FULL-RATE FULL-DIVERSITY SPACE-FREQUENCY CODING FOR MIMO OFDM SYSTEMS

Maxime Guillaud

Eurecom Institute
2229 Route des Crêtes, BP. 193
06904 Sophia Antipolis Cedex – France
e-mail : `guillaud@eurecom.fr`
Tel : +33 4 93 00 26 47

Dirk T.M. Slock

Eurecom Institute
2229 Route des Crêtes, BP. 193
06904 Sophia Antipolis Cedex – France
e-mail : `slock@eurecom.fr`
Tel : +33 4 93 00 26 06

ABSTRACT

The use of OFDM modulations over MIMO wideband wireless channels opens the way for promising data rates over the air, enabling high-throughput wireless LANs. We propose a coding method that enables the efficient use of the formidable diversity provided by MIMO-OFDM frequency-selective channels, without sacrificing throughput, and while maintaining a reasonable decoding complexity. We show that this scheme exploits all the available (transmit and receive) space diversity through the use of linear precoding, as well as frequency diversity through convolutive coding. The corresponding decoder design is detailed.

1. INTRODUCTION

Recent advances in Multiple-Input-Multiple-Output (MIMO) transmission systems [1] have sparked a new interest for wireless channel diversity. Once considered as a drawback of wireless channels, multiple propagation paths are now regarded as a valuable resource that needs to be exploited in order to achieve the highest transmission rates. The application of MIMO methods to wideband channels yields an unprecedented amount of channel diversity, and makes its actual exploitation with a reasonable complexity a challenging task.

We propose a coding strategy for MIMO Orthogonal Frequency Division Multiplexing (OFDM) systems, that exploits full spatial (transmit and receive) diversity, as well as frequency diversity [2], through the combination of binary codes and linear precoding. Unlike many coding schemes that optimize the error probability under the assumption of Maximum Likelihood decoding [3], the proposed scheme was designed to make decoding relatively simple, while preserving most of the diversity available from the channel. It accomodates any number of transmit and receive antennas, unlike previously proposed schemes [4] that rely on the Alamouti design [5], and as such can only be applied to systems with 2 Tx antennas. Nevertheless, it does not sacrifice throughput, since no redundancy is added by the linear precoding operations, contrary to most diversity schemes (we will denote this a full-rate precoding).

The coding and decoding algorithms are detailed, and an analysis of the achieved diversity will be presented, as well as design rules. We will assume throughout this article that perfect channel state information (CSI) is available to the receiver.

Eurecom's research is partially funded by its industrial partners : Ascom, Bouygues Telecom, Cegetel, France Telecom, Hitachi Europe, Motorola, ST Microelectronics, Swisscom, Texas Instruments and Thales Communications. This work was also partially supported by the french National Telecommunications Research Network (RNRT) project ANTIPODE.

1.1. Notations

In the sequel, we consider a system with N transmit (Tx) antennas and M receive (Rx) antennas, using OFDM modulation over P active frequency bands (or tones). The constellation size is unimportant to this analysis. Furthermore, we assume that P is an integer multiple of N ($P = NG$).

For the sake of simplicity, in order to hide the multipath issues in the OFDM system, we only study the problem from the frequency domain point of view, and assume that the cyclic prefix addition/removal [6] is done transparently. Let us denote by

$$\mathbf{s}_n = (s_{n,1},\ \ldots\ ,\ s_{n,P})\,, \quad n \in [1\ldots N]$$

the frequency-domain representation of the OFDM symbol transmitted over antenna n. Similarly, we denote by

$$\mathbf{r}_m = (r_{m,1},\ldots,r_{m,P})\,, \quad m \in [1\ldots M]$$

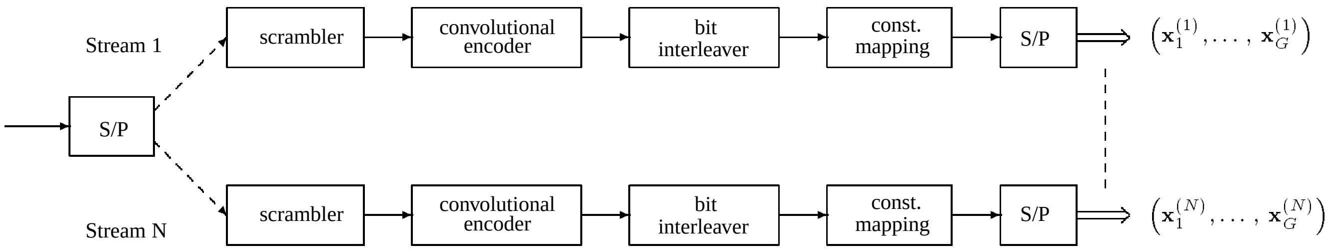

Figure 1: Initial stream separation

the frequency-domain representation of the signal received by antenna number m.

The MIMO channel is represented, in frequency domain, by one complex scalar per Tx–Rx antenna pair, per frequency subband : $h^p_{m,n}$ denotes the gain between transmit antenna number n and receive antenna number m, on frequency band $p \in [1 \ldots P]$. These random variables are assumed to be uncorrelated between different Tx–Rx antennas pairs (this can be achieved by adjusting the distance between elements of the antennas), but in general correlated between subbands of the same Tx–Rx antennas pair. The channel is assumed to be perfectly known at the receiver, and unknown to the transmitter.

Superposition of the signals at the receiver yields

$$r_{m,p} = \sum_{n=1}^{N} h^p_{m,n} s_{n,p} \qquad (1)$$

2. CODING

In a way similar to concatenated codes, the coding operations are split in two parts :

- **outer code** : several independent binary codes (block or convolutive) operating in parallel on different set of bits (which we will denote by ”streams”)
- **inner code** : linear precoding and carefully designed tone assignment scheme. These linear operations must preserve the independence of the streams in order to keep the decoding process simple.

The first step is pictured in figure 1 : the incoming stream of data bits is split into N separate streams. Each of those streams is independently scrambled, encoded by the binary code, bit-interleaved and symbol-mapped. Those symbols are then serial-to-parallel converted, generating N parallel streams of N symbols each. The second step, namely the linear precoding, is described in depth in the following paragraphs.

2.1. Linear precoding

During one OFDM symbol, the outer code as sketched on figure 1 outputs G consecutive $N \times 1$ vectors $\left(\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_G^{(k)}\right)$ for each stream $k = 1 \ldots N$. They are independently precoded by a $N \times N$ matrix $\mathbf{Q}$ (more details on $\mathbf{Q}$ will be given later) :

$$\mathbf{y}_g^{(k)} = \begin{pmatrix} y_{g,1}^{(k)} \\ \vdots \\ y_{g,N}^{(k)} \end{pmatrix} = \mathbf{Q}\mathbf{x}_g^{(k)} \qquad (2)$$

2.1.1. Tone Assignment

The assignment of the linearly precoded values onto the OFDM tones is a permutation of the NP input values onto the P tones of each of the N Tx antennas, according to

$$s_{a(n),i(k,g,n)} = y_{g,n}^{(k)} \qquad (3)$$

where a is any permutation of $[1 \ldots N]$, and i is such that

- $(k, g, n) \mapsto (a(n), i(k, g, n))$ is a bijective mapping of $[1 \ldots N] \times [1 \ldots G] \times [1 \ldots N]$ onto $[1 \ldots N] \times [1 \ldots P]$
- $\forall (k, g, n_1, n_2),\ i(k, g, n_1) = i(k, g, n_2) \Rightarrow n_1 = n_2$

This definition ensures that each $\mathbf{y}_g^{(k)}$ is spread over all N transmit antennas (see figure 2), on a set of N different tones. This guarantees that interference from other streams using the same tone on different Tx antennas will be uncorrelated with the stream being considered. A wide range of functions i meet this constraint. The lack of a satisfactory model for the correlation between tones prevents us from narrowing down this definition.

The case of a is easier : due to the symmetry of our model with respect to the Tx antenna, we can choose $a(n) = n$ without loss of generality.

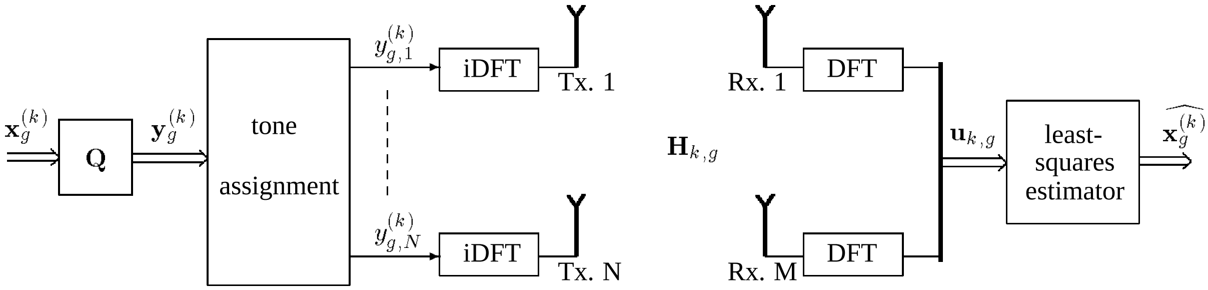

Figure 2: Linear processing operations

2.1.2. Diversity advantage

Let us consider a single error event at the linear processing stage, under the matched-filter bound assumptions (all streams but stream number k are muted) : the decoded vector $\mathbf{x}_g^{(k)'}$ differs from the transmitted one $\mathbf{x}_g^{(k)}$ only in the j-th coefficient, $j = 1 \dots N$

$$\mathbf{E} = \mathbf{x}_g^{(k)} - \mathbf{x}_g^{(k)'} = (0, \dots 0, e_j, 0, \dots 0)^T \quad (4)$$

where $e_j \neq 0$ is any difference between two complex symbols of the constellation. We form a vector that gathers the MN channel coefficients

$$\mathbf{C}_{k,g} = \left(h_{1,1}^{i(k,g,1)} \dots h_{M,1}^{i(k,g,1)}, \dots, h_{1,N}^{i(k,g,N)} \dots h_{M,N}^{i(k,g,N)}\right)^T$$

The received signals corresponding to the transmission of $\mathbf{y}_g^{(k)} = \mathbf{Q}\mathbf{x}_g^{(k)}$ and $\mathbf{y}_g^{(k)'} = \mathbf{Q}\mathbf{x}_g^{(k)'}$ can be written respectively

$$\mathbf{u}_{k,g} = \left(\mathrm{diag}(\mathbf{y}_g^{(k)}) \otimes \mathbf{I}_M\right) \mathbf{C}_{k,g} \quad (5)$$

and

$$\mathbf{u}_{k,g}' = \left(\mathrm{diag}(\mathbf{y}_g^{(k)'}) \otimes \mathbf{I}_M\right) \mathbf{C}_{k,g} \quad (6)$$

where $\otimes \mathbf{I}_M$ denotes the Kronecker product with the $M \times M$ identity matrix. Combining (5) and (6) yields

$$\mathbf{u}_{k,g} - \mathbf{u}_{k,g}' = (\mathrm{diag}(\mathbf{Q}(\mathbf{x}_g^{(k)} - \mathbf{x}_g^{(k)'})) \otimes \mathbf{I}_M)\mathbf{C}_{k,g} \quad (7)$$

which lets us write the the squared Euclidean distance between the received samples as

$$\begin{aligned} & \mathrm{d}\left(\mathbf{u}_{k,g}, \mathbf{u}_{k,g}'\right)^2 \\ = \quad & \mathbf{C}_{k,g}^\dagger \left(\mathrm{diag}(\mathbf{QE})^\dagger \otimes \mathbf{I}_M^\dagger\right) \left(\mathrm{diag}(\mathbf{QE}) \otimes \mathbf{I}_M\right) \mathbf{C}_{k,g} \\ = \quad & \mathbf{C}_{k,g}^\dagger \left((\mathrm{diag}(\mathbf{QE})^\dagger \mathrm{diag}(\mathbf{QE})) \otimes \mathbf{I}_M\right) \mathbf{C}_{k,g} \end{aligned} \quad (8)$$

As shown in [7], assuming that the channel coefficients in $\mathbf{C}_{k,g}$ are complex, zero-mean, independent Gaussian random variables, the rank of $\left((diag(\mathbf{QE})^\dagger diag(\mathbf{QE})) \otimes \mathbf{I}_M\right)$ determines the diversity advantage of this coding scheme. It is obvious from equation (8) that if $\mathbf{Q}$ contains no zero, our scheme achieves diversity MN. In this case, the design criterion for the choice of $\mathbf{Q}$ should maximize

$$\min_{j, e_j} \det \left((\mathrm{diag}(\mathbf{QE})^\dagger \mathrm{diag}(\mathbf{QE})) \otimes \mathbf{I}_M\right)$$

under the following constraints

- $\mathbf{Q}$ must be full rank
- $||\mathbf{Q}|| = 1$

A satisfactory solution is the Vandermonde matrix [8]

$$\mathbf{Q} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & \theta_1 & \dots & \theta_1^{N-1} \\ 1 & \theta_2 & \dots & \theta_2^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & \theta_N & \dots & \theta_N^{N-1} \end{pmatrix} \quad (9)$$

where $\theta_k = e^{j\frac{\pi}{N}(1+2k)},\ k = 1 \dots N$.

3. DECODING

3.1. Inner decoding

The first step in the decoder is a joint inversion of the linear precoder and the MIMO channel, through least-squares estimation of $\mathbf{x}_g^{(k)}$.

We first rewrite equation (1) as

$$\forall (m, k, g, f),\ r_{m,i(k,g,f)} = \sum_{n=1}^{N} h_{m,n}^{(i(k,g,f))} s_{n,i(k,g,f)} \quad (10)$$

$$= h_{m,f}^{(i(k,g,f))} s_{f,i(k,g,f)} + \sum_{n \neq f} h_{m,n}^{(i(k,g,f))} s_{n,i(k,g,f)} \quad (11)$$

Thanks to the definition of i, the second term of the sum is uncorrelated with $\mathbf{y}_g^{(k)}$. We denote this noise term $n_{m,k,g,f}$.

In order to decode $\mathbf{x}_g^{(k)}$, we need to form the following matrices

$$\mathbf{y}_g^{(k)} = \left(s_{1,i(k,g,1)}, \ldots, s_{N,i(k,g,N)}\right)^T$$

$$\mathbf{u}_{k,g} = \begin{pmatrix} r_{1,i(k,g,1)} \\ \vdots \\ r_{M,i(k,g,1)} \\ \vdots \\ r_{1,i(k,g,N)} \\ \vdots \\ r_{M,i(k,g,N)} \end{pmatrix} \qquad \mathbf{n}_{k,g} = \begin{pmatrix} n_{1,k,g,1} \\ \vdots \\ n_{M,k,g,1} \\ \vdots \\ n_{1,k,g,N} \\ \vdots \\ n_{M,k,g,N} \end{pmatrix}$$

and

$$\mathbf{H}_{k,g} = \begin{pmatrix} h_{1,1}^{i(k,g,1)} & & & \\ \vdots & 0 & \ldots & 0 \\ h_{M,1}^{i(k,g,1)} & & & \\ & h_{1,2}^{i(k,g,2)} & \ddots & \vdots \\ & \vdots & & \\ 0 & h_{M,2}^{i(k,g,2)} & & 0 \\ & & \ddots & \\ \vdots & \ddots & & h_{1,N}^{i(k,g,N)} \\ 0 & \ldots & 0 & \vdots \\ & & & h_{M,N}^{i(k,g,N)} \end{pmatrix}$$

using these notations, we obtain the following expression

$$\mathbf{u}_{k,g} = \mathbf{H}_{k,g}\mathbf{y}_g^{(k)} + \mathbf{n}_{k,g} = \mathbf{H}_{k,g}\mathbf{Q}\mathbf{x}_g^{(k)} + \mathbf{n}_{k,g} \qquad (12)$$

Thus, $\mathbf{x}_g^{(k)}$ can be obtained through least-squares estimation

$$\widehat{\mathbf{x}_g^{(k)}} = \left(\mathbf{Q}^\dagger \mathbf{H}_{k,g}^\dagger \mathbf{H}_{k,g}\mathbf{Q}\right)^{-1} \left(\mathbf{H}_{k,g}\mathbf{Q}\right)^\dagger \mathbf{u}_{k,g} \qquad (13)$$

3.2. Outer decoding

Thanks to the design rules presented in section 2.1.1, we have shown how $\left(\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_G^{(k)}\right)$ can be estimated independently for each stream k. This allows for separate decoding of each stream, using any decoder suitable for the used code, and hence for interference cancelation between streams (since streams have been regarded as interference to each other throughout the linear processing). All the classical interference cancellation schemes can be used (onion peeling, parallel interference cancellation (PIC), etc ...).

4. CONCLUSION

We introduced a coding scheme for MIMO OFDM systems based on the concatenation of parallel binary codes and linear precoding, especially suitable for applications where the throughput can not be sacrificed to redundancy, but where a fairly large number of Tx and Rx antennas are available.

5. REFERENCES

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, March 1998.

[2] H. Bölcskei and A. J. Paulraj, "Space-frequency coded broadband OFDM systems," in *Proc. IEEE Wireless Communications and Networking Conference*, September 2000, vol. 1, pp. 1–6.

[3] B. Lu and X. Wang, "Space-time code design in OFDM systems," in *Proc. IEEE Globecom Conference*, San Francisco, CA, November 2000.

[4] Z. Liu, G. B. Giannakis, S. Barbarossa, and A. Scaglione, "Transmit-antennae space-time block coding for generalized OFDM in the presence of unknown multipath," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 7, pp. 1352–1364, July 2001.

[5] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[6] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications: Where Fourier meets Shannon," *IEEE Signal Processing Magazine*, vol. 17, no. 3, pp. 29–48, May 2000.

[7] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 744–765, March 1998.

[8] A. Medles and D. T. M. Slock, "Linear convolutive space-time precoding for spatial multiplexing MIMO systems," in *Proc.* 39^{th} *Annual Allerton Conference on Communication, Control, and Computing*, Champaign, IL, October 2001.