Semi-Blind Maximum-Likelihood Multichannel Estimation with Gaussian Prior for the Symbols using Soft Decisions

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Abstract: We present Maximum-Likelihood (ML) approaches to semi-blind estimation of multiple FIR channels. The first approach, DML, is based on a deterministic model. The second one, GML is based on a Gaussian model in which the input symbols are considered as Gaussian random variables: this model leads to better and more robust performance than DML. Algorithms are presented to solve DML and GML and the significant improvement of GML w.r.t. DML is demonstrated. A soft decision strategy is also presented to improve ML performance: the most reliable decisions taken at the output of an equalizer built from a semi-blind ML channel estimate are treated as known symbols and semi-blind ML is reiterated with an augmented number of known symbols.

I   Introduction

Blind multichannel identification has received considerable interest over the last decade. In particular, second-order methods have raised a lot of attention, due to their ability to perform channel identification with relatively short data bursts. These methods suffer from several drawbacks though. They leave an ambiguity in the channel determination (in a single-user context, they can only determine the channel up to a phase factor) and cannot identify certain ill-conditioned channels. This motivates the development of various other methods to alleviate this problem. Semi-blind estimation techniques exploit the knowledge of certain input symbols and appear superior to purely blind and training sequence methods as much for their performance as for their ability to perform identification for any channel for few known symbols [1].

We present two semi-blind Maximum-Likelihood (ML) methods to estimate multiple FIR channels. The first one, DML, is based on a deterministic model and combines a blind ML criterion with a training sequence criterion: a low-complexity solution is proposed. The other method, GML, based on a Gaussian model in which the symbols are considered as Gaussian, is solved by the method of scoring. The significant improvement of GML w.r.t. DML is demonstrated. A soft decision strategy is also proposed in which the decisions on the most reliable

symbol estimates given by an equalizer built from the semi-blind ML channel estimate are considered as correct. The corresponding symbols are taken as known and semi-blind ML is reiterated with an augmented number of known symbols. Simulations illustrate the different algorithms.

II   Problem Formulation

We consider a single-user multichannel model: this model results from the oversampling of the received signal and/or from reception by multiple antennas. Consider a sequence of symbols $a(k)$ received through m channels of length N and coefficients $\boldsymbol{h}(i)$:

$$\boldsymbol{y}(k) = \sum_{i=0}^{N-1} \boldsymbol{h}(i)a(k-i) + \boldsymbol{v}(k), \tag{1}$$

$\boldsymbol{v}(k)$ is an additive independent white Gaussian circular noise with $r_{\boldsymbol{vv}}(k-i) = \mathrm{E}\,\boldsymbol{v}(k)\boldsymbol{v}(i)^H = \sigma_v^2 I_m \delta_{ki}$.

The symbol constellation is assumed known. When the input symbols are real, it will be advantageous to consider separately the real and imaginary parts of the channel and received signal as:

$$\begin{bmatrix} \mathrm{Re}(\boldsymbol{y}(k)) \\ \mathrm{Im}(\boldsymbol{y}(k)) \end{bmatrix} = \sum_{i=0}^{N-1} \begin{bmatrix} \mathrm{Re}(\boldsymbol{h}(i)) \\ \mathrm{Im}(\boldsymbol{h}(i)) \end{bmatrix} a(k-i) + \begin{bmatrix} \mathrm{Re}(\boldsymbol{v}(k)) \\ \mathrm{Im}(\boldsymbol{v}(k)) \end{bmatrix} \tag{2}$$

Let's rename $\boldsymbol{y}(k) = [\mathrm{Re}^H(\boldsymbol{y}(k))\ \mathrm{Im}^H(\boldsymbol{y}(k))]^H$, and idem for $\boldsymbol{h}(i)$ and $\boldsymbol{v}(k)$; we get again (1), but this time, all the quantities are real. The number of channels gets doubled, which has for advantage to increase diversity. Note that the monochannel case does not exist for real input constellations.

Assume we receive M samples, concatenated in the vector $\boldsymbol{Y}_M(k)$:

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(h)\, A_{M+N-1}(k) + \boldsymbol{V}_M(k) \tag{3}$$

$\boldsymbol{Y}_M(k) = [\boldsymbol{y}^H(k-M+1) \cdots \boldsymbol{y}^H(k)]^H$, similarly for $\boldsymbol{V}_M(k)$, and $A_M(k) = [a^H(k-M-N+2) \cdots a^H(k)]^H$, where $(.)^H$ denotes Hermitian transpose. $\mathcal{T}_M(h)$ is a block Toeplitz matrix filled out with the channel coefficients grouped in the vector h. We assume that some symbols are known: A_k contains the M_k known symbols and A_u, the M_u unknown symbols. We shall

$$\boldsymbol{Y} \;=\; \mathcal{T}(h)\,A + \boldsymbol{V} = \mathcal{T}_k(h)A_k + \mathcal{T}_u(h)A_u + \boldsymbol{V} \qquad (4)$$

III Semi-Blind ML

A Deterministic ML (DML)

In the deterministic model both input symbols and channel coefficients are considered as deterministic. We are interested in the joint estimation of h and the unknown symbols (decoupled from the estimation of σ_v^2). DML maximizes the probability density function of the observations $f_{Y|A,h}(\boldsymbol{Y}) = f_{V|A,h}(\boldsymbol{Y} - \mathcal{T}(h)A)$ and the DML criterion is:

$$\min_{A_u,h} \|\boldsymbol{Y} - \mathcal{T}(h)A\|^2 \qquad (5)$$

optimizing w.r.t. the unknown symbols, we get:

$$A_u = \left(\mathcal{T}_u^H(h)\mathcal{T}_u(h)\right)^{-1}\mathcal{T}_u^H(h)\left(\boldsymbol{Y} - \mathcal{T}_k(h)A_k\right) \qquad (6)$$

which is the output of the non-causal Minimum-Mean-Squared-Error (MMSE) Zero-Forcing (ZF) decision-feedback equalizer with feedback of the known symbols. Substituting (6) in (5) we get the following minimization criterion for h:

$$\min_{h}\left(\boldsymbol{Y} - \mathcal{T}_k(h)A_k\right)^H P_{\mathcal{T}_u(h)}^{\perp}\left(\boldsymbol{Y} - \mathcal{T}_k(h)A_k\right) \qquad (7)$$

where $P_{\mathcal{T}_u(h)}^{\perp} = I - \mathcal{T}_u(h)\left(\mathcal{T}_u^H(h)\mathcal{T}_u(h)\right)^{-1}\mathcal{T}_u^H(h)$ is the projection on the orthogonal complement of the column space of $\mathcal{T}_u(h)$.

B Gaussian ML (GML)

In the Gaussian Model [2],[3], the channel coefficients are still considered as deterministic but the input symbols as Gaussian random variables (to take their second-order statistics into account). This hypothesis, although false, allows to robustify the estimation problem and improves performance w.r.t. DML [2, 1].

In the Gaussian model for (4), $\boldsymbol{V} \sim \mathcal{N}(0, C_{VV})$ is independent of $A \sim \mathcal{N}(A^o, C_{AA})$. A^o is the prior mean for the symbols. In the Gaussian case, the estimation of the channel can be done without the estimation of the unknown input symbols: GML considers the joint estimation of h and the coefficients of C_{VV}. $\boldsymbol{Y} \sim \mathcal{N}(\mathcal{T}(h)A^o, C_{YY})$, $C_{YY} = \mathcal{T}(h)C_{AA}\mathcal{T}^H(h) + C_{VV}$ and the GML criterion is $\max\limits_{h,\sigma_v^2} f_{Y|h,\sigma_v^2}(\boldsymbol{Y})$, or:

$$\min_{h,\sigma_v^2}\left\{\ln\det C_{YY} + \left(\boldsymbol{Y} - \mathcal{T}(h)A^o\right)^H C_{YY}^{-1}\left(\boldsymbol{Y} - \mathcal{T}(h)A^o\right)\right\} \qquad (8)$$

We will specialize this general model to the semi-blind case as follows: $A^o = \mathcal{P}\begin{bmatrix} A_k \\ 0 \end{bmatrix}$ and $C_{AA} = \mathcal{P}\begin{bmatrix} \epsilon I & 0 \\ 0 & \sigma_a^2 I \end{bmatrix}\mathcal{P}^H$, where $\mathcal{P}$ is a permutation matrix such that $A = \mathcal{P}[A_k^H \; A_u^H]^H$ and ϵ is arbitrarily small. Furthermore, as already mentioned, we take $C_{VV} = \sigma_v^2 I$.

In this section we assume that the known symbols are grouped and at the beginning of the burst ($\mathcal{P} = I$). We propose a semi-blind criterion mixing a deterministic and a Gaussian point of view: it combines the blind DML criterion and a training sequence criterion.

A Blind DML

In the blind case, criterion (7) becomes:

$$\min_{h} \boldsymbol{Y}^H P_{\mathcal{T}(h)}^{\perp}\boldsymbol{Y}. \qquad (9)$$

We consider here that the blind DML identifiability conditions are verified [1]: the channel is irreducible, the input symbols are persistently exciting and the burst is sufficiently long. The channel is then identifiable up to a scale factor and the regularizing constraint $\|h\| = 1$ is assumed. $P_{\mathcal{T}(h)}^{\perp}$ is the orthogonal projection onto the noise subspace (the orthogonal complement of the column space of $\mathcal{T}(h)$).

The key to a computationally attractive solution of the DML problem is a linear parameterization of the noise subspace. We consider here a linear parameterization in terms of channel coefficients. Let $\mathbf{H}^{\perp}(z)$ be such a parameterization; it verifies $\mathbf{H}^{\perp}(z)\mathbf{H}(z) = 0$ and $\mathcal{T}(h^{\perp})\mathcal{T}(h) = 0$; $\mathcal{T}(h^{\perp})$ is filled with the coefficients of $\mathbf{H}^{\perp}(z)$ and spans the noise subspace. An example is [4]:

$$\mathbf{H}^{\perp}(z) = \begin{bmatrix} -H_2(z) & H_1(z) & 0 & \cdots & 0 \\ 0 & -H_3(z) & H_2(z) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ H_m(z) & 0 & \cdots & 0 & -H_1(z) \end{bmatrix}. \qquad (10)$$

Since $P_{\mathcal{T}(h)}^{\perp} = P_{\mathcal{T}^H(h^{\perp})}$, (9) can be written as:

$$\min_{\|h\|=1} \boldsymbol{Y}^H \mathcal{T}^H(h^{\perp})\mathcal{R}^+ \mathcal{T}(h^{\perp})\boldsymbol{Y} \qquad (11)$$

where $\mathcal{R} = \mathcal{T}(h^{\perp})\mathcal{T}^H(h^{\perp})$ and $(.)^+$ denotes the Moore-Penrose pseudo-inverse ($\mathcal{T}(h^{\perp})$ may not be full-row rank). $\mathcal{T}(h^{\perp})$ being linear in h, a matrix $\mathcal{Y}$ filled out with the elements of the observation vector $\boldsymbol{Y}$ can be found such that $\mathcal{Y}h = \mathcal{T}(h^{\perp})\boldsymbol{Y}$. Then (11) becomes:

$$\min_{\|h\|=1} h^H \mathcal{Y}^H \mathcal{R}^+ \mathcal{Y}h \qquad (12)$$

B Blind Pseudo-Quadratic DML (PQDML)

The principle of PQDML has been first applied to sinusoids in noise estimation [5] and then to blind DML in [6]. The gradient of the DML cost function (12) may be arranged as $\mathcal{P}(h)h$, where $\mathcal{P}(h)$ is (ideally) positive semi-definite with a single singularity. The ML solution verifies $\mathcal{P}(h)h = 0$, which is solved under constraint $\|h\| = 1$ by the PQDML strategy as follows: in a first step $\mathcal{P}(h)$ is considered constant, and as $\mathcal{P}(h)$ is positive semi-definite, the problem becomes quadratic: h is chosen in [6] as the eigenvector corresponding to the smallest absolute eigenvalue of $\mathcal{P}(h)$. This solution is used to reevaluate $\mathcal{P}(h)$

finding the right $\mathcal{P}(h)$, especially with the positive semi-definite constraint. In our problem:

$$\mathcal{P}(h) = \mathcal{Y}^H \mathcal{R}^+ \mathcal{Y} - \mathcal{B}^H \mathcal{B} \qquad (13)$$

$\mathcal{T}^H(h^\perp)\mathcal{B} = \mathcal{B}^* h^*$ with $\mathcal{B} = \left[\mathcal{T}(h^\perp)\mathcal{T}^H(h^\perp)\right]^+ \mathcal{T}(h^\perp)\mathcal{Y}$ $((.)^*$ denotes complex conjugate). Asymptotically, the effect of the second term is to remove the noise contribution present in the first one, then $\mathcal{P}(h) = \mathcal{X}^H \mathcal{R}^+ \mathcal{X}$, where $\mathcal{X}h = \mathcal{T}(h^\perp)X$, $X = \mathcal{T}(h)A$; this $\mathcal{P}(h)$ admits h as singular eigenvector. Asymptotically, there is global convergence: any initialization of $\mathcal{P}(h)$ results in a consistent PQDML channel estimate and the second iteration finds the global minimizer.

The matrix $\mathcal{P}(h)$ is indefinite for finite M, and applying directly the PQDML strategy will not work as stated in [6], except for high SNR. We introduce an arbitrary λ, such that PQDML becomes the following minimization problem:

$$\min_{\|h\|=1, \lambda} h^H \left\{ \mathcal{Y}^H \mathcal{R}^+ \mathcal{Y} - \lambda \mathcal{B}^H \mathcal{B} \right\} h \qquad (14)$$

with semi-definite positivity constraint on the central matrix. h is the minimal generalized eigenvector of $\mathcal{Y}^H \mathcal{R}^+ \mathcal{Y}$ and $\mathcal{B}^H \mathcal{B}$, and λ the minimal generalized eigenvalue. Asymptotically, there is global convergence for h, as described previously, and for λ (to 1).

The stationary points of PQDML are the same as those of DML, this is why PQDML has the same performance as DML. Asymptotically PQDML gives the global ML minimizer.

C   Semi-blind Criterion

Let's decompose Y as $Y = [Y_{TS}^H \ Y_B^H]^H$. Y_B groups all the observations where unknown symbols only appear. Y_{TS} groups all the observations containing known symbols, and especially the $N-1$ observations where a mixture of both known and unknown symbols appear.

The symbols in Y_B are treated as deterministic and blind DML is applied to it and solved by PQDML. In Y_{TS}, the known symbols are treated as deterministic and the unknown symbols as i.i.d. Gaussian random variables of mean 0 and variance σ_a^2 and GML is applied to it. In the GML criterion, we neglect the first term (the determinant term): the resulting criterion corresponds to the optimally Weighted Least-Squares (WLS) problem $\|Y_{TS} - \mathcal{T}_{TS}(h)A_{TS}^o\|_{C_{YY}^{-1}}^2$ (A_{TS}^o is the mean of the symbols, $C = C_{Y_{TS}Y_{TS}}$), solved by initializing the denominator and solving the LS problem.

As Y_{TS} and Y_B are decoupled in terms of noise, the mixed ML criterion will be the sum of DML for Y_B and WLS for Y_{TS}:

$$\min_{h, \sigma_v^2} Y_B^H P_{\mathcal{T}_B^H(h^\perp)} Y_B + \sigma_v^2 \|Y_{TS} - \mathcal{T}_{TS}(h)A_{TS}^o\|_{C^{-1}}^2 \quad (15)$$

Only the information coming from the unknown symbols present in Y_{TS} is lost, which is negligible as the number of observations Y_B will be usually large. At each iteration, $h =$

$$\sigma_v^2 \left(\mathcal{Y}_B^H \mathcal{R}_B^+ \mathcal{Y}_B - \lambda \mathcal{B}_B^H \mathcal{B}_B + \sigma_v^2 A_{TS}^H C^{-1} A_{TS} \right)^{-1} A_{TS}^H C^{-1} Y_{TS}$$
$$(16)$$

fers the advantage to be of low complexity.

V   The Method of Scoring for GML

It appears difficult to find low complexity methods to solve GML: we propose the iterative method of scoring. Compared to PQDML used to solve DML, it represents an increase in complexity but at the same time a gain in performance that can be significant, as will be seen in the simulations (and predicted by the performance studies). Furthermore, it offers more robustness to ill-conditioned channels.

The method of scoring consists in an approximation of the Newton-Raphson algorithm which finds an estimate $\theta^{(i)}$ at iteration i from $\theta^{(i-1)}$, the estimate at iteration $i-1$, as:

$$\theta^{(i)} = \theta^{(i-1)} - \left[\frac{\partial}{\partial \theta^*} \left(\frac{\partial c(\theta)}{\partial \theta^*} \right)^H \Bigg|_{\theta^{(i-1)}} \right]^{-1} \frac{\partial c(\theta)}{\partial \theta^*} \Bigg|_{\theta^{(i-1)}}$$
$$(17)$$

where $c(\theta)$ is the cost function and θ contains the parameters to estimate. The method of scoring approximates the second derivative by its expected value, which is here the Fisher Information Matrix (FIM). This approximation is justified in the blind case by the law of large numbers as the number of data is generally large. In the semi-blind case, the number of known symbols being finite, this approximation is not valid anymore but it will turn out to work very well in our simulations. Here $c(\theta) = \ln f_{Y|h, \sigma_v^2}(Y)$. The expression of the FIM for GML can be found in [2]. This iterative algorithm requires an initialization close to the global maximum, and may fall in local minima if not correctly initialized.

The GML formulation has for advantage to take into account known symbols even if they are not grouped. This is difficult to do with methods combining a blind and a training sequence criterion. This property will be particularly useful for the soft decision strategy explained below.

Note that the method of scoring could also be applied to solve DML in case the known symbols are not grouped. In this case, the expression of FIM [2] needs to be averaged over the unknown symbols.

VI   Soft Decision Strategy

The soft decision strategy which is particularly well connected to the general semi-blind context, is as follows:

1. From an estimate of the channel, an equalizer is built that gives estimates of the unknown symbols. The most reliable estimates are selected and hard decisions on them are considered as known symbols. The non reliable symbols are still considered as unknown.

2. Semi-blind estimation is again applied with this augmented number of known symbols. Steps 1 and 2 can be reiterated.

where all the equalizer outputs would be considered as error-free and then as known. Some algorithms exploiting the finite alphabet nature of the input symbols [7] follow that scheme: step 2 is replaced by a training sequence based channel estimation where the training symbols are the hard decisions. This kind of algorithms requires a good channel initialization and because the decision step may not be error-free, fall easily in local minima. This could be avoided by the soft decision process.

Consider a MMSE-ZF equalizer as in (6) based on the true channel. It gives as estimates for the unknown symbols:

$$
\begin{aligned}
\hat{A}_u &= \left(\mathcal{T}_u^H(h)\mathcal{T}_u(h)\right)^{-1}\mathcal{T}_u^H(h)\left(\boldsymbol{Y} - T_k(h)A_k\right) \\
&= A_u + \left(\mathcal{T}_u^H(h)\mathcal{T}_u(h)\right)^{-1}\mathcal{T}_u^H(h)\boldsymbol{V}.
\end{aligned}
\tag{18}
$$

Then for each unknown symbol: $\hat{a}(k) = a(k) + v'(k)$ where $v'(k)$ is a centered Gaussian random variable, linear combination of elements of $\boldsymbol{V}$. Figure 1 shows the distribution of $\hat{a}(k)$.

We will consider here only the case of a BPSK; the principle of soft decisions could be extended to other constellations. The reliable symbol estimates will verify $|\hat{a}(k)| \geq \alpha$; they will be all the more reliable as α is large: see figure 1 with $\alpha = 1$, in which case, as $v'(k)$ is centered, approximately half the symbol estimates would be considered as reliable.

This soft decision strategy introduces however correlations between $a(k)$ ($= \text{dec}(\hat{a}(k))$) and $v'(k)$ and then between the noise $\boldsymbol{V}$ and the symbols A originally independent. Figure 2 shows the joint distribution of $a(k)$ and $v'(k)$ for the reliable and non-reliable $\hat{a}(k)$: in both cases, $a(k)$ and $v'(k)$ are correlated (for $\alpha = 1$, the marginal distribution of $v'(k)$ remains approximately unchanged).

Simulations proved GML and DML to be very sensitive to these modifications in the correlations: you get better performance when you do not add the hard decisions to the list of the known symbols. The repercussions of these correlations in the formulations of DML and GML are as follows:

- For DML, in $f_{V|A,h}(\boldsymbol{Y} - \mathcal{T}(h)A)$, correlations between A and V are to be taken into account: $v'(k)|a(k)$ does not have a Gaussian distribution anymore, but half a Gaussian.

- For GML, in a Gaussian approximation for $f_{Y|h}$, the correlations between A and $\boldsymbol{V}$ have to be taken into account.

The incorporation of these modifications in DML and GML are the subject of ongoing studies. As an alternative approach, we considered another type of interval of reliability: a symbol estimate is considered as reliable if $|\hat{a}(k) - \text{dec}(\hat{a}(k))| \leq \beta$ (figure 3). With this choice of interval, the correlation between symbols and noise disappears (as long as the interval is sufficiently small): see figure 4. The marginal distribution of the noise has changed though, and namely the variance of $v'(k)$ associated to the reliable or non-reliable symbols is different. Again, ongoing studies try to take into account these modifications. Simulations showed that DML or GML as in (7) and (9) were not very sensitive to them: we kept DML and GML in their original formulation in this paper.

Note that the MMSE-ZF equalizer could be replaced by an MMSE equalizer which gives a higher output SNR. At the out-

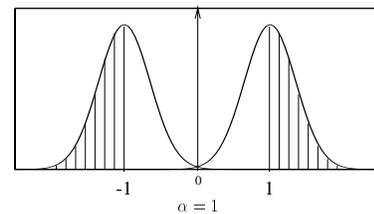account, as well as its bias [8].

Figure 1: Distribution of the symbol estimates at the output of the MMSE ZF equalizer: reliable decisions such that $|\hat{a}(k)| \geq 1$
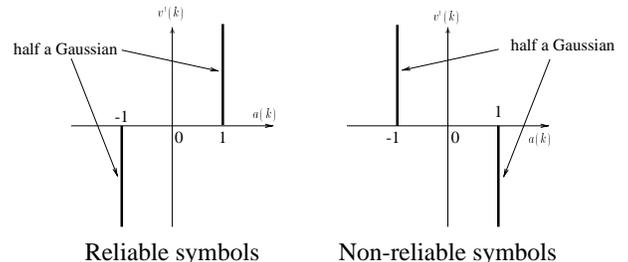

Figure 2: Joint distribution of $\boldsymbol{v}'(k)$ and $a(k)$ for the reliable (left) and non reliable symbols (right)
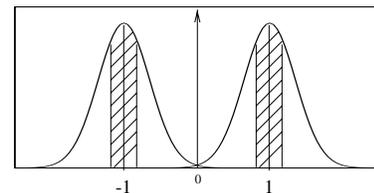

Figure 3: Distribution of the symbol estimates at the output of the MMSE ZF equalizer: reliable decisions such that $|\hat{a}(k) - \text{dec}(\hat{a}(k))| \geq \beta$

VII Simulations

We propose to test the different algorithms in the following sequence:

1. Initialization: the channel is estimated blindly by SRM [9] up to a scale factor that we determine thanks to the training symbols. When the channel is too ill-conditioned to be correctly estimated blindly, training sequence estimation is done instead.

2. Semi-blind PQDML.

3. Semi-blind GML solved by the method of scoring.

4. A MMSE-ZF equalizer is built from the previous channel estimate and soft decisions are taken. GML based on the soft decisions solved by the method of scoring.

These different steps are of increasing complexity but also of increasing performance. We assume σ_v^2 known in our simulations. We do not show the performance of the method of scoring

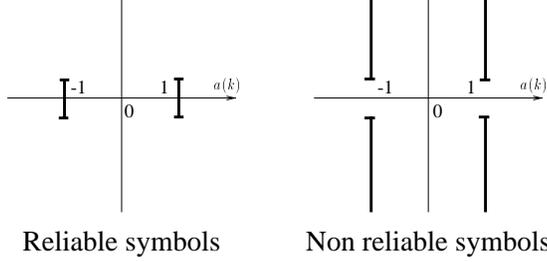Reliable symbols   Non reliable symbols

Figure 4: Joint distribution of $v'(k)$ and $a(k)$ for the reliable (left) and non reliable decisions (right)

for DML: in our simulations it performed similarly to PQDML of step 1, and, based on soft decisions, it performed worse than soft GML of step 4. The MMSE equalizer was also tested instead of the MMSE ZF, and gave worse performance.

Simulations are presented in figure 5, where the channel is real with coefficients chosen randomly ($m = 2$, $N = 3$), and the SNR is 5 dB, 10 dB and 20 dB; 3 symbols are initially known. The normalized MSE (NMSE) of the channel estimate is averaged over 100 Monte-Carlo runs of the noise for a given input burst. Three iterations of PQDML and GML are done (though usually 1 suffices), only the result of the final one is shown. The last step, soft GML, is compared to semi-blind GML based on the same number of known symbols (*i.e.* the initially known symbols augmented with the hard decisions) randomly dispersed over the burst in order to see the impact of the correlation changes explained in the previous section.

PQDML improves channel estimation w.r.t. SRM and GML w.r.t. PQDML: their performance is close to the (deterministic or Gaussian) CRBs at 10 dB and 20 dB. The soft decision step improves again the results, but not as much as if the hard decisions would have been located at random positions in the burst. In this example, the advantage of the soft decisions may not be obvious: indeed after semi-blind GML at step 2, the estimation of the channel is already very good and the improvement brought by the soft decision step can only be marginal.

To demonstrate, in another example, the good performance of GML w.r.t. DML, we show in figure 6, the case of subchannels with a nearly common zero $[\boldsymbol{h}(0)\ \boldsymbol{h}(1)] = \begin{bmatrix} 1 & 1 \\ 1 & 0.98 \end{bmatrix}$ at 10 dB and with 3 known symbols; the initialization is done by training sequence estimation. For that ill-conditioned channel, PQDML fails whereas GML performs remarkably well.

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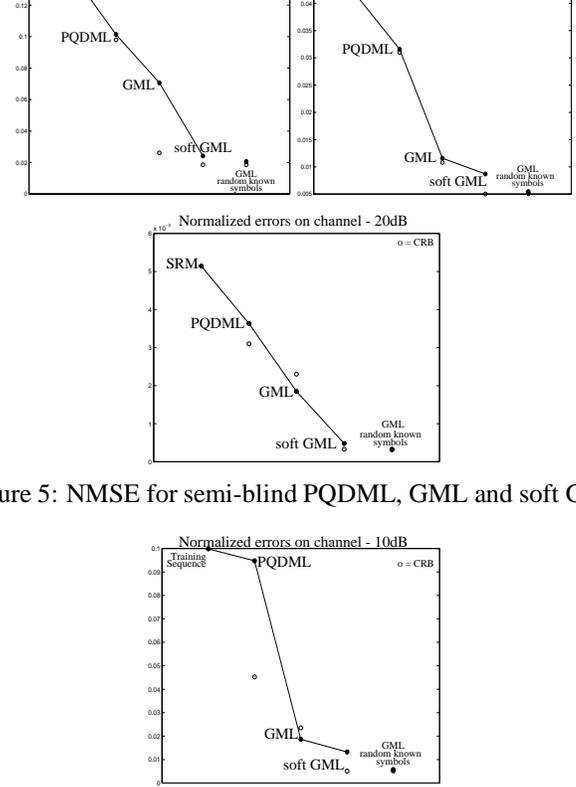
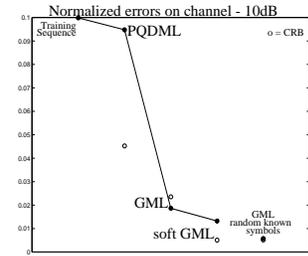Figure 5: NMSE for semi-blind PQDML, GML and soft GML

Figure 6: NMSE for semi-blind PQDML, GML and soft GML

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