\mathcal{H}_{\underline{KM}}) \to \mathcal{L}(\mathcal{H}_{\mathcal{Y}})$, where $\mathcal{H}_{\underline{KM}} = \otimes_{j=1}^2 \mathcal{H}_{K_j} \otimes \mathcal{H}_{M_j}$, $\mathcal{H}_{K_j} = \text{span}\{|k_j\rangle : k_j \in [K_j]\}$ and $\mathcal{H}_{M_j} = \text{span}\{|m_j\rangle : m_j \in [M_j]\}$, where $\left\langle \hat{k}_j \middle| k_j \right\rangle = \delta_{k_j\hat{k}_j}$ and $\langle\hat{m}_j|m_j\rangle = \delta_{m_j\hat{m}_j}$. We formalize a simulation protocol.

Defn 2. *A $(\underline{K}, \underline{M}, \underline{\theta}, \Delta)$ one-shot (POVM simulation) protocol consists of (i) bank of K_j POVMs $\theta_{k_j} = \{\theta_{k_j,m_j} \in \mathcal{P}(\mathcal{H}_{X_j}) : m_j \in M_j\}$ for $k_j \in \mathcal{K}_j$ each with at most M_j outcomes for $j \in [2]$, and a (decoder) POVM $\Delta = \{\Delta_y \in \mathcal{P}(\mathcal{H}_{\underline{\mathcal{KM}}}) : y \in \mathcal{Y}\}$. $\lambda_{\mathcal{Y}}$'s action on $\rho_{\underline{X}}$ can be η-simulated with (communication) cost $(\underline{R}, \underline{C})$ if there exists a $(\underline{K}, \underline{M}, \underline{\theta}, \Delta)$ one-shot POVM simulation protocol such that $\log K_j \le C_j$, $\log M_j \le R_j$ for $j \in [2]$ and*

$$\left\| \left(i_R \otimes \left[\mathscr{E}^{\Delta} \circ \{\mathscr{E}^{\theta_1} \otimes \mathscr{E}^{\theta_2}\} - \mathscr{E}^{\lambda_{\mathcal{Y}}}\right]\right)(\phi_{\rho_{\underline{X}}}) \right\|_1 \le \eta. \quad (2)$$

$\lambda_{\mathcal{Y}}$'s action on $\rho_{\underline{X}}$ can be perfectly simulated with (communication) cost $(\underline{R}, \underline{C})$ if for every $\eta > 0$, there exists N_η such that for all $n \ge N_\eta$, $\otimes_{t=1}^n \lambda_{\mathcal{Y}}$'s action on $\otimes_{t=1}^n \rho_{\underline{X}}$ can be η-simulated with cost $(n\underline{R}, n\underline{C})$. We define

$$\mathscr{C}_\eta(\rho_{\underline{X}}, \lambda_{\mathcal{Y}}) \triangleq \left\{ \begin{matrix} (\underline{R}, \underline{C}) : \lambda_{\mathcal{Y}}\text{'s action on } \rho_{\underline{X}} \text{ can be} \\ \eta\text{-simulated with cost } (\underline{R}, \underline{C}) \end{matrix} \right\}, \text{ and}$$

$$\mathscr{C}(\rho_{\underline{X}}, \lambda_{\mathcal{Y}}) \triangleq \left\{ \begin{matrix} (\underline{R}, \underline{C}) : \lambda_{\mathcal{Y}}\text{'s action on } \rho_{\underline{X}} \text{ can be} \\ \text{perfectly simulated with cost } (\underline{R}, \underline{C}) \end{matrix} \right\}. \quad (3)$$

III. Likelihood POVMs and Analysis Tools

As we discussed in Sec. II and Defn. 2, designing POVMs for the simulation protocol are central to solving the MCP. These designed POVMs perform quantum covering and are central in several fundamental problems [6]–[10]. To comprehend this design problem, let $\rho \in \mathcal{D}(\mathcal{H})$ denote a state and $\lambda_{\mathcal{Y}} = \{\lambda_y : y \in \mathcal{Y}\}$ a POVM. A n-fold product measurement on tensor state $\boldsymbol{\rho} \triangleq \rho^{\otimes n}$ results in post-measurement QC state

$$\Upsilon_{XY}^n \triangleq \sum_{y^n \in \mathcal{Y}^n} \sqrt{\boldsymbol{\rho}} \lambda_{y^n} \sqrt{\boldsymbol{\rho}} \otimes |y^n\rangle\langle y^n|.$$ For large n, Υ_{XY}^n tends to state $\Gamma_{XY}^n \triangleq \sum_{y^n \in T_\delta(p_Y)} 2^{-nH(Y)} \left[2^{-nH(X|Y)} \Pi_{y^n}\right] \otimes |y^n\rangle\langle y^n|$

where the PMF $p_Y(y) = \text{tr}(\lambda_y \rho)$, Π_{y^n} is a projector of dimension $2^{nH(X|Y)}$, where $H(\cdot), H(\cdot|\cdot)$ are Von-Neumann entropies. The goal is to design an $n-$letter simulation POVM θ which when performed on $\boldsymbol{\rho}$, results in a post-measurement state Γ_{XY}^n. Ideally, one would employ the operators $\theta_{y^n} = \frac{\sqrt{\boldsymbol{\rho}} \lambda_{y^n} \sqrt{\boldsymbol{\rho}}}{p_Y(y^n)} : y^n \in T_\delta(p_Y)$ for the simulation POVM design. However, $\sum_{y^n \in T_\delta(p_Y)} \theta_{y^n}$ might dominate $I_{\mathcal{H}}^{\otimes n}$.[1]. This has led to 'coating' θ_{y^n} with several projectors - typical, conditional-typical, cut-off etc. This coating results in several difficulties, starting from its very definition[2], inapplicability to one-shot, involved analysis for even the point-to-point scenario etc.[3]

A. Likelihood POVMs : Definition and Context

Given a collection (a codebook) $c_k = (y^n(m, k) \in \mathcal{Y}^n : m \in [M])$, we define the *likelihood POVM* $\theta_k = \{\theta_{k,m} \in \mathcal{P}(\mathcal{H}^{\otimes n}) : m \in [M]\}$ with M outcomes as

$$\theta_{k,m} \triangleq \frac{S_k^{-\frac{1}{2}} \sqrt{\boldsymbol{\rho}} \lambda_{y^n} \sqrt{\boldsymbol{\rho}} S_k^{-\frac{1}{2}}}{M p_Y(y^n)}, \text{ where } S_k \triangleq \sum_{m \in [M]} \frac{\sqrt{\boldsymbol{\rho}} \lambda_{y^n} \sqrt{\boldsymbol{\rho}}}{M p_Y^n(y^n)}. \quad (4)$$

[1]Note the normalization by $p_Y^n(y^n)$

[2] [13] defines the simulation POVM through a 3 page description starting in [13, Eqn. 22 on Pg. 16] through [13, Eqn. 32 on Pg. 19]

[3]It is possibly for this reason that the natural distributed scenario (Fig. 1) has not been addressed till 2017 and surprising still that [11], [28] in 2020 employs the same simulation POVM designed by Winter [4].

Clearly, θ_k is a POVM[4] and indeed the most natural from the above discussion. Though well known, its analysis for the MCP is very involved. Among others, one issue is that when we randomize over the codebook c_k, the presence of inverse term $S_k^{-\frac{1}{2}}$ in defn. θ_k, now a random object, lends the likelihood POVMs not analyzable. It is for these analysis difficulties that despite awareness[5] and simplicity, the likelihood POVMs, to the best of the author's knowledge have not been employed for the MCP.

A New Approach via a Proxy State: Starting from [30], [31] we have developed a new approach at analyzing the likelihood POVMs. Instead of analyzing the action of the likelihood POVMs on the given state $\phi_{\boldsymbol{\rho}}$ - a purification of $\boldsymbol{\rho}$ - as is convention, we analyze the likelihood POVM's action on a *proxy state*. Specifically, we analyze it on the canonical purification φ_{S_k} of S_k in (4). An informed reader recognizes that S_k approaches ρ. In fact, this yields one of our rate bounds. However, purifications of close states need not be close. Utilizing the freedom in purifications and the closeness of canonical purifications [4, App. A], we developed [30], [31] a new approach to the MCP in the single terminal case.

B. Challenges in Distributed Scenario

To simulate a separable POVM in a distributed scenario, we need to design atleast 2 likelihood POVMs both acting on components of the same entangled state. Refer to. With limited freedom on purifications (owing to entanglement) and the availability of bounds on closeness of only canonical purifications, we now need $\varphi_{S_{k_1}}$ and $\varphi_{S_{k_2}}$ to be separately close to the same corresponding components of a single purification of $\boldsymbol{\rho}_{X_1X_2}$, the latter possibly $\varphi_{\boldsymbol{\rho}_{12}}$. Next, the known results in regards to closeness of canonical purifications [4, App. A] relies on the two operators whose closeness we study, to be states, i.e. unit trace. Due to the two codebooks - c_1, c_2 in proof of Thm. 1- being independently distributed - $S_{k_1} \otimes S_{k_2}$ is not guaranteed unit trace.

C. Techniques, Tools and Contributions

We develop an intricate sequence of steps to handle the above mentioned challenge. Carefully respecting non-commutativity, utilizing the structure of S_{k_j} and leveraging the equivalence of purifications [5, Thm. 5.1.1], we develop a robust technique to analyze likelihood POVMs. We indicate these steps in Sec. IV-B, but reserve a detailed step-by-step analysis to [30].

We highlight the power, universality and simplicity of the above developed techniques. With this work, we have now achieved the best known inner bounds for single POVM simulation in point-to-point, multiple POVM simulation in distributed and multiple POVM simulation in a distributed decoder setup. Moreover, we are able to handle both one-shot (See Sec. IV and in particular Rem. 1) and structured likelihood POVMs (Sec. V) in a unified approach. This is on top of the likelihood POVMs simplicity we alluded to.

In generalizing the above approach, the distributed scenario offers

IV. Inner Bounds Via Unstructured IID POVMs

A. Distributed POVM Simulation in One-Shot

We present our first contribution - a new inner bound to the distributed MCP in the one-shot regime. In the following for $j \in [2]$ and $\jmath$ denotes complement index, i.e., $\{j, \jmath\} = \{1, 2\}$.

Defn 3. *For POVM $\lambda_{\mathcal{Y}} = \{\lambda_y \in \mathcal{P}(\mathcal{H}_{\underline{X}}) : y \in \mathcal{Y}\}$, we let $\tau(\lambda_{\mathcal{Y}})$ denote the collection of all triples $(\underline{\mathcal{W}}, \mu, p)$, where $(\underline{\mathcal{W}}, \mu, p)$ represents (i) POVMs $\mu_{\mathcal{W}_j} \triangleq \{\mu_{w_j} \in \mathcal{P}(\mathcal{H}_{X_j}) : w_j \in \mathcal{W}_j\}$ with outcome set $\mathcal{W}_j$ for $j \in [2]$ and (ii) stochastic matrix $p_{Y|\underline{W}} = p_{Y|W_1W_2}$ satisfying* (1)*. For a triple $(\underline{\mathcal{W}}, \mu, p) \in \tau(\lambda_{\mathcal{Y}})$, we let $\sigma^{R\underline{X}\underline{W}Y}_{\underline{\mathcal{W}},\mu,p} \triangleq \sigma^{RX_1X_2W_1W_2Y}_{\underline{\mathcal{W}},\mu,p} \triangleq (i_R \otimes \left[\overline{\mathscr{E}}^{\mu_{\mathcal{W}_1}} \otimes \overline{\mathscr{E}}^{\mu_{\mathcal{W}_1}}\right] \otimes \overline{\mathscr{E}}_p^{Y|\underline{W}})\{\varphi_{\rho_{12}}\}$ and associate the set $\mathscr{A}_1(\underline{\mathcal{W}}, \mu, p)$ of all quadraples $(R_1, R_2, C_1, C_2) \in \mathbb{R}^4_{\geq}$ satisfying*

$$R_j > \mathcal{I}(W_j; R) - \mathcal{I}(W_1; W_2),\ R_1 + R_2 > \mathcal{I}(\underline{W}; R) - \mathcal{I}(W_1; W_2),$$

$$R_j + C_j > \mathcal{I}(W_j; R, Y) - \mathcal{I}(W_1; W_2), \sum_{j=1}^{2} R_j + C_j > \mathcal{I}(\underline{W}; RY)$$

$$R_1 + R_2 + C_j > \mathcal{I}(W_j; R, Y) + \mathcal{I}(W_{\jmath}; R) - \mathcal{I}(W_1; W_2),$$

for $j = 1, 2$, where $\underline{W} = W_1, W_2$ and $\mathcal{I}(W_1; W_2) \triangleq \mathcal{D}_H^{\frac{\eta}{8}}(P_{W_1W_2} \| P_{W_1} \times P_{W_2}) + \log\left(\frac{\eta}{8}\right)$, $\mathcal{I}(W_j; R) \triangleq \mathcal{D}_{\max}^{\frac{\eta}{16}}(\sigma_{W_jR} \| \sigma_R \otimes \sigma_{W_j}) + \log\left(\frac{64}{\eta^2}\right)$, $\mathcal{I}(W_j; RY) \triangleq \mathcal{D}_{\max}^{\frac{\eta}{16}}(\sigma_{W_jRY} \| \sigma_{RY} \otimes \sigma_{W_j}) + \log\left(\frac{64}{\eta^2}\right)$, $\mathcal{I}(\underline{W}; RY) \triangleq \mathcal{D}_{\max}^{\frac{\eta}{16}}(\sigma_{\underline{W}RY} \| \sigma_{RY} \otimes \sigma_{\underline{W}}) + \log\left(\frac{64}{\eta^2}\right)$, $\mathcal{I}(\underline{W}; R) \triangleq \mathcal{D}_{\max}^{\frac{\eta}{16}}(\sigma_{\underline{W}R} \| \sigma_R \otimes \sigma_{\underline{W}}) + \log\left(\frac{64}{\eta^2}\right)$ all information quantities are evaluated wrt state $\sigma^{RX_1X_2W_1W_2Y}_{\underline{\mathcal{W}},\mu,p}$. Here, $\mathcal{D}_H^{\epsilon}(\cdot||\cdot)$ is smooth hypothesis testing divergence and $\mathcal{D}_{\max}^{\epsilon}(\cdot||\cdot)$ is the smooth max divergence. See [32] for these definitions.

Thm 1. *The action of $\lambda_{\mathcal{Y}}$ on $\rho_{X_1X_2}$ can be η-simulated in one-shot with communication cost (R_1, R_2, C_1, C_2) if there exists a $(\underline{\mathcal{W}}, \mu, p) \in \tau(\lambda_{\mathcal{Y}})$ for which $(R_1, R_2, C_1, C_2) \in \mathscr{A}_1(\underline{\mathcal{W}}, \mu, p)$.*

Proof. Here, we provide the main steps, highlighting the novel elements. See [30] for a complete proof. We adopt a few notational simplifications. We let $\mathcal{H}_j = \mathcal{H}_{X_j}$ for $j \in [2]$, $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$, $\rho_{12} = \rho_{\underline{X}} = \rho_{X_1X_2}$, $\rho_1 = \mathrm{tr}_{X_2}\{\rho_{\underline{X}}\}, \rho_2 = \mathrm{tr}_{X_1}\{\rho_{\underline{X}}\}$, $|\varphi_{\rho_{12}}\rangle \in \mathcal{H}_R \otimes \mathcal{H}_{12}$ denote the canonical purification of ρ_{12} and hence $\mathcal{H}_R = \mathcal{H}_{12}$. We now specify the POVMs and decoder that define our POVm simulation protocol. Towards that end, let $\sigma^{R\underline{X}\underline{W}Y}_{\underline{\mathcal{W}},\mu,p}$ for some $(\underline{\mathcal{W}}, \mu, p) \in \tau(\lambda_{\mathcal{Y}})$ and let $(R_1, R_2, C_1, C_2) \in \mathscr{A}(\underline{\mathcal{W}}, \mu, p)$.

POVM Simulation Protocol : Throughout, $K_j = 2^{C_j}, M_j = 2^{R_j}, B_j = 2^{\beta_j} \in \mathbb{N}$ and $[K_j], [M_j], [B_j]$ denote common randomness, message and bin index sets respectively. Since,

[4]with the appended operator $\theta_{k,-1} \triangleq I_{\mathcal{H}}^{\otimes n} - \pi_k$ where π_k is a projector on range space of S_k, if needed

[5]As noted in [13], the classical version of MCP is the channel synthesis problem [29] and the above likelihood POVM is a quantum version of Cuff's encoder [29]. In spite of its simplicity, [11], [13], aware of [29] have not adopted the same.

the two POVM outcomes are correlated, a Slepian-Wolf [33] binning can reduce message rates R_1, R_2. The outcomes are therefore binned and the bin index $b_j \in [B_j]$ is not communicated to the Rx. For $j \in [2]$, let $c_j = (w_j(k_j, m_j, b_j) \in \mathcal{W}_j : (k_j, m_j, b_j) \in [K_j] \times [M_j] \times [B_j])$ be codes and let $\mu_{k_j,m_j,b_j} \triangleq \mu_{w_j(k_j,m_j,b_j)} \in \mathcal{P}(\mathcal{H}_j)$. For $j \in [2]$, we let

$$S_{k_j} \triangleq \sum_{m_j=1}^{M_j} \sum_{b_j=1}^{B_j} \frac{\sqrt{\rho_j}\mu_{k_j,m_j,b_j}\sqrt{\rho_j}}{K_j M_j B_j p_{W_j}(w_{k_j,m_j,b_j})}, \theta_{k_j,m_j} \triangleq \sum_{b_j=1}^{B_j} \theta_{k_j,m_j,b_j}$$

$$\theta_{k_j,m_j,b_j} \triangleq \frac{S_{k_j}^{-\frac{1}{2}}\sqrt{\rho_j}\mu_{k_j,m_j,b_j}\sqrt{\rho_j}S_{k_j}^{-\frac{1}{2}}}{K_j M_j B_j p_{W_j}(w_{k_j,m_j,b_j})}, \theta_{k_j} \triangleq \begin{Bmatrix} \theta_{k_j,m_j} \in \mathcal{P}(\mathcal{H}_j) \\ \text{for } m_j \in [M_j] \end{Bmatrix}$$

where $p_{W_j}(w_{k_j,m_j,b_j}) = \operatorname{tr}(\rho_j \mu_{k_j,m_j,b_j})$. θ_{k_j} is a POVM.[6] The decoder CPTP map is standard hypothesis-testing based one-shot decoder used in channel coding [34].

B. Breakdown of Error $\|\cdot\|_1$

A POVM simulation protocol must accomplish two tasks - a *QC covering* of the post measurement quantum-classical space and a *Q-covering* of the post measurement quantum space. In addition, the two distributed outcomes are correlated, permitting distributed compression via binning. We shall see the error in (2) splits into 3 corresponding terms.

To fish out the binning error term, we introduce a proxy protocol that communicates the bin indices. Let the proxy likelihood POVMs $\overline{\theta}_{k_j} = \{\theta_{k_j,m_j,b_j} : k_j \in [K_j], m_j \in [M_j], b_j \in [B_j]\}$ communicate the bin index and the corresponding proxy decoder $\overline{\Delta}$ utilize the same. Save for this change, proxy protocol is identical to the proposed protocol.

Towards identifying the 3 error terms, let target $T \triangleq (i_R \otimes \mathscr{E}^{\lambda})(\varphi_{\rho_{12}})$, $\mathcal{S}_{\underline{k}} \triangleq (i_R \otimes \mathscr{E}^{\Delta_{\underline{k}}} \circ [\mathscr{E}^{\theta_{k_1}} \otimes \mathscr{E}^{\theta_{k_2}}])(\varphi_{\rho_{12}})$. (2) is

$$\left\| T - \sum_{\underline{k}} \frac{\mathcal{S}_{\underline{k}}}{\underline{K}} \right\|_1 \leq A + \sum_{\underline{k}} \frac{B_{\underline{k}} + C_{\underline{k}} + D_{\underline{k}}}{\underline{K}}, \ A \triangleq \left\| T - \sum_{\underline{k}} \frac{\overline{\mathcal{S}}_{\underline{k}}^{12}}{\underline{K}} \right\|_1$$

$$B_{\underline{k}} \triangleq \left\| \overline{\mathcal{S}}_{\underline{k}}^{12} - \mathcal{S}_{\underline{k}}^{12} \right\|_1, C_{\underline{k}} \triangleq \left\| \mathcal{S}_{\underline{k}}^{12} - \mathcal{S}_{\underline{k}}^{2} \right\|_1, D_{\underline{k}} \triangleq \left\| \mathcal{S}_{\underline{k}}^{2} - \mathcal{S}_{\underline{k}} \right\|_1$$

$$\mathcal{S}_{\underline{k}}^2 \triangleq \sum_{m_2,b_2} \mathscr{E}^{\Delta_{\underline{k}}} \circ \operatorname{tr}_2 \circ \mathcal{E}^{\theta_{k_1}} \{ J_{k_2 m_2 b_2}(\varphi_{\rho_{12}} \otimes |m_2\rangle\langle m_2|) J_{k_2 m_2 b_2}^{\dagger} \},$$

$$\mathcal{S}_{\underline{k}}^{12} \triangleq \sum_{\underline{m},\underline{b}} \mathscr{E}^{\Delta_{\underline{k}}} \circ \operatorname{tr}_{12} \left\{ J_{\underline{k},\underline{m},\underline{b}}(\varphi_{\rho_{12}} \otimes |\underline{m}\rangle\langle \underline{m}|) J_{\underline{k},\underline{m},\underline{b}}^{\dagger} \right\},$$

$$\overline{\mathcal{S}}_{\underline{k}}^{12} \triangleq \sum_{\underline{m},\underline{b}} \mathscr{E}^{\overline{\Delta}_{\underline{k}}} \circ \operatorname{tr}_{12} \left\{ J_{\underline{k},\underline{m},\underline{b}}(\varphi_{\rho_{12}} \otimes |\underline{m}\ \underline{b}\rangle\langle \underline{m}\ \underline{b}|) J_{\underline{k},\underline{m},\underline{b}}^{\dagger} \right\},$$

$$J_{k_j m_j b_j} \triangleq \frac{(I_R \otimes I_1 \otimes \sqrt{\mu_{k_j,m_j,b_j}})}{M_j B_j p_{W_j}(w_{k_j,m_j,b_j})}, J_{\underline{k},\underline{m},\underline{b}} = J_{k_1 m_1 b_1} J_{k_2 m_2 b_2}.$$

An informed reader will recognize $C_{\underline{k}}, D_{\underline{k}}$ correspond to Q-covering, $B_{\underline{k}}$ corresponds to the binning error event and the first term in the above split corresponds to CQ-covering mentioned earlier. The central challenge is in handling $C_{\underline{k}}$. Before discussing $C_{\underline{k}}$, we address the rest. The analysis of $B_{\underline{k}}$ is straightforward and we refer to [30] for proof of the same. A is handled in Lemma 1 directly by the change-of-measure covering lemma (Sec. A). This brings us to $D_{\underline{k}}$ and $C_{\underline{k}}$. The novelty is in transforming $\mathcal{S}_{\underline{k}}^2$ in two different ways appropriately to handle $D_{\underline{k}}$ and $C_{\underline{k}}$. We only indicate steps here and provide full details in [30].

[6] If $I_{\mathcal{H}_j} > \sum_{m_j} \theta_{k_j,m_j} (= \pi_{S_{k_j}})$, we choose $\theta_{k_j,0,0} = I_{\mathcal{H}_j} -$ and ensure θ_{k_j} appended with $\theta_{k_j,0,0}$ is a POVM. The latter has no effect on the analysis and we have assumed $I_{\mathcal{H}_j} > \sum_{m_j} \theta_{k_j,m_j}$ for simplicity.

Upper bound on $D_{\underline{k}}, C_{\underline{k}}$: Let $\mathcal{U}_{R1} \triangleq \mathcal{U}_{RX_1} : \mathcal{H}_R \otimes \mathcal{H}_{X_1} \rightarrow \mathcal{H}_R \otimes \mathcal{H}_{X_1}$ and $\mathcal{R}_{R2} \triangleq \mathcal{R}_{RX_2} : \mathcal{H}_R \otimes \mathcal{H}_{X_2} \rightarrow \mathcal{H}_R \otimes \mathcal{H}_{X_2}$ be isometries such that $(\mathcal{U}_{RX_1} \otimes I_{X_2}) |\varphi_{\rho_1}\rangle \otimes |\varphi_{\rho_2}\rangle = (\mathcal{R}_{RX_2} \otimes I_{X_1}) |\varphi_{\rho_1}\rangle \otimes |\varphi_{\rho_2}\rangle = |\varphi_{\rho_{12}}\rangle$ from [5, Thm. 5.1.1], $\mathscr{U}_1(A) = \mathcal{U}_{\underline{X}1} A \mathcal{U}_{\underline{X}1}^{\dagger}$ and $\mathscr{R}_2(B) = \mathcal{R}_{\underline{X}2} B \mathcal{R}_{\underline{X}2}^{\dagger}$. The identities

$$K_2 \mathscr{E}^{\theta_{k_2}} \left\{ (\mathscr{U}_1 \otimes i_2)\{\varphi_{\rho_1} \otimes \varphi_{S_{k_2}}\} \right\} = \sum_{m_2,b_2} \operatorname{tr}_2 \left\{ J_{k_2 m_2 b_2}(\varphi_{\rho_{12}} \otimes |m_2\rangle\langle m_2|) J_{k_2 m_2 b_2}^{\dagger} \right\} \quad (5)$$

$$K_1 \mathscr{E}^{\theta_{k_1}} \left\{ (\mathscr{R}_2 \otimes i_1)\{\varphi_{S_{k_1}} \otimes \varphi_{\rho_2}\} \right\} = \sum_{m_1,b_1} \operatorname{tr}_1 \left\{ J_{k_1 m_1 b_1}(\varphi_{\rho_{12}} \otimes |m_1\rangle\langle m_1|) J_{k_1 m_1 b_1}^{\dagger} \right\} \quad (6)$$

are central to analyzing $C_{\underline{k}}, D_{\underline{k}}$. We verify (5), (6) in [30], we show that truth of (5), (6) imply C_k, D_k reduce to familiar Q-covering terms. Towards this, note that (5) implies

$$\mathcal{S}_{\underline{k}}^2 = \sum_{m_2,b_2} \mathscr{E}^{\Delta_{\underline{k}}} \circ \operatorname{tr}_2 \circ \mathcal{E}^{\theta_{k_1}} \{ J_{k_2 m_2 b_2}(\varphi_{\rho_{12}} \otimes |m_2\rangle\langle m_2|) J_{k_2 m_2 b_2}^{\dagger} \} \quad (7)$$

$$= K_2 \mathscr{E}^{\Delta_{\underline{k}}} \circ [\mathcal{E}^{\theta_{k_1}} \otimes \mathcal{E}^{\theta_{k_1}}] \{ (\mathscr{U}_1 \otimes i_2)\{\varphi_{\rho_1} \otimes \varphi_{S_{k_2}}\} \} \text{ and} \quad (8)$$

$$\mathcal{S}_{\underline{k}} = \mathscr{E}^{\Delta_{\underline{k}}} \circ [\mathcal{E}^{\theta_{k_1}} \otimes \mathcal{E}^{\theta_{k_1}}] \{ (\mathscr{U}_1 \otimes i_2)\{\varphi_{\rho_1} \otimes \varphi_{\rho_2}\} \} \text{ since} \quad (9)$$

$\varphi_{\rho_{12}} = (\mathscr{U}_1 \otimes i_2)\{\varphi_{\rho_1} \otimes \varphi_{\rho_2}\}$.

Ananlysis of Terms and Resulting Rate Bounds : Perusing (8), (9), we have $D_{\underline{k}} = \left\| \mathcal{S}_{\underline{k}}^2 - \mathcal{S}_{\underline{k}} \right\|_1 \leq \left\| \varphi_{\rho_2} - K_2 \varphi_{S_{k_2}} \right\|_1 \leq \sqrt[4]{\|\rho_2 - K_2 S_{k_2}\|_1}$, where the last inequality is due to closeness of canonical purifications [4, App. A]. This gives us our first rate bound $R_2 + B_2 > \mathcal{I}(W_2; R)$. The handling of $C_{\underline{k}}$ results in $\left\| \mathcal{S}_{\underline{k}}^{12} - \mathcal{S}_{\underline{k}}^2 \right\|_1 \leq \left\| \varphi_{\rho_2} - K_2 \varphi_{S_{k_2}} \right\|_1 \leq \sqrt[4]{\|\rho_1 - K_1 S_{k_1}\|_1}$. This gives our second rate bound $R_1 + B_1 > \mathcal{I}(W_1; R)$. As we mentioned earlier $B_{\underline{k}}$ is the standard classical Slepian-Wolf binning term and using the hypothesis-testing based one-shot decoder, we obtain the bound $B_1 + B_2 < \mathcal{I}(W_1; W_2)$.

Remark 1. If one studies the pre-Fourier-Motzkin bounds, they can be divided into three categories. The first is the pure Q-covering bounds, the one we mentioned above in regards to $R_j + B_j$. These bounds ensure only covering the post-measurement quantum state. The second category is the classical binning bound. The last is the joint quantum-classical covering that covers both the RY QC space. Recall R is reference and Y is the classical outcome. In the distributed case, the first and third categories result in three bounds each. The first category yields bounds on $R_j + B_j$ for $j \in 2$ and $R_1 + R_2 + B_1 + B_2$. The third category yield bounds on $R_j + B_j + C_j$ for $j \in 2$ and $R_1 + R_2 + C_1 + C_2 + B_1 + B_2$. It turns out that, given the bounds $R_1 + B_1$ and $R_2 + B_2$, the bound on $R_1+R_2+B_1+B_2$ is superfluous. However, this is not true with the third category of bounds wrt QC covering of RY space. Here the $R_1+R_2+C_1+C_2+B_1+B_2$ is not superfluous. Since [35] performs time-sharing, they are unable to achieve the one-shot rates. Again, they are able to achieve the one-shot rates in the restricted feedback case [35, Thm. 3]. It turns out in this case, the bound $R_1+R_2+C_1+C_2+B_1+B_2$ becomes superfluous.

The bound on $A_{\underline{k}}$ is derived by direct application of QCL in Lemma 1. Specifically, by choosing $\mathcal{Y} = \mathcal{H}_{X_1} \otimes \mathcal{H}_{X_2} \otimes \mathcal{H}_Y$, classical set $\mathcal{X} = \mathcal{W}_1 \times \mathcal{W}_2$, $\sigma = (i_R \otimes \mathscr{E}^{\lambda_\mathcal{Y}})\{\varphi_{X_1X_2}\} = (i_R \otimes \mathscr{E}_p^{Y|\underline{W}}) \circ \left[\mathscr{E}^{\theta_1} \otimes \mathscr{E}^{\theta_2}\right]\{\varphi_{\rho_{X_1X_2}}\}$, $q_{W_1W_2}(w_1,w_2) = \mathrm{tr}\big(\rho_{\underline{X}}[\mu_{w_1} \otimes \mu_{w_2}]\big)$ and $p_{W_1W_2}(w_1,w_2) = \mathrm{tr}(\rho_{X_1}[\mu_{w_1}])\,\mathrm{tr}(\rho_{X_2}[\mu_{w_2}])$. While we have a bipartite covering here, this can be straightforwardly managed using Lemma 1 to yield three bounds on $R_j+C_j+B_j$ for $j=1,2$ and R_1 $\square$

C. Inner Bounds via IID POVMs in Asymptotic Regime

We state the following inner bound achievable via unstructured POVMs based on IID random codes. The proof is identical to that of Thm. 1 except for the bounds we obtain. See [30] for a detailed proof. In the following $j \in [2]$ and $\jmath$ denotes complement index, i.e., $\{j,\jmath\} = \{1,2\}$.

Defn 4. *For a triplet $(\underline{\mathcal{W}},\mu,p) \in \tau(\lambda_\mathcal{Y})$ in Defn. 3, let $\mathscr{A}(\underline{\mathcal{W}},\mu,p)$ be the set of all $(R_1,R_2,C_1,C_2) \in \mathbb{R}^4_{\geq}$ satisfying*

$$R_j > I(W_j;R) - I(W_1;W_2),\ R_1+R_2 > I(\underline{W};R) - I(W_1;W_2),$$
$$R_j+C_j > I(W_j;R,Y) - I(W_1;W_2), \sum_{j=1}^{2} R_j+C_j > I(\underline{W};RY)$$
$$R_1+R_2+C_j > I(W_j;R,Y) + I(W_\jmath;R) - I(W_1;W_2),$$

for $j=1,2$, where $\underline{W} = W_1,W_2$ and all information quantities are evaluated wrt state $\sigma^{RX_1X_2W_1W_2Y}_{\underline{\mathcal{W}},\mu,p}$.

Thm 2. *The action of $\lambda_\mathcal{Y}$ on $\rho_{X_1X_2}$ can be perfectly simulated with communication cost (R_1,R_2,C_1,C_2) if there exists a $(\underline{\mathcal{W}},\mu,p) \in \tau(\lambda_\mathcal{Y})$ for which $(R_1,R_2,C_1,C_2) \in \mathscr{A}(\underline{\mathcal{W}},\mu,p)$.*

V. Inner Bounds via Structured POVMs

Defn 5. *For POVM $\lambda_\mathcal{Y} = \{\lambda_y \in \mathcal{P}(\mathcal{H}_{\underline{X}}) : y \in \mathcal{Y}\}$, we let $\tau_\oplus(\lambda_\mathcal{Y})$ denote the collection of all quadraples $(\mathcal{W} = \mathcal{F}_q, \mu^1_\mathcal{W}, \mu^2_\mathcal{W}, p_{Y|W_1\oplus W_2})$, where (i) $\mathcal{W} = \mathcal{F}_q$ is the finite field of size prime power q, (ii) $\mu^j_\mathcal{W} = \{\mu^j_w \in \mathcal{P}(\mathcal{H}_{X_j}) : w \in \mathcal{W}\}$ is a POVM on component j with outcome set $\mathcal{W} = \mathcal{F}_q$ and $p_{Y|W_1\oplus W_2}(y|w_1 \oplus w_2)$ is a stochastic matrix such that*

$$\lambda_y = \sum_{w_1,w_2} p_{Y|W_1\oplus W_2}(y|w_1\oplus w_2)\mu_{w_1} \otimes \mu_{w_2} = \lambda_y\ \forall y \in \mathcal{Y}. \tag{10}$$

For quadraples $(\mathcal{W} = \mathcal{F}_q, \mu^1_\mathcal{W}, \mu^2_\mathcal{W}, p_{Y|W_1\oplus W_2}) \in \tau_\oplus(\lambda_\mathcal{Y})$, let $\mathscr{A}_\oplus(\mathcal{W},\mu,p)$ be the set of all $(R_1,R_2,C_1,C_2) \in \mathbb{R}^4_{\geq 0}$ such that

$$R_j > I(W_j;R) + I(Z;W_\jmath) - I(W_1;W_2)$$
$$R_j + C_j > I(W_j;RY) + I(Z;W_\jmath) - I(W_1;W_2)$$
$$\sum_{j=1}^{2} R_j + C_j > I(\underline{W};RY) + \sum_{j=1}^{2} I(W_j;Z) - I(W_1;W_2)$$

where $\jmath \in [2]$ denotes complement index, i.e., $\{j,\jmath\} = \{1,2\}$, $\underline{W} = W_1,W_2$, $Z = W_1 \oplus W_2$ and all information quantities are computed with respect to state $\sigma^{RW_1W_2ZY} = (i_R \otimes \overline{\mathscr{E}}^{YZ|W_1W_2} \circ [\mathscr{E}^{\mu^1} \otimes \mathscr{E}^{\mu_2}])\{\varphi_{\rho_{X_1X_2}}\}$, where $p_{YZ|W_1W_2}(y,z|w_1,w_2) = p_{Y|W_1W_2}(y|w_1,w_2)\mathbb{1}_{z=w_1\oplus w_2}$.

Thm 3. *The action of $\lambda_\mathcal{Y}$ on $\rho_{X_1X_2}$ can be perfectly simulated with communication cost (R_1,R_2,C_1,C_2) if there exists a quadraple denote the collection of all quadraples $(\mathcal{W} = \mathcal{F}_q, \mu^1_\mathcal{W}, \mu^2_\mathcal{W}, p_{Y|W_1\oplus W_2}) \in \tau_\oplus(\lambda_\mathcal{Y})$ for which $(R_1,R_2,C_1,C_2) \in \mathscr{A}_\oplus(\underline{\mathcal{W}},\mu,p)$.*

Appendix A
Change of Measure Quantum Covering Lemma

We formulate and prove a slightly general quantum covering [5, Chap. 17] that permits choosing the random density operators according to a different distribution than that is used in the averaging.

Lemma 1. *Let $\mathcal{H}$ be a finite dimensional Hilbert space of dimension d, $\mathcal{X}$ be a finite set and for each $x \in \mathcal{X}$, let $\rho_x \in \mathcal{D}(\mathcal{H})$ be density operators. Let p_X and q_X be two distributions on $\mathcal{X}$ and $\sigma = \sum_{u\in\mathcal{X}} q_X(u)\rho_u$. Suppose $C = (X^n(m) \in \mathcal{X}^n : 1 \leq m \leq M)$ be a collection of M pairwise independent and identically distributed vectors with $\mathbf{P}(X^n(m) = x^n) = \prod_{t=1}^n p_X(x_t)$ for each $m \in [M]$. Then, there exists $\eta > 0$ such that for sufficiently large $n \in \mathbb{N}$, we have (11) below A detailed proof is provided in attached appendix.*

$$\mathbb{E}_\mathbf{P}\left\{\left\|\sigma^{\otimes n} - A\right\|_1\right\} \leq \exp\left\{-n\left[\frac{\log M}{n} + \sum_{x\in\mathcal{X}} q_x(x)S(\rho_x) - D(q_X\|p_X)[1+4\eta] - S(\rho)\right]\right\}, \tag{11}$$

where $A \triangleq \dfrac{1}{M}\sum_{m=1}^{M} A(m)$, $A(m) \triangleq \dfrac{q_X^n(X^n(m))}{p_X^n(X^n(m))}\rho_{X^n(m)}$, $\rho_{x^n} = \bigotimes_{t=1}^{n} \rho_{x_t}$ *for any* $x^n = (x_1,\cdots,x_n) \in \mathcal{X}^n$.

References

[1] H. J. Groenewold, "A Problem of Information Gain by Quantal Measurements," International Journal of Theoretical Physics, vol. 4, no. 5,

pp. 327–338, Sept. 1971.
[2] S. Massar and S. Popescu, "Amount of information obtained by a quantum measurement," Phys. Rev. A, vol. 61, p. 062303, May 2000. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.61.062303
[3] A. Winter and S. Massar, "Compression of quantum-measurement operations," Phys. Rev. A, vol. 64, p. 012311, Jun 2001. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.64.012311
[4] A. Winter, ""Extrinsic"and "Intrinsic" Data in Quantum Measurements: Asymptotic Convex Decomposition of Positive Operator Valued Measures," Communications in mathematical physics, vol. 244, no. 1, pp. 157–185, 2004.
[5] M. M. Wilde, Quantum Information Theory, 2nd ed. Cambridge University Press, 2017.
[6] C. H. Bennett, I. Devetak, A. W. Harrow, P. W. Shor, and A. Winter, "The quantum reverse shannon theorem and resource tradeoffs for simulating quantum channels," IEEE Transactions on Information Theory, vol. 60, no. 5, pp. 2926–2959, 2014.
[7] N. Datta, M.-H. Hsieh, and M. M. Wilde, "Quantum rate distortion, reverse shannon theorems, and source-channel separation," IEEE Transactions on Information Theory, vol. 59, no. 1, pp. 615–630, 2013.
[8] M. M. Wilde, N. Datta, M.-H. Hsieh, and A. Winter, "Quantum rate-distortion coding with auxiliary resources," IEEE Transactions on Information Theory, vol. 59, no. 10, pp. 6755–6773, 2013.
[9] J. Radhakrishnan, P. Sen, and N. A. Warsi, "One-shot private classical capacity of quantum wiretap channel: Based on one-shot quantum covering lemma," 2017. [Online]. Available: https://arxiv.org/abs/1703.01932
[10] I. Devetak, "The private classical capacity and quantum capacity of a quantum channel," IEEE Transactions on Information Theory, vol. 51, no. 1, pp. 44–55, 2005.
[11] T. A. Atif, M. Heidari, and S. S. Pradhan, "Faithful simulation of distributed quantum measurements with applications in distributed rate-distortion theory," IEEE Transactions on Information Theory, vol. 68, no. 2, pp. 1085–1118, 2022.
[12] S. Chakraborty, A. Padakandla, and P. Sen, "Centralised multi link measurement compression with side information," in 2022 IEEE International Symposium on Information Theory (ISIT), 2022, pp. 61–66.
[13] M. M. Wilde, P. Hayden, F. Buscemi, and M.-H. Hsieh, "The information-theoretic costs of simulating quantum measurements," Journal of Physics A: Mathematical and Theoretical, vol. 45, no. 45, p. 453001, 2012.
[14] A. Anshu, R. Jain, and N. A. Warsi, "Convex-split and hypothesis testing approach to one-shot quantum measurement compression and randomness extraction," IEEE Transactions on Information Theory, vol. 65, no. 9, pp. 5905–5924, 2019.
[15] J. Radhakrishnan, P. Sen, and N. Warsi, "One-shot marton inner bound for classical-quantum broadcast channel," IEEE Transactions on Information Theory, vol. 62, no. 5, pp. 2836–2848, 2016.
[16] N. Datta, J. M. Renes, R. Renner, and M. M. Wilde, "One-shot lossy quantum data compression," IEEE Transactions on Information Theory, vol. 59, no. 12, pp. 8057–8076, 2013.
[17] A. Anshu, R. Jain, and N. A. Warsi, "Convex-split and hypothesis testing approach to one-shot quantum measurement compression and randomness extraction," IEEE Transactions on Information Theory, vol. 65, no. 9, pp. 5905–5924, 2019.
[18] M. M. Wilde, "Position-based coding and convex splitting for private communication over quantum channels," Quantum Information Processing, vol. 16, no. 10, Sept. 2017. [Online]. Available: http://dx.doi.org/10.1007/s11128-017-1718-4
[19] H.-C. Cheng and L. Gao, "Tight analysis of convex splitting with applications in quantum information theory," in 2023 IEEE International Symposium on Information Theory (ISIT), 2023, pp. 192–197.
[20] A. Anshu, R. Jain, and N. A. Warsi, "On the near-optimality of one-shot classical communication over quantum channels," Journal of Mathematical Physics, vol. 60, no. 1, p. 012204, Jan 2019. [Online]. Available: http://dx.doi.org/10.1063/1.5039796
[21] N. Datta and R. Renner, "Smooth entropies and the quantum information spectrum," IEEE Transactions on Information Theory, vol. 55, no. 6, pp. 2807–2815, 2009.
[22] J. Korner and K. Marton, "How to encode the modulo-two sum of binary sources (corresp.)," IEEE Transactions on Information Theory, vol. 25, no. 2, pp. 219–221, 1979.
[23] A. Padakandla and S. S. Pradhan, "Achievable Rate Region for Three User Discrete Broadcast Channel Based on Coset Codes," IEEE Transactions on Information Theory, vol. 64, no. 4, pp. 2267–2297, April 2018.
[24] S. S. Pradhan, A. Padakandla, and F. Shirani, "An algebraic and probabilistic framework for network information theory," Foundations and Trends® in Communications and Information Theory, vol. 18, no. 2, pp. 173–379, 2020. [Online]. Available: http://dx.doi.org/10.1561/0100000083
[25] D. Krithivasan and S. Pradhan, "Distributed source coding using abelian group codes: A new achievable rate-distortion region," IEEE Trans. on Info. Th., vol. 57, no. 3, pp. 1495–1519, March 2011.
[26] T. A. Atif and S. S. Pradhan, "Distributed quantum faithful simulation and function computation using algebraic structured measurements," 2021.
[27] R. Ahlswede and A. Winter, "Strong converse for identification via quantum channels," IEEE Transactions on Information Theory, vol. 48, no. 3, pp. 569–579, 2002.
[28] T. A. Atif and S. S. Pradhan, "Distributed quantum faithful simulation and function computation using algebraic structured measurements," in 2021 IEEE International Symposium on Information Theory (ISIT), 2021, pp. 124–129.
[29] P. Cuff, "Distributed channel synthesis," IEEE Transactions on Information Theory, vol. 59, no. 11, pp. 7071–7096, Nov 2013.
[30] A. Padakandla, "Optimal Simulation of Quantum Measurements via the Likelihood POVMs," arXiv preprint arXiv:2109.12586, 2021.
[31] ——, "Optimal simulation of quantum measurements via the likelihood povms," in International Zurich Seminar on Information and Communication (IZS 2022), Proceedings, March 2022, pp. 59–63.
[32] A. Anshu, R. Jain, and N. A. Warsi, "Building blocks for communication over noisy quantum networks," IEEE Transactions on Information Theory, vol. 65, no. 2, pp. 1287–1306, 2019.
[33] D. Slepian and J. Wolf, "Noiseless coding of correlated information sources," Information Theory, IEEE Transactions on, vol. 19, no. 4, pp. 471 – 480, July 1973.
[34] L. Wang and R. Renner, "One-shot classical-quantum capacity and hypothesis testing," Physical Review Letters, vol. 108, no. 20, May 2012. [Online]. Available: http://dx.doi.org/10.1103/PhysRevLett.108.200501
[35] T. A. Atif and S. S. Pradhan, "Distributed quantum faithful simulation and function computation using algebraic structured measurements," CoRR, vol. abs/2101.02360, 2021. [Online]. Available: https://arxiv.org/abs/2101.02360